THE ASTROPHYSICAL JOURNAL

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS

Agents
THE BAKER & TAYLOR COMPANY
NEW YORK

THE CAMBRIDGE UNIVERSITY PRESS
LONDON AND EDINBURGH

THE ASTROPHYSICAL JOURNAL

An International Review of Spectroscopy and Astronomical Physics

EDITORS

GEORGE E. HALE
Mount Wilson Solar Observatory of the Carnegie Institution of Washington

EDWIN B. FROST
Yerkes Observatory of the University of Chicago

HENRY G. GALE
Ryerson Physical Laboratory of the University of Chicago

VOLUME XXXVII

JANUARY–JUNE, 1913

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS

Published
January, March, April, May, June, 1913

Composed and Printed By
The University of Chicago Press
Chicago, Illinois, U.S.A.

CONTENTS

NUMBER I

NUMBER II

NUMBER III

NUMBER IV

NUMBER V

THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XXXVII JANUARY 1913 NUMBER 1

THE SPECTROSCOPIC SYSTEM 9 *CAMELOPARDALIS*

BY OLIVER JUSTIN LEE

INTRODUCTION

The unique behavior of the sharp calcium line K as shown in radial velocity investigations of a certain class of stars was first noted by Hartmann in a discussion of the spectrum and the orbit of δ *Orionis*.[1] The large range, something over 200 km, indicated by the broad lines of this star made the comparatively constant velocity of the K line of calcium stand out prominently. Hartmann's conclusion that his range of values for K, 21 km, was due to general errors of observation has been questioned by Frost and Adams.[2] His hypothesis that the calcium line K as photographed in the spectrum of δ *Orionis* is due to a calcium cloud lying in the line of sight at some point between the solar system and this star has not been disproved and constitutes a part of the history of stellar spectroscopy.

Since that time considerable attention has been given to stars of the early spectral types which have sharp H and K lines, and a list of objects known to belong to this class would at the present time include a hundred or more stars. From statistical evidence Frost has proposed the presence of sharp H and K lines with the

[1] *Astrophysical Journal*, **19**, 268, 1904.

[2] *Ibid.*, **19**, 351, 1904.

other lines in the spectrum broad and diffuse as criterion that the star is probably binary.[1]

A number of stars of this class have been shown to give anomalous results for the calcium lines when these are compared with results from the other lines. Three detailed investigations only besides that by Hartmann have been published on this type of star. Jordan[2] has derived the orbits of both components of *o Persei*. He measured the K line on 60, and the H line on 22 of the 70 plates discussed in his paper. Considerations on the motion of the center of gravity of the system and the mean velocity shown by the calcium lines "seem to indicate that this line probably has its origin in the system." Daniel and Schlesinger[3] in discussing the calcium line problem in connection with an orbit determination of *β Scorpii* admit the possibility of a small oscillation with the same period as that of the orbital revolution. Duncan[4] in discussing the orbit of *β Scorpii* from plates obtained by Slipher finds that the constant calcium velocity differs by 10 km from that of the center of gravity of the system. He concludes that this evidence supports the hypothesis of a detached calcium cloud in the line of sight.

Shortly after the appearance of Hartmann's paper Frost and Adams announced that measures on the broad lines of the star *9 Camelopardalis* showed it to have variable radial velocity.[5] Assured by the character of the two calcium lines H and K and also by the good agreement of their measurements upon them, these observers asserted that the two lines actually vary, the variation differing in phase and magnitude from that shown by the broad lines. Their original values as published are given for convenience in Table I. The calcium values are attributed, probably as a convenience, to the second component of the system. Frost and Adams interpreted the variation of these lines as real, as distinct from the variation in the broad lines, and placed its source in the star system itself. Since this announcement was made a small

[1] *Astrophysical Journal*, **29**, 234, 1909.

[2] *Publications of the Allegheny Observatory*, **2**, 63, 1910.

[3] *Ibid.*, **2**, 127, 1912.

[4] *Publications of the Astronomical and Astrophysical Society of America*, **2**, 82, 1912.

[5] *Astrophysical Journal*, **19**, 350, 1904.

number of stars giving similar results has been found, such as *37 ϕ' Orionis*, *ξ Persei*,[1] and *ν Geminorum*.[2] Since this paper was completed Cannon[3] has published an orbit of *ξ Persei* from measures on H and K.

TABLE I

Plate	Date	G.M.T.	Taken by	Velocity		No. of Lines		Velocity Mean
				Frost	Adams	Frost	Adams	
				km	km			km
I B 194....	1903 Nov. 17...	$15^h\ 9^m$	A	+14	+ 6	5	3	+10
229....	Dec. 26...	21 59	F	+ 3	− 2	3	4	+ 1
254....	1904 Jan. 2...			+10	+14	2	3	+12
	Second comp.	21 30	A	− 9	− 6	2	2	− 8
285....	Feb. 26...			+ 9	+13	6	4	+11
	Second comp.	17 10	A	+ 2	+ 3	2	2	+ 3
288....	Mar. 8...			− 6	− 7	3	5	− 7
	Second comp.	17 21	F	+ 8	+ 6	2	2	+ 7
296....	Mar. 19...			− 1	+ 4	3	4	+ 2
	Second comp.	16 2	A	−12	−12	2	2	−12

Several years ago Professor Frost suggested to the writer the desirability of an investigation of the anomalous calcium type of spectroscopic binary and the present paper is the outcome of such an investigation. Evidently where the variation to be inquired into is as small as it is in all stars of this type, it is important to choose one in the spectrum of which both the H and K lines are strong and sharp and therefore measurable with some degree of accuracy. The star *9 Camelopardalis* fulfils this requirement. The $H\epsilon$ line of hydrogen is so weak that it interferes in no way with the H line of calcium. Another consideration which favors the choice of this star is its position in the sky. Its declination is +60°, which enables the observer to follow it comfortably for 12 hours on a single night, if necessary, and its right ascension is 5^h, which brings the star into opposition about December 1, when the nights are at their longest. Unfortunately, the broad lines do not give a large range of velocities. However, since the calcium anomaly led to this investigation it was early determined that the advantages given above outweighed this disadvantage.

[1] *Ibid.*, **30**, 62, 1909.

[2] *Ibid.*, **32**, 300, 1910.

[3] *Journal of the Royal Astronomical Society of Canada*, **6**, 188, 1912.

SPECTRUM OF 9 *Camelopardalis*

($\alpha = 4^h\ 44^m.1$; $\delta = +66°\ 10'$; Mag. = 4.34; H.R. Type B)

Vogel and Wilsing[1] give a list of nineteen lines which they were able to measure and identify in this spectrum. Table II shows what lines are strong enough to be readily seen on our Bruce plates. The lines with asterisks alone were used in obtaining radial velocities, and the lines which have double asterisks were given most weight.

TABLE II

Element	λ		Element	λ	
H+He........	3889.5	H_ζ	*He*...........	4143.92*	
Ca.........	3933.82	K	*H*............	4340.63**	H_γ
Ca.........	3968.62	H	*He*...........	4388.10*	
H.........	3970.21*	H_ϵ	*He*...........	4471.65**	
He........	4026.34*		*C*............	4647.6	Lockyer
Si.........	4089.10**		*C*............	4650.8	Lockyer
N.........	4097.47*	Lockyer	*He*...........	4713.25	
H.........	4101.90	H_γ	*H*............	4861.63*	H_β
Si.........	4116.40**		*He*...........	4922.10	

An idea of the appearance of the spectrum may be obtained from the enlarged spectrogram shown in Plate I. More will be said about the individual lines later in the discussion.

THE OBSERVATIONS

Plates from the first up to and including No. 2671 were made with the Seed "non-halation" double-coat emulsion; all the remaining plates but one were taken with the "Seed 30." For No. 2969 the fine-grained "Seed 23" was used. All the plates used in this series were taken with the usual one-prism arrangement of the Bruce spectrograph. In order to make the calcium lines stand out sharply, and to narrow the broad lines down to the cores, the majority of the plates were strongly overexposed. The average exposure time was 88 minutes. Table III gives all the necessary data for the plates used in this paper. Unless both calcium lines could be well measured the plate has been omitted. Under the column "Taken by" are placed the initials of the observer by whom

[1] *Pub. Astrophys. Observatoriums zu Potsdam*, **12**, 34, 1902.

TABLE III

Plate	Taken by	Date	G.M.T.	Julian Day	Velocity *Ca* Lines	Wt.	Velocity Broad Lines	No. of Lines
I B 194.....	A	1903 Nov. 17	15h 9m	2416436.63	− 2	1	+ 6	5
229.....	F	Dec. 26	21 59	6475.92	− 4	2	0	4
254.....	A	1904 Jan. 2	21 30	6482.90	−10	4	+ 9	3
285.....	A	Feb. 26	17 10	6537.72	− 4	5	+ 8	5
288.....	F	Mar. 8	17 21	6548.72	+ 4	3	+16	3
296.....	A	Mar. 19	16 2	6559.67	−14	4	+ 1	6
654.....	B	1905 Dec. 25	21 49	7205.91	+ 1	2	+ 4	5
663.....	B	1906 Jan. 26	20 10	7237.84	− 2	4	+ 6	6
867.....	B	Sept. 24	21 24	7478.89	−10	1	− 2	5
893.....	F B	Oct. 31	18 46	7515.78	+ 1	5	− 7	5
1159.....	B	1907 Sept. 2	21 52	7821.91	......	..	+ 9	6
1181.....	L	Sept. 23	22 25	7842.93	− 5	4	+13	6
1189.....	L	Sept. 24	22 12	7843.92	......	..	− 1	5
1200.....	F	Oct. 11	20 34	7860.86	− 4	4	+27	5
1210.....	B	Oct. 18	19 56	7867.83	− 4	4	+26	6
1234.....	P	Oct. 22	19 23	7871.81	− 6	3	−11	6
1241.....	B	Nov. 22	17 19	7902.72	−10	4	− 7	5
1245.....	L	Nov. 23	15 42	7903.65	......	..	0	7
1256.....	L	Nov. 27	21 55	7907.91	......	..	+18	6
1261.....	B	Nov. 30	17 39	7910.74	− 3	3	+18	6
1812.....	L	1908 Oct. 30	22 46	8245.88	− 7	2	− 3	6
1821.....	B	Nov. 2	20 27	8248.85	+ 7	1	− 5	6
1850.....	L	Nov. 10	22 58	8256.96	+ 4	3	− 1	6
2113.....	F L	1909 Aug. 23	21 28	8542.89	−11	4	− 2	7
2134.....	L	Sept. 10	22 9	8560.92	......	..	− 9	5
2170.....	L	Oct. 29	22 48	8609.95	− 6	1	−14	8
2203.....	L	Nov. 29	15 28	8640.64	......	..	+ 4	10
2278.....	F	1910 Feb. 7	14 49	8710.62	− 3	1	+ 2	6
2611.....	L	Dec. 29	15 46	9035.66	+ 3	4	− 9	8
2671.....	B Ar	1911 Feb. 10	12 55	9078.54	0	1	+10	9
2808.....	B	Aug. 25	21 46	9274.91	+ 3	2	+44	6
2814.....	Ar	Aug. 28	21 23	9277.89	− 5	1	+14	6
2831.....	L	Sept. 15	21 37	9295.90	+10	3	+12	5
2836.....	B	Sept. 18	22 00	9298.92	− 2	5	+32	6
2839.....	B	Oct. 23	19 1	9333.79	+ 5	5	+ 8	5
2843.....	F	Oct. 31	18 29	9341.77	− 6	5	+37	5
2846.....	F	Nov. 7	15 30	9348.65	+ 6	4	+ 4	5
2850.....	L	Nov. 7	22 18	9348.93	0	3	− 3	7
2857.....	L	Nov. 18	15 9	9359.63	+ 3	4	+ 9	9
2867.....	F	Nov. 25	21 40	9366.90	0	5	+18	8
2871.....	L	Nov. 26	14 33	9367.61	+11	4	+ 7	8
2875.....	B	Nov. 26	21 17	9367.89	− 3	4	+10	9
2880.....	B	Nov. 30	15 29	9371.65	− 1	4	+16	6
2884.....	F L	Dec. 3	14 48	9374.62	− 2	5	+15	8
2896.....	L	Dec. 4	21 46	9375.91	+ 4	5	+11	7
2902.....	F B	Dec. 12	16 16	9383.68	+ 3	4	+10	6
2905.....	L	Dec. 12	21 9	9383.88	0	4	+ 9	9
2906.....	L	Dec. 12	22 46	9383.95	+ 7	4	+ 6	7
2918.....	L	Dec. 24	12 1	9395.50	+ 2	1	+ 8	7
2919.....	L	Dec. 24	14 7	9395.59	− 2	2	+ 6	7
2929.....	B L	1912 Jan. 2	12 20	9404.51	+ 7	2	+12	5
2930.....	L B	Jan. 2	14 5	9404.59	+ 5	4	+15	4
2932.....	B	Jan. 2	16 48	9404.75	− 1	1	+ 4	3

TABLE III—*Continued*

Plate	Taken by	Date	G.M.T.	Julian Day	Velocity *Ca* Lines	Wt.	Velocity Broad Lines	No. of Lines
I B 2933.....	B L	1912 Jan. 2	$18^h 34^m$	2419404.78	+ 7	2	+ 3	3
2934.....	L	Jan. 2	20 40	9404.86	+ 6	5	+14	4
2938.....	L B	Jan. 6	12 48	9408.53	− 3	3	+ 7	7
2939.....	L	Jan. 6	14 42	9408.61	− 4	5	+11	5
2940.....	L	Jan. 6	16 29	9408.69	− 4	3	+ 6	6
2941.....	L B	Jan. 6	18 10	9408.76	− 7	2	+ 2	7
2942.....	B	Jan. 6	20 06	9408.84	− 4	2	− 7	4
2946.....	B F	Jan. 9	12 12	9411.51	− 3	3	+10	8
2947.....	F	Jan. 9	13 58	9411.58	− 7	4	+ 5	5
2949.....	F L	Jan. 9	17 39	9411.74	− 2	4	+11	5
2953.....	L B	Jan. 11	18 29	9413.77	− 6	5	+ 1	5
2954.....	B	Jan. 11	20 30	9413.86	− 6	4	− 6	8
2957.....	B L	Jan. 15	12 08	9417.50	− 3	5	+18	4
2958.....	B L	Jan. 15	14 00	9417.58	+ 4	2	−26	6
2959.....	B	Jan. 15	15 50	9417.66	− 4	4	− 4	5
2960.....	B L	Jan. 15	17 26	9417.73	− 5	4	− 9	4
2961.....	L	Jan. 15	18 56	9417.79	− 2	5	− 3	6
2962.....	L	Jan. 15	20 40	9417.86	− 3	3	− 2	7
2965.....	B	Jan. 23	12 18	9425.51	+ 4	4	+14	6
2966.....	B	Jan. 23	14 3	9425.58	+ 2	5	+19	5
2967.....	B	Jan. 23	16 5	9425.67	+ 3	4	+17	5
2969.....	B L	Jan. 23	19 41	9425.82	− 3	5	− 1	5
2981.....	B	Feb. 8	12 48	9441.53	+ 1	2	− 3	4
2983.....	B	Feb. 8	16 48	9441.70	− 3	3	0	5

the plate was secured: A=Adams; Ar=Arbogast; B=Barrett; F=Frost; L=Lee; P=Parkhurst. The efficient and friendly assistance of Mr. F. R. Sullivan, engineer in charge of the telescope, is gratefully acknowledged.

The calcium lines on the plates have been measured twice and in some cases three times. These measurements were made so far apart that no prejudice of memory would prevent them from being independent. The weights given were derived in the following manner: (*a*) All the plates were examined consecutively without referring to their measures and the weight 0 to 5 assigned to them based upon the completeness of exposure, spark and other instrumental adjustments, freedom of the calcium lines from grain distortion, etc. (*b*) The measures were then examined and from them each plate was again weighted as follows:

For H and K within 2 km and means of double measures within 1 km, weight = 5

For H and K within 3 km and means of double measures within 1.5 km, weight = 4

For H and K within 4 km and means of double measures within 2 km, weight = 3

For H and K within 6 km and means of double measures within 2 km, weight = 2

For H and K within 7 km and means of double measures within 2.5 km, weight = 1

For H and K more than 10 km apart, regardless of agreement of means, weight = 0

Now the weight given for each observation is the result of combining its (*a*) and (*b*) in the ratio of $\frac{2}{5}$ and $\frac{3}{5}$ respectively. For an illustration, consider the measures of Plate No. 663, Table IV. The first measure of H and K yielded -1.4 and -3.0 km respectively for the two lines; the second measure gave in the same way -2.7 and -0.9 km. In each pair the agreement is within 2 km, and the means -2.2 and -1.8 agree far within 1 km. Hence the (*b*) weight of this plate is 5. The appearance of the plate and of the lines was, however, not so reassuring, so that the (*a*) weight 3 was given to it. The final weight of the plate was rounded off to 4. Since the two lines are in general of the same character in this star, this procedure is obviously justifiable. This mode of weighting the observation from each plate was used in preference to that of taking the sum of the line weights for the weight of the observation. Line weights will relate properly the different values from one plate, but unless the observer completes a series of plates of one star without measuring other types of spectra in between, he will hardly be able to keep his standards of weighting constant. Obviously this must be done to obtain a fair comparison of plate values from line weights. It is needless to say that weights are arbitrary and in observations of this kind do nothing more than indicate a probability that one observation is better than another.

Table IV contains the individual measures of the H and K lines. For the sake of keeping the series as uniform as possible only the writer's measures of the Yerkes Observatory plates have been used. All the measures with a few exceptions were made and the weights assigned before any of the results hereafter described were derived. Plates of this star with measures were kindly sent from Allegheny and Ottawa upon request. I remeasured these,

TABLE IV

Plate	First Measure			Second Measure			Mean Velocity
	K	H	Velocity$_1$	K	H	Velocity$_2$	
I B 194	0.0	+0.1	0.0	−5.9	−1.1	−3.5	−1.8
229	−0.7	−0.8	−5.2	−4.5	−2.0	−3.2	−4.2
254	−11.2	−10.7	−10.9	−11.6	−6.4	−8.5	−9.7
285	−4.7	−3.4	−4.0	−2.6	−5.9	−3.2	−3.6
288	+4.7	+2.1	+3.4	+6.1	+1.8	+4.2	+3.8
296	−16.7	−12.0	−14.4	−14.3	−11.5	−12.9	−13.6
654	+0.9	−0.8	0.0	+1.4	+3.0	+2.2	+1.1
663	−1.4	−3.0	−2.2	−2.7	−0.9	−1.8	−2.0
867	−6.9	−12.5	−8.8	−3.6	−18.3	−10.9	−9.8
893	−1.7	+1.5	−0.1	+2.4	+2.3	+2.3	+1.1
1181	−4.7	−4.4	−4.6	−7.2	−5.0	−6.1	−5.4
1200	−1.0	−6.8	−3.9	−2.0	−6.5	−4.2	−4.0
1210	−2.4	−3.0	−2.7	−6.3	−3.9	−5.1	−3.9
1234	−1.4	−9.4	−5.4	−8.9	−5.7	−7.3	−6.4
1241	−8.5	−9.0	−8.8	−11.6	−10.1	−10.9	−9.8
1261	0.0	−6.5	−2.6	−2.9	−4.8	−4.2	−3.4
1812	−8.3	−1.7	−5.7	−7.9	−8.2	−8.1	−6.9
1821	+14.3	+5.9	+8.0	+10.0	+3.3	+6.7	+7.4
1850	+4.0	+5.6	+4.8	+3.1	+5.2	+4.1	+4.4
2113	−10.6	−11.6	−11.1	−9.8	−12.0	−10.9	−11.0
2170	−10.3	−1.3	−6.7	−10.5	+1.3	−4.6	−5.6
2278	−6.3	+1.1	−3.3	−8.5	+1.1	−3.7	−3.5
2611	+1.5	+6.3	+3.9	+1.2	+4.6	+2.9	+3.4
2671	−5.5	+3.7	−0.9	−0.3	+3.3	+1.5	+0.3
2808	+0.5	+4.2	+2.1	+0.1	+5.6	+3.8	+3.0
2814	−12.3	−3.3	−6.9	−5.4	−1.6	−3.5	−5.2
2831	+11.2	+11.0	+11.1	+8.1	+9.4	+9.2	+10.2
2836	−2.5	−0.6	−1.5	−2.5	−0.7	−1.9	−1.7
2839	+4.9	+5.4	+5.1	+4.7	+4.7	+4.7	+4.9
2843	−6.6	−3.8	−5.2	−7.1	−4.6	−5.8	−5.5
2846	+3.2	+9.0	+5.4	+4.8	+9.1	+6.2	+5.8
2850	+0.2	−1.3	−0.8	+0.8	−0.1	+0.4	−0.2
2857	+1.9	+4.9	+3.4	+0.9	+5.0	+3.0	+3.2
2867	−0.4	−1.4	−0.8	−0.4	0.0	−0.2	−0.5
2871	+9.3	+15.6	+10.9	+10.3	+10.6	+10.4	+10.6
2875	−3.2	−3.5	−3.3	−4.2	−1.3	−3.4	−3.4
2880	−3.8	+2.3	−1.2	−4.3	+1.8	−1.7	−1.4
2884	−4.0	+1.7	−2.6	−3.2	−0.7	−2.4	−2.5
2896	+3.6	+5.1	+4.2	+3.9	+4.4	+4.2	+4.2
2902	+1.2	+3.8	+2.5	+1.2	+6.2	+3.7	+3.1
2905	−2.4	+2.9	+0.2	−2.4	+3.2	+0.4	+0.3
2906	+6.7	+6.0	+6.3	+8.3	+6.8	+7.4	+6.8
2918	−1.1	+9.0	+2.9	−3.1	+6.1	+0.6	+1.8
2919	−3.4	+3.9	−1.0	−4.2	+1.3	−2.0	−1.5
2929	+7.7	+1.3	+5.1	+9.5	+8.8	+9.2	+7.2
2930	+5.1	+7.2	+6.0	+3.6	+6.3	+4.8	+5.4
2932	−7.4	+3.2	−0.3	−8.5	+3.1	−0.8	−0.6
2933	+3.1	+8.6	+7.2	+2.6	+8.7	+6.7	+7.0
2934	+4.3	+6.9	+5.6	+4.9	+7.3	+6.2	+5.9
2938	−5.1	+0.6	−3.0	−6.3	+0.7	−3.3	−3.2
2939	−3.7	−5.1	−4.4	−5.2	−4.4	−3.7	−4.0
2940	−7.6	−0.3	−3.9	−7.1	−2.0	−5.1	−4.5

TABLE IV—*Continued*

Plate	First Measure			Second Measure			Mean Velocity
	K	H	Velocity₁	K	H	Velocity₂	
I B 2941	− 9.9	− 5.0	− 8.0	− 9.0	− 0.2	− 6.2	− 7.1
2942	− 7.8	− 1.6	− 4.3	− 9.7	− 0.6	− 4.5	− 4.4
2946	− 4.4	+ 0.8	− 1.8	− 7.4	+ 0.8	− 3.3	− 2.6
2947	− 6.2	− 8.0	− 7.3	− 6.5	− 9.2	− 7.6	− 7.4
2949	− 4.4	+ 1.4	− 2.5	− 4.6	+ 1.5	− 2.3	− 2.4
2953	− 5.1	− 6.8	− 6.0	− 5.1	− 7.2	− 6.2	− 6.1
2954	− 8.3	− 4.8	− 7.0	− 8.3	− 1.5	− 6.0	− 6.5
2957	− 3.7	− 2.1	− 2.9	− 1.5	− 4.1	− 2.7	− 2.8
2958	− 0.9	+ 8.0	+ 2.7	+ 1.4	+ 8.6	+ 4.3	+ 3.5
2959	− 5.6	− 2.2	− 4.1	− 5.2	− 1.1	− 3.4	− 3.8
2960	− 5.9	− 6.6	− 6.2	− 5.6	− 2.5	− 4.4	− 5.3
2961	− 0.4	− 2.9	− 1.7	− 0.4	− 4.0	− 1.9	− 1.8
2962	− 5.9	+ 0.2	− 3.9	− 2.6	+ 1.1	− 1.4	− 2.6
2965	+ 4.4	+ 4.8	+ 4.6	+ 6.0	+ 3.0	+ 4.3	+ 4.4
2966	− 0.8	+ 2.9	+ 1.3	+ 0.5	+ 5.0	+ 3.1	+ 2.2
2967	+ 3.8	+ 0.4	+ 1.8	+ 3.8	+ 4.3	+ 4.1	+ 3.0
2969	− 4.0	− 2.1	− 3.2	− 1.3	− 2.4	− 1.9	− 2.6
2981	− 2.2	+ 4.7	+ 1.9	− 2.6	+ 3.7	+ 1.0	+ 1.4
2983	− 5.7	− 2.0	− 3.8	− 5.7	+ 0.5	− 2.6	− 3.2

but the necessity of having uniform material soon became apparent so that they were not utilized. All the reductions have been made with the aid of tables constructed by the writer in the manner proposed by Schlesinger.[1]

To see if the relatively very sharp H and K lines might nevertheless show double lines, a careful examination of them was made on the plates. The conclusions were: (*a*) The calcium lines are simple in this star; (*b*) a considerable difference in the width of the lines can be accounted for as a function of fulness of exposure, development, and accidental grain structure of the plate.

The labor involved in deriving the period of variation of the calcium lines has been considerable. At first, simple variations from the sine curve were tried, and then various curves derivable from elliptic motion. The period was not found until 45 or more probable periods, ranging from 0.4 to 20 days in length, had been tested. Where the range of oscillation is so small, single observations are often misleading, and therefore it was necessary to ascertain that the greater part of the observations were not satisfied before rejecting the period. The adopted value of the period of

[1] *Publications of the Allegheny Observatory*, 1, 9, 1910.

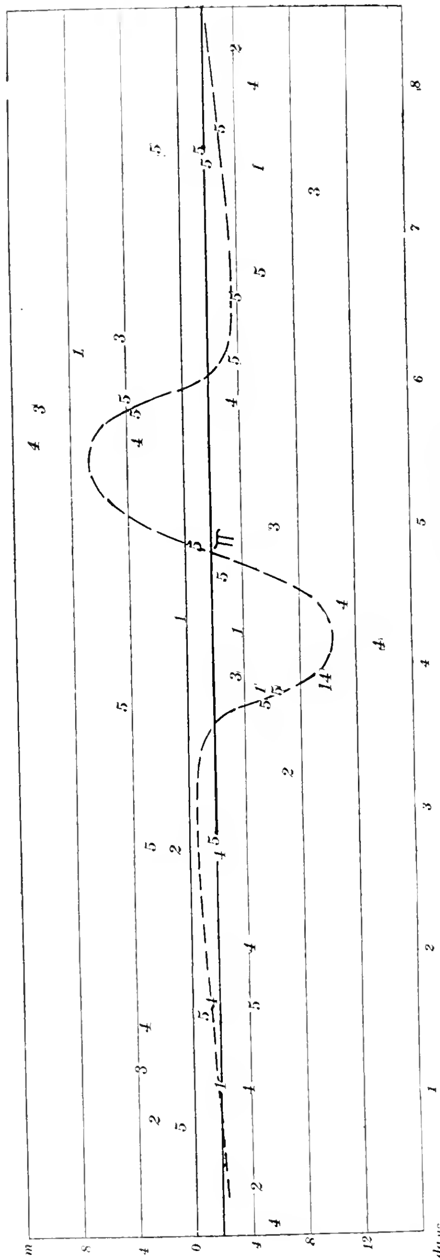


FIG. 1.—Velocity-curve of *9 Camelopardalis*

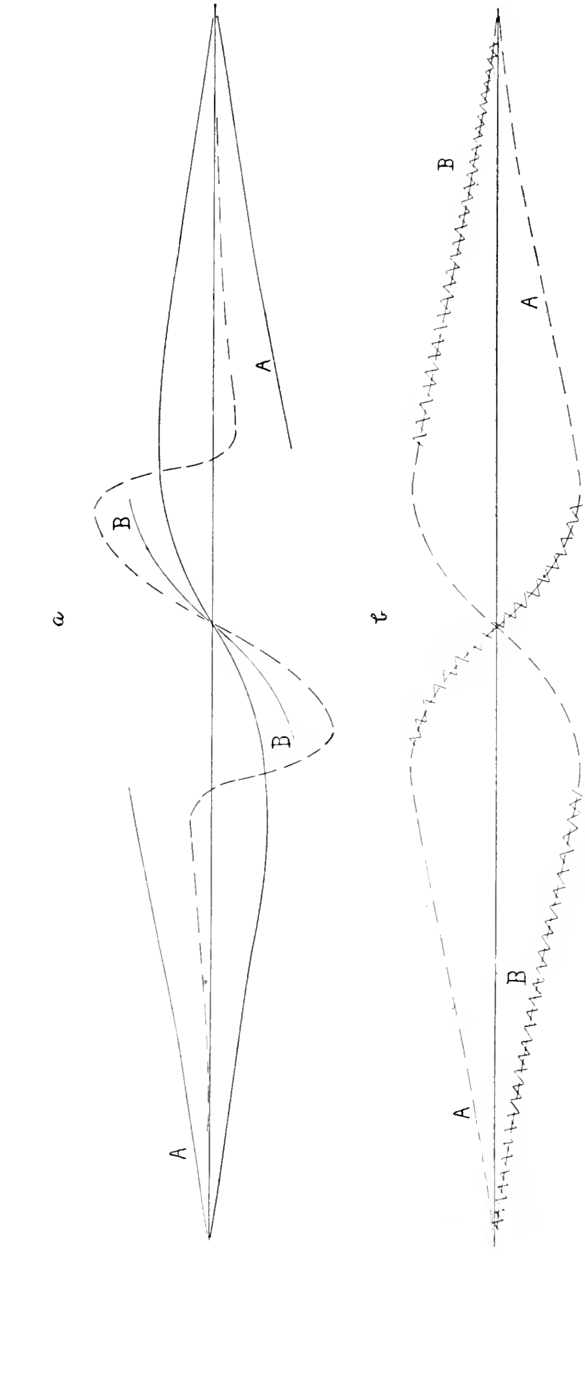


Fig. 2.—*a*: Smooth curve is velocity-curve of primary
AA and BB are uneclipsed portions of velocity-curve of the calcium masses
Dash-curve is algebraic sum of the other curves
b: Velocity-curves of calcium masses, crossed curve representing eclipsed portions

variation is 7.9957 days. Since observations extend over an interval of about 3000 days, or 375 complete periods, the length of the period is defined rather precisely. A change of one unit in the last place is sufficient to distort the curve appreciably. In the velocity-curve (Fig. 1) it has seemed desirable to give each observation used rather than the normal point with respect to phase. Also, instead of giving a dot or circle for each observation, a number has been used which indicates its weight. When several plates were taken in one night the mean of all or of a group is given as an observation.

Let the γ axis be drawn parallel to the time axis, and so adjusted that the areas above and below it shall be equal. For all practical purposes the curve is now symmetrical about the point π. Obviously the velocity-curve is not even approximately that of one component of a binary. It should be borne in mind that it would be as difficult to explain this velocity-curve by assuming a two-body system merely, were the parts of the curve before the low minimum and after the high maximum to approach closely to the γ axis, as it is when they actually cross it. Hence a small error in tracing the curve leaves the problem essentially as it is. While the assumption that the system is triple or quadruple might explain the form of the velocity-curve, it would not account for the sharp calcium-line anomaly without identifying the source of these lines with one of the bodies of the system. Thus it will be necessary to assume a physical situation which not only produces the observed velocity-curve, but which also takes account of the difference in the appearance of the H and K lines and the other lines in the spectrum.

In the development which follows, the reader will bear in mind that the indicative mood is used merely as a convenience.

Fig. 3 gives an outline of the system which is assumed. It is a two-body system with certain added elements. The symmetry of the chief minimum with the chief maximum portion of the curve suggests that the axis of the elliptical orbit lies approximately in the line of sight. The periastron point in the orbit of the primary lies nearest to the observer, and the inclination is taken to be nearly 90°. The secondary body is small as compared with the primary,

and its orbit has not been drawn. The center of gravity of the system lies close to the surface of the primary. The primary is a star of an early spectral type, the atmosphere of which contains large areas that have rapidly changing density-gradients. The calcium vapor here, as in the sun, lies at a greater distance from the generating body than the vapors of the other elements.[1] While this outer envelope is continuous about the two components of the binary, the spectrographically effective part of it are the appendages shown in Fig. 3.

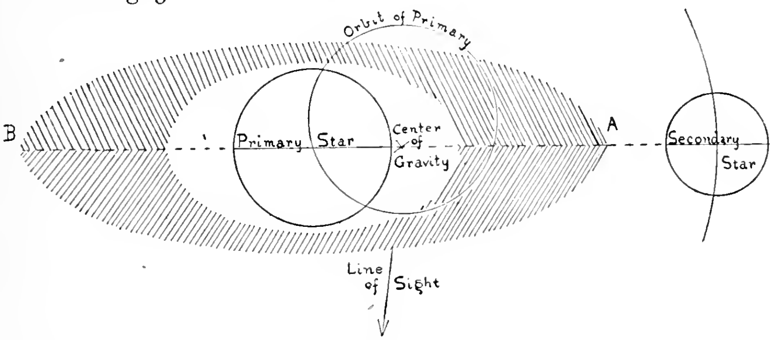


FIG. 3.—Equatorial section of the system assumed

On account of the greater distance of these vapor masses from the primary, the material in them is in a relatively quiescent state, and since it is not subject to the radical changes in vapor-pressure and temperature which agitate the lower-lying sources of the other lines in the spectrum, it gives the narrow H and K lines observed.

[1] Charles E. St. John (*Astrophysical Journal*, **32**, 80, 1910) speaking of chromospheric calcium says: "The high velocity of the calcium vapor producing the K_3 line points to a higher elevation of this layer of calcium vapor than of the hydrogen effective in the production of the $H\alpha$ line."

Sir Norman Lockyer, in reporting observations of the eclipse of 1898, gives these chromospheric heights of the various chemical elements (quoted in Abbot, *The Sun*, p. 171): *Ca* (K), 9700 km; Hydrogen, 7200 km; Helium, 5400 km; *Ca* (4227), 3200 km; *Fe*, 2300 to 100 km for iron lines of different classes.

See Abbot, *The Sun*, pp. 155–172, for a suggestive description of cloud formations in the solar atmosphere.

Hale and Ellerman, using the K line of calcium, photographed a prominence to a height of 452,000 km (*Astrophysical Journal*, **1**, 434, 1895). Fox, using the H line, obtained a photograph of an eruptive prominence which reached 316,900 km (*ibid.*, **26**, 155, 1907). Slocum photographed another which reached 319,500 km, before running off the edge of the plate (*ibid.*, **32**, 128, 1910).

The secondary star tends to draw this high-level calcium into two regions, one on each side of the primary. Possibly the centers of these regions coincide with two of the three points of zero relative velocity which lie in the line joining the two stars. Hence the secondary star controls the positions of these masses as the orbital velocities of the two stars vary. That is to say, the varying angular velocity of revolution of the calcium clouds is not proportional to the angular velocity of rotation of the primary on its axis, which probably takes place once for every revolution in its orbit, but it is proportional to the velocity of the primary in its orbit. The motion of the calcium vapor is not of the nature of a tidal wave which would in general give no radial velocity effect, but it is a motion of translation of the cloud of vapor. The calcium masses exist as bodies practically separate from the star, and the friction between them and the lower strata of the stellar atmosphere is insignificant. Therefore motion of translation with respect to the surface of the primary is possible.

Fig. 4 shows the primary and the spectrographically effective centers of the calcium masses in critical positions, but all reduced in size relative to the orbit so as to avoid making the figure confusing to the eye. Obviously the secondary star is found at all times at the end of a line drawn from the center of the primary through the center of gravity, and terminating at the outer orbit. From the motion of the primary in its elliptical orbit, the revolution of the masses about the primary, and their alternate eclipse by it, it is proposed to duplicate the observed velocity-curve. In order to show this in a fairly accurate way, use has been made of the principle of the hodograph which may be stated as follows: the path being an ellipse described about the focus, S, under the law of the inverse square, the hodograph is the auxiliary circle, the other focus, H, being the origin, and HQ drawn perpendicular to the tangent at P, where P is any point on the ellipse, and Q the corresponding point on the hodograph. Fig. 5 is double: the part below the line π 10 is half of an ellipse of eccentricity 0.30, in which adjacent focal radii are drawn so as to include equal areas. A few of these near the periastron point have been divided into halves. The part of the figure above the π 10 line is half of the auxiliary

circle and the lengths of its radii drawn as stated above are proportional to the velocities in the ellipse. The data derived from the hodograph and their transformations for use in the radial velocity-curves are given in Table V.

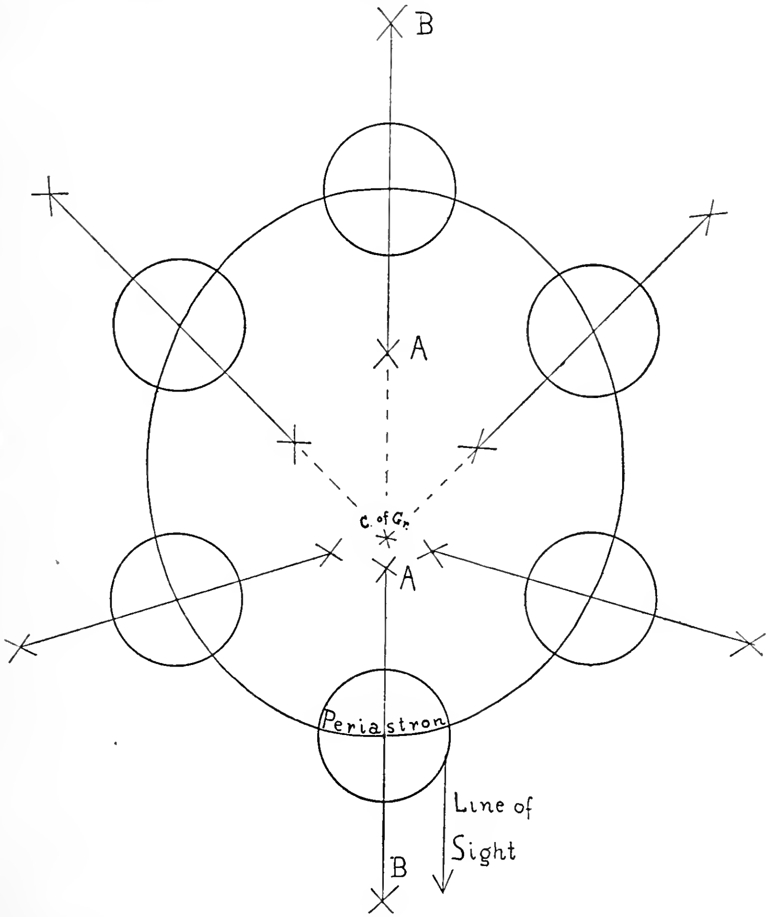


FIG. 4.—Primary star and calcium masses in critical positions

The orbital motion of the primary is always tangential to the ellipse and the projections upon the line of sight are given in Table V under (4). The velocities of the calcium masses are derived regarding the center of the primary as a stationary point. By our assumption on p. 14 their angular velocities also are

proportional to the numbers in (3), Table V. This element of motion is directed, not tangentially to the ellipse, but perpendicularly to the radius vector, and the projections upon the line of sight given in (6), Table V, are taken accordingly. The proportionality factors by which the quantities in (4) and (6) are converted to those in (5) and (7), respectively, not only fix the scale

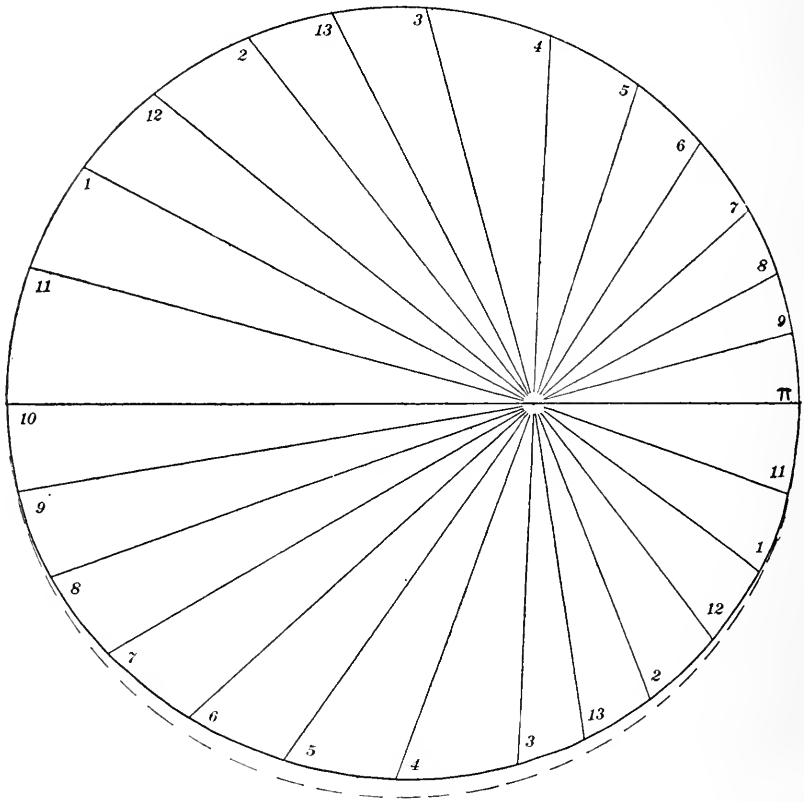


FIG. 5.—One-half of orbital ellipse and hodographic circle

of the curves in Fig. 2, but also determine to a degree the relative scale in the system which has been assumed. Obviously in the case under consideration, these factors are arbitrary within certain limits. In Fig. 2*a*, plotted from (5), Table V, is shown as a smooth line the radial velocity-curve of the orbital motion of the primary alone. In Fig. 2*b*, drawn from data in (7), Table V, are shown the

radial components of the velocities of the calcium masses relative to the center of the primary star regarded as a stationary point. The crossed portions of these curves represent the parts of the orbits of the calcium masses described about the primary during eclipse by the primary. The uncrossed parts in *b*, drawn as *AA* and *BB* in *a*, are corrections to be applied to the smooth curve in *a*. The algebraic sum of these is the dash-curve in *a*, which is, as will readily be seen, a close duplicate of the observed

TABLE V

Orbital Motion of Primary					Motion of Calcium Masses	
No.	Time from Periastron	Nos. Proportional to Velocity in Ellipse	Projection upon Line of Sight	$(4)\times\frac{1}{14}$	Velocity about Center of Primary	$(6)\times\frac{1}{10}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Days					
π	0.0	131	0	0.0	0	0.0
11	0.2	130	34	2.4	43	4.3
1	0.4	127	60	4.3	76	7.6
12	0.6	122	78	5.6	97	9.7
2	0.8	116	92	6.6	108	10.8
13	1.0	110	97	6.9	109	10.9
3	1.2	102	99	7.1	100	10.0
4	1.6	92	92	6.6	85	8.5
5	2.0	84	79	5.6	69	6.9
6	2.4	78	65	4.6	52	5.2
7	2.8	72	48	3.4	37	3.7
8	3.2	69	32	2.3	22	2.2
9	3.6	67	17	1.2	10	1.0
10	4.0	66	0	0.0	0	0.0

velocity-curve given in Fig. 1. The parts of the composed curve which represent the shifting of the spectroscopically predominant factor from *A* to *B* and from *B* to *A*, Fig. 2*b*, by the eclipse operation cannot be definitely drawn. At these points, corresponding to the steep drops in the curve *to* the primary minimum and *from* the primary maximum, the peculiarities of the physical situation show to greatest effect, and the correction to the velocity-curve of the star is for a time indeterminate. This uncertainty is also shown in the observed velocity-curve by the

observational values at these points. On considering the complexity of the physical situation, the velocities at these times could hardly be expected to follow a curve very closely.

Four of the orbital elements, P, K, μ, and γ, come directly from the observations and from the velocity-curve; the other four elements arise from the assumed orbit. The orbital elements are

$$\begin{aligned} P &= 7.9957 \text{ days} \\ e &= 0.30 \\ \mu &= 45^\circ\!.024 \\ K &= 9.0 \text{ km} \\ \omega &= 90^\circ \text{ or } 270^\circ \text{ closely} \\ T &= \text{J.D. } 2416480.35 \\ \gamma &= -2.25 \text{ km} \\ i &= 90^\circ \text{ closely} \end{aligned}$$

Reasonable physical and orbital differences in stars of this class easily give rise to a number of anomalies of calcium such as have been observed. Consider a binary system in which the two bodies are comparable in light, and hence probably of approximately the same mass. When the time of rotation and the period of revolution are equal, the mutually facing sides of the two bodies are much hotter than the sides facing away, and the incandescent calcium collected from both stars into some point between them is spectrographically predominant over the outside calcium masses. In the case of stars of equal mass, the inside zero-velocity point coincides with their center of gravity, and if the inner calcium mass follows this zero-velocity point, it will have the same constant velocity as the center of gravity of the system. When the two stars are unequal in mass, as assumed in *9 Camelopardalis*, the difference in conditions of vaporization on the sides of the primary toward and away from the secondary is not so great and the development of an outside as well as of an inside calcium mass, both of which are in evidence spectrographically, is possible. Also, with increasing disparity of mass of the two stars, the inside zero-velocity point recedes toward the star of less mass. If the calcium cloud follows this point, it acquires a variable velocity, which agrees in phase with that of the secondary star.

DETERMINATION OF THE MASSES

Assuming that the spectroscopically effective calcium clouds lie about the zero-velocity points on the common radius vector of the two stars, the masses of the two stars may be determined. The mean distance of the center of the primary from the center of gravity of the system is 398,000 km. From Fig. *2a*, it is seen that the ratio of the velocities of the primary and the inside calcium body around the center of gravity of the system is $\frac{40}{53}$. Hence this is the ratio of their distances from this point. Let $1-\mu$ and μ be

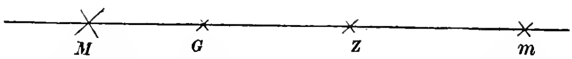


FIG. 6.—Relative positions of stars and inner calcium mass

the relative masses of the two stars M and m. In Fig. 6, let M, G, Z, and m represent respectively the primary, center of gravity of the system, inside zero-velocity point, and the secondary star. Also let $\overline{MZ}=r_1$, and $\overline{mZ}=r_2$. Since moments about G are equal,

$$M(\overline{MG})=m(\overline{mG})$$

or

$$M\tfrac{40}{53}r_1=m(1-\tfrac{40}{53}r_1) \tag{1}$$

where $r_1=1-r_2$. Since $\mu=\dfrac{m}{M+m}$, (1) becomes after solving

$$r_1=1-\tfrac{93}{40}\mu\,. \tag{2}$$

With the same units and notation, the following equation has been derived:[1]

$$r_2^5-(3-\mu)r_2^4+(3-2\mu)r_2^3-\mu r_2^2+2\mu r_2-\mu=0\,. \tag{3}$$

In order to obtain an approximate value for r_2 expand (3) as a power series in $\mu^{\frac{1}{3}}$ to three terms, and equate to (2) and the following equation results:

$$0.6934\mu^{\frac{1}{3}}-0.1603\mu^{\frac{2}{3}}+2.2950\mu-1=0\,. \tag{4}$$

Let $z=\mu^{\frac{1}{3}}$ and substitute $\left(z-\dfrac{q}{3p}\right)$ for z to eliminate the second term (p and q being the coefficients in (4) of the first and second terms). This gives, after dividing through by p,

$$z^3+0.3005z-0.4200=0. \tag{5}$$

[1] Moulton, *Celestial Mechanics*, p. 198, *b*.

Solving (5) by Cardan's method, z is found to equal 0.6164, and μ to equal 0.2342. The corresponding value for r_2 obtained from (2) is 0.4537. A brief calculation shows that the rigorous equation (3) is closely satisfied with $\mu=0.2596$, and $r_2=0.3936$. Hence

$$\frac{M}{m}=2.85\,.$$

The ratio of the masses and the focal distance of the primary being known, the distance between the centers of the two stars is found to be 1,538,000 km.

Making use of Schlesinger's convenient formula[1]

$$(M+m)\sin^3 i=\frac{4}{10^{20}}\frac{(a_M \sin i+a_m \sin i)^3}{P^2}$$

where a_M and a_m are expressed in kilometers and P in days, and remembering that in the present case sin i is nearly equal to unity, it follows that

$$M+m=0.00230\odot\ .$$

Since the chief source of the incandescent calcium is the primary star, it is more probable that the effective calcium mass lies in a position which is a compromise between the place where it is generated and the zero-velocity point. This would increase the distance between the centers of the two stars and hence increase the mass of the system.

The parallax of this star has been derived by Slocum and Mitchell[2] and is $+0''.026\pm0''.005$.

THE BROAD LINES

The results so far have been derived without reference to the evidence given by the broad lines in the spectrum. Measures of these depend upon the lines denoted by asterisks in Table I, and the results of the double measures of them are given in Table VI. All the settings have been made upon the middle of the lines.

[1] *Publications of the Allegheny Observatory*, 1, 145, 1910.

[2] Given at the August 1912 meeting of the Astronomical and Astrophysical Society of America.

TABLE VI

Plate	Velocity₁	No. of Lines	Velocity₂	No. of Lines	Velocity
I B 194	+ 7.6	2	+ 3.9	5	+ 6
229	− 2.5	3	+ 3.3	4	0
254	+ 2.9	3	+15.3	3	+ 9
285	+ 4.2	4	+11.3	5	+ 8
288	+16.5	2	+14.8	3	+16
296	+ 0.1	6	+ 1.2	5	+ 1
654	+ 7.1	5	+ 0.4	3	+ 4
663	+ 5.7	6	+ 6.7	3	+ 6
867	− 4.0	3	− 1.0	5	− 2
893	−10.1	4	− 3.9	5	− 7
1159	+ 5.7	6	+15.6	6	+ 9
1181	+13.7	4	+12.2	6	+13
1189	− 8.0	5	+ 5.5	5	− 1
1200	+28.3	5	+26.4	5	+27
1210	+24.6	6	+26.6	6	+26
1234	−10.4	6	−11.5	6	−11
1241	−10.5	7	− 4.0	5	− 7
1245	+ 5.4	6	− 5.6	7	0
1256	+20.2	6	+16.2	6	+18
1261	+15.2	5	+20.3	6	+18
1812	− 3.0	6	− 2.7	5	− 3
1821	− 6.0	5	− 3.8	6	− 5
1850	+ 2.3	6	− 3.8	3	− 1
2113	+ 0.4	7	− 4.9	2	− 2
2134	− 8.7	5	− 8.5	5	− 9
2170	− 7.4	8	−20.7	3	−14
2203	− 0.2	10	+ 7.4	10	+ 4
2278	+ 4.7	6	0.0	4	+ 2
2511	+ 6.5	7	+ 9.4	3	+ 8
2596	+ 9.9	8	+ 6.2	4	+ 8
2611	+ 2.9	8	−20.7	4	− 9
2671	+10.4	9	+ 9.3	5	+10
2808	+39.8	5	+47.9	6	+44
2814	+11.1	5	+16.7	6	+14
2831	+13.5	5	+ 9.8	3	+12
2836	+29.5	6	+34.6	3	+32
2839	+ 8.8	5	+ 6.6	4	+ 8
2843	+35.9	5	+38.2	4	+37
2846	+ 6.8	5	+ 1.8	2	+ 4
2850	− 2.3	7	− 3.4	2	− 3
2857	+11.9	9	+ 5.7	6	+ 9
2867	+19.8	8	+16.8	7	+18
2871	+ 7.5	7	+ 6.5	8	+ 7
2875	+ 9.7	7	+10.3	9	+10
2880	+16.4	5	+16.2	6	+16
2884	+17.2	8	+13.1	8	+15
2896	+10.6	7	+11.6	7	+11
2902	+ 8.3	6	+10.8	5	+10
2905	+10.2	9	+ 7.0	4	+ 9
2906	+ 8.5	7	+ 2.8	4	+ 6
2918	+ 8.0	7	+ 8.0	4	+ 8
2919	+ 7.0	7	+ 5.7	3	+ 6
2929	+15.0	5	+ 6.7	2	+12
2930	+12.0	4	+17.7	2	+15

TABLE VI—*Continued*

Plate	Velocity₁	No. of Lines	Velocity₂	No. of Lines	Velocity
I B 2932	+ 6.2	3	+ 2.5	2	+ 4
2933	+ 7.8	3	− 1.5	2	+ 3
2934	+15.9	4	+ 8.2	2	+14
2938	+ 7.7	7	+ 7.0	5	+ 7
2939	+13.3	5	+ 8.5	2	+11
2940	+ 8.3	6	+ 3.8	4	+ 6
2941	+ 7.0	7	− 4.0	2	+ 2
2942	− 7.3	4	− 7.7	1	− 7
2946	+12.0	8	+ 9.0	4	+10
2947	+ 5.1	5	+ 4.9	4	+ 5
2949	+13.7	5	+ 8.6	3	+11
2953	+ 0.2	5	+ 1.3	4	+ 1
2954	− 5.5	8	− 6.6	4	− 6
2957	+16.8	4	+20.6	4	+18
2958	−23.9	6	−31.1	2	−26
2959	− 2.2	5	− 5.2	4	− 4
2960	− 8.3	4	− 9.4	4	− 9
2961	− 3.9	6	− 2.7	4	− 3
2962	− 1.7	7	− 2.1	3	− 2
2965	+11.7	5	+16.4	6	+14
2966	+18.9	5	+18.2	5	+19
2967	+13.3	5	+20.0	5	+17
2969	+ 1.0	5	− 2.8	5	− 1
2981	− 5.5	4	0.0	4	− 3
2983	0.0	3	+ 0.1	5	0

Early in the process of measurement, attempts were made to find in the diffuse unsymmetrical lines consistent evidence of the second component. Later, after most of the plates had been obtained, they were examined again. The results were: (*a*) There are no plates showing double lines; (*b*) the lines even of the same element change radically in character from plate to plate, chiefly noticeable as unsymmetrical distortions of the order of 10 to 20 times the size of grain defects on the plate; (*c*) unless measures on the centers of the lines will give their period of oscillation, it cannot be found. Table VII shows the individual line measures and remeasures for six plates.

No period has so far been found from them, either independently of the calcium period or in attempting to find some period which is commensurable with that of the calcium lines. The conclusion of the matter seems to be: the broad lines are due to two unequal components whose velocities are of low order of magnitude. The sources of these lines exist in a disturbed state which produces

TABLE VII

λ	229		296		2203		2808		2919		2938	
	First Measure	Second Measure	First Measure	Second Measure	First Measure	Second Measure	First Measure	Second Measure	First Measure	Second Measure	First Measure	Second Measure
4026	......	......	$+16^{1/2}$	......	$+1^{1}$	$+15^{1}$	......	......	$+23^{1/2}$	......	$+32^{1/2}$	......
4089	$+27^{0}$	$+24^{1/2}$	$+32^{1/2}$	$+32^{1}$	-6^{1}	-9^{1}	$+25^{1}$	$+27^{2}$	$+12^{1}$	$+15^{2}$	$+20^{1}$	$+16^{1}$
4097	$+8^{1}$	0^{1}	$+27^{1/2}$	$+30^{1}$	0^{1}	$-19^{1/4}$	$-7^{1/2}$	-10^{1}	$+53^{0}$	$+44^{0}$	$+4^{1/2}$	$+10^{1}$
4101	......	......	......	......	$-5^{1/2}$	......	......	......	$+30^{1/2}$	......	......	......
4116	......	......	$+37^{1/2}$	$+33^{1}$	-4^{1}	$+7^{1}$	$+18^{1/4}$	......	$+18^{1}$	......	$+24^{1}$	$+24^{1/2}$
4143	......	......	......	......	$-3^{1/2}$	$-5^{1/2}$	......	$+22^{1/2}$	$-2^{1/2}$	......	......	......
4340	$+3^{1}$	$+3^{1}$	$+18^{1}$	$+18^{1}$	-14^{2}	$+10^{1}$	$+36^{1}$	$+18^{1}$	$+10^{1}$	$+14^{1}$	$+10^{2}$	$+25^{1/2}$
4388	......	......	......	......	$-1^{1/2}$	$+5^{1}$	......	......	......	......	......	......
4471	-1^{1}	$+13^{2}$	$+3^{1/2}$	$+12^{2}$	-2^{3}	$+6^{2}$	$+40^{2}$	$+30^{3}$	$+5^{2}$	$+4^{2}$	$+20^{2}$	$+16^{2}$
4481	......	......	......	......	......	$+4^{1}$	$+12^{1}$	......	......	......	......	......
4861	......	......	......	......	-6^{2}	-12^{1}	$+14^{1}$	$+16^{1/2}$	-22^{0}	......	$+17^{1}$	......
Weighted mean	+ 3.3	+ 9.1	+21.7	+22.8	− 4.8	+ 2.8	+27.3	+19.2	+11.7	+10.4	+17.1	+16.4
Reduction to sun	−5.81		−21.6		+4.61		+20.6		−4.7		−9.4	
Velocity	− 2.5	+ 3.3	+ 0.1	+ 1.2	− 0.2	+ 7.4	+47.9	+39.8	+ 7.0	+ 5.7	+ 7.7	+ 7.0

The small figures are line weights.

spectrum distortions that mask the possible double structure of the lines. Table VI shows a preponderance of positive velocities over those carrying the minus sign, which probably means that the broad line curve, whatever it is, lies more above the zero-velocity line than below it. This introduces a difference in the position of the γ axis as derived from the calcium lines and as derived from the broad lines. The very low pressure in the calcium masses and the relatively great pressure in the lower strata of the stellar atmosphere may give a positive and a negative correction respectively, which is sufficiently large to account for the discrepancy.

The results of this investigation may be summed up as follows:

1. A period and a velocity-curve have been derived from the H and K lines in the spectrum of *9 Camelopardalis*.

2. A physical situation has been assumed which explains the observed velocity-curve and which also takes account of the sharpness of the calcium lines as contrasted with the width and diffuseness of the other lines in the spectrum.

3. With one assumption the ratio of the masses and the sum of the masses has been computed to be 2.85 and 0.0023$\odot$ respectively.

4. The proposed system has been shown to be sufficiently elastic to explain in part at least other calcium anomalies that have been observed.

5. No positive results have been obtained from the broad lines.

The general aspects of the scheme proposed in this paper for stars of this class have developed in informal discussions with Professor Frost, to whose encouragement its presentation at this time is largely due. I wish to express my appreciation to Mr. Barrett for assistance in getting the spectrograms, many of them obtained on very cold winter nights.

Yerkes Observatory
July 1912

A HIGH-LEVEL MEASUREMENT OF SOLAR RADIATION

By FRANK W. VERY

In the *Comptes rendus* of the Institute of France for 1897 and 1898, M. Violle described an actinometer for *ballons sondes*, recommended by him to the Congress at Rome in 1879. "It is a ball of [thin] copper, blackened exteriorly and containing interiorly a thermometric apparatus whose indications can be recorded at a distance on a registering cylinder."[1] Tested in the ascension of June 4, 1898, at a height of 13,700 m, with the barometer at 118 mm and the air temperature 65° Centigrade below zero, this instrument recorded in sunshine −12°C., or a temperature-excess $\theta = 53^\circ$ higher than the air temperature. M. Violle has been good enough to give me further details of the apparatus and method, for which I gratefully acknowledge my indebtedness. It is sufficient to say that the sun's rays are alternately excluded and caused to shine upon the instrument by the presence or absence of a double aluminum shield operated by clockwork.

The excess of temperature registered on this occasion is about three times as great as that of the blackened copper shell of a somewhat similar arrangement, the Violle conjugate bulbs, a rather sluggish instrument which, however, agrees very well with a black-bulb sun-thermometer (after applying the proper convection factor for the difference in size) when the air is perfectly calm and the conditions are steady.[2] The Violle blackened copper sphere at sea-level seldom records an excess of more than 17°

Centigrade Temperatures	Noon	12:30 P.M.	2:00 P.M.	1:00 P.M.	2:30 P.M.	3:00 P.M.
Reduced ordinary black-bulb—Air, $\theta_1 =$	16°.9	18°.5	20°.4 (max.)	18°.8	16°.6	18°.3
Violle blackened shell —Air, $\theta_2 =$	15.2	15.2	17.3	18.5 (max.)	17.4	18.3
Violle—black-bulb $\theta_2 - \theta_1 =$	−1.7	−3.3	−3.1	−0.3	+0.8	±0.0

[1] *Comptes rend.*, **126**, 1748, 1898.

[2] See Winslow Upton, *Report of Observations Made on the Expedition to Caroline Island to Observe the Total Solar Eclipse of May 6, 1883*, observation of the afternoon of May 2, 1883, "when the air was almost perfectly still."

or 18° C. above air temperature, representing a solar radiation, equivalent to about 1.2 cal./sq. cm. min. If the excesses were directly proportional to the radiation, the sounding-balloon reading would mean an observed thermal equivalent of 3.6 cal./sq. cm. min., but the convection losses being smaller in the rarefied air, this quantity must be reduced. It was with the design of developing a method which would be applicable to observations of static temperatures, that I undertook a thorough investigation of the theory of the Violle actinometer, presented to the Astronomical and Astrophysical Society of America at its meeting in December 1911. The conclusion then reached was that it is possible to use either the static or the dynamic method with this instrument, and with equal accuracy.

The problem of the reduction of M. Violle's high-level observation, however, is not so simple as that of the actinometer; because, instead of having a water-jacket whose temperature can be measured accurately, we are obliged to be contented with an estimate of the temperature of the environment. A similar difficulty attends the use of the Violle conjugate bulbs, where it is customary to take the difference between temperatures of bulb and air as the temperature of excess. This excess is 4° or 5° larger than would be given by the same thermometer in a water-jacket, exposed to the sun's rays through an aperture not much wider than is sufficient to include the solar angle; and this follows partly because the freely exposed instrument receives solar radiation diffused from the sky and less perfectly reflected from the earth's surface, and also partly because the temperature of the sunlit surface, even if grass-covered, is several degrees above that of the air. The heat received by the thermometric apparatus, directly from the solar radiation and indirectly from the sun's rays reflected by the surroundings, is exactly equal, when static equilibrium is attained, to the heat abstracted by contact with the air plus that removed as equivalent radiation through the air to space. The diffusion of sunlight by the sky is somewhat smaller at the height of 14 km and the radiation to space more free than at sea-level. From either of these causes the registered heat at the upper level is too small when comparison is made with a read-

ing at sea-level. On the other hand, the temperature-excess in sunshine is slightly larger when the basal temperature from which the excess is reckoned is lower.

The variations produced by these conflicting secondary influences may cause an uncertainty of a few degrees in the observed excess, but accepting the measured temperatures as the best that can be obtained under conditions which are not quite so advantageous as we could wish, though better than some to which we must submit at sea-level, we may compute the corresponding radiation by Stefan's law with Kurlbaum's constant. The blackened copper both radiates and absorbs a little less than an ideal black body and these two errors partially compensate. Whatever error remains from this cause affects equally the reading of a sun-thermometer under surface conditions. We have, then, radiation in calories per square centimeter per second at the height of 13,700 m

$$R = 1.267 \times (10)^{-12} \{(261)^4 - (208)^4\}$$
$$= 0.005878 - 0.002372 = 0.003506.$$

It is not necessary in the static method to know the water-equivalent of the radiating mass, because the condition of the radiating surfaces and of the surrounding medium alone affect the comparison; but among the conditions as to surface we do need the dimensions of the apparatus, since the radius of curvature of the sphere (in this case 2.5 cm) enters into the computation of the convection. Inserting this value in a formula which represents the result of comparisons of thermometer readings in inclosures with air at ordinary pressure and exhausted,[1] I obtain the following provisional value of the combined losses per sq. cm. of surface and second of time:

Convection+molecular penetration....	0.00843
Radiation..........................	0.00351
Sum..........................	0.01194

Since the sum of the losses per second represents emission from a spherical surface, this number must be multiplied by four to give

[1] *Publications of the Astronomical and Astrophysical Society of America*, **2**, 90.

the solar radiation in the sectional beam which produces the heat of the steady interchange, and this being further multiplied by 60 to reduce to minutes, gives

2.86 gram calories per sq. cm per minute

for the observed solar radiation-equivalent.

The transference of heat through the air between surfaces separated by a narrow space occurs mainly through penetration of gaseous molecules, the mass movements of convection currents being impeded by viscosity and adhesion of the air to the neighboring solid surfaces. There is greater freedom of motion in a wider inclosure, and more of the motion of individual molecules is transferred to simultaneous group movements in convection currents. The complete formulas take into account the actual pressure and temperature, as well as the dimensions of the apparatus, and give a sum of penetration and convection which is probably not very different from that in the free air,[1] although the relative importance of the two constituents is no doubt changed. The measures on which the computation is founded were necessarily obtained in a narrowly circumscribed inclosure within which a vacuum could be produced. In a larger inclosure the convection would have been greater and the penetration smaller, the radiation remaining unchanged. It is not easy to foresee just how these changes would affect a body in free air, and I can only call the result provisional, but it certainly points to a value of the solar constant at least as great as 3, and probably very nearly 3.5 cal./sq. cm min.

M. Violle's sounding-balloon observation is a notable one, but above its altitude of nearly 14 km are many more kilometers of absorbent atmosphere. Blair and the Mount Weather observers found the temperature still increasing up to nearly 30 km where $-41°$ C. was recorded, or $24°$ higher than the air temperature in M. Violle's measurement. Aqueous vapor in appreciable quantities is found at these great altitudes, showing that the chief atmospheric absorbent is still present, and thus there is no reason

[1] In calm free air, the excess of a black-bulb sun-thermometer above the temperature of surrounding surfaces does not differ much from that of the same thermometer in an actinometer inclosure over the temperature of the water-jacket, after eliminating the effect of sky radiation.

to doubt that aqueous vapor is chiefly responsible for the absorption of solar radiation by which the great temperature-inversion of the upper non-adiabatic layer is produced, just as the presence of an excess of aqueous vapor has been found to be associated with minor temperature-inversions in the lower atmosphere. On September 11, 1910, the Weather Bureau sounding-balloon ascen-

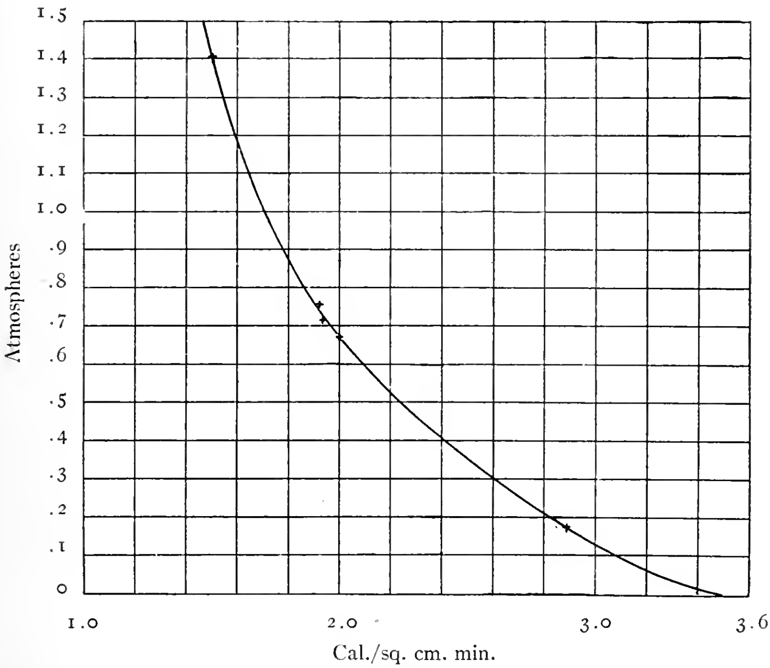


FIG. 1

sion from Huron, S.D., gave a temperature of $-25\overset{\circ}{.}5$ C. and a relative humidity of 56 per cent at an altitude of 24,899 m.

These observations extend the limits of the aqueous atmosphere to much greater altitudes than some meteorologists have thought to be possible. The form of the temperature-curve indicates that the approximately isothermal layer, and therefore the aqueous atmosphere whose absorption of solar radiation is the chief factor in producing it, extends to perhaps something like 80 km. Thus it is highly probable that the increment of solar radiation, observed

up to 14 km, will continue with only a gradually diminishing rate up to the limits of the "isothermal" layer.

In conclusion, an independent estimate of the solar constant is given, taking observations of solar radiation at three very different heights in the atmosphere, namely, a thermal equivalent of 1.5 cal./sq. cm min. at sea-level for transmission under best winter conditions by an air mass of 1.4 atmospheres, 2.00 cal. corresponding to an air mass of 0.674 and an altitude of 4420 m, and 2.86 cal. in the isothermal layer at 13,700 m. The middle value is Keeler's observation on the summit of Mount Whitney, and there are also plotted for comparison Nanry's reading at Mountain Camp, Mt. Whitney, from Langley's *Solar Researches* with trifling corrections by myself, together with an entirely new reduction by me of Violle's measurement at the summit of Mont Blanc. All of these measurements were made with the Violle actinometer, an instrument of unsurpassed precision.

The sounding-balloon observation is less precise but furnishes a precious indication of the power of the solar rays at an altitude never before reached in such measures. The high-level observations were made in summer, but under conditions of local temperature and humidity which resemble those of the winter observations chosen to represent sea-level. Thus we have a fairly homogeneous series. A smooth curve passed through these three points (Fig. 1) meets the axis of X at 3.5 cal./sq. cm min., which is the solar constant of radiation.

Westwood Astrophysical Observatory
Westwood, Mass.
July 31, 1912

A CRITERION OF ACCURACY IN MEASUREMENTS OF ATMOSPHERIC TRANSMISSION OF SOLAR RADIATION

By FRANK W. VERY

It is to James Forbes that we owe the first recognition that the absorption of solar radiation by the earth's atmosphere not only varies with the changing composition of the atmosphere from summer to winter, but is inconstant even during a single day of cloudless sky, and that the coefficient of absorption of the transmitted rays, from which the more absorbable rays have been sifted out, goes on diminishing more and more as the atmospheric mass already traversed becomes greater. At least this is what ordinarily takes place. We shall see that there are exceedingly rare exceptions to this law which are of the utmost importance.

By a further extension of his principle, Forbes arrived at the conclusion that the intensity of the transmitted rays tends to reach a definite value after passing through an infinite mass of atmospheric air. This variability of the coefficient of absorption by a unit layer of air was attributed to the non-homogeneity of the solar radiations which are unequally absorbed, so that the sifted sunbeam becomes progressively richer in more transmissible radiations the greater the atmospheric mass which they have traversed. The importance of the principle in question was recognized by Radau in his admirable little treatise, *Actinométrie*, and has been discussed at length in the writings of Crova and Langley.

In addition to this variation of radiant absorption by a unit layer with changing air mass, attributable to the non-homogeneity of the solar radiation, there is a further variation due to the systematic change in the absorbent quality of the air by which a unit layer, receiving identical radiation, transmits this radiation less freely in the middle of the day. The cause of this variation of absorbent quality has been discussed in my paper "On the Need of Adjustment of the Data of Terrestrial Meteorology and of

Solar Radiation, and on the Best Value of the Solar Constant."[1] In the present communication I propose to consider the means by which we may recognize this variation of atmospheric quality and in some cases apply corrections for the same.

If it were possible to measure strictly homogeneous radiations, and if occasions could be found when the atmospheric quality remains unchanged between high-sun and low-sun observations, we could apply Bouguer's equation

$$R=Ap^{\epsilon},$$

in which R = solar radiation received normally on the unit of area at the surface of the earth, A = the solar constant, p = the coefficient of transmission, assumed to remain unchanged during the day, and ϵ = the air mass taken as unity for sea-level and normal pressure, so that, if B is the observed barometer reading at the place of observation and ζ the sun's zenith distance, $\epsilon=(B/760)$ sec ζ, until zenith distances greater than 56° are reached, beyond which a small correction is required.

In the *Annales de chimie et de physique* for 1888 (Ser. 6, t. 14, p. 541–574), M. Crova described the results obtained with his actinograph thus:

The constant character of the numerous curves which we have studied consists in a permanent fluctuation of the intensity of solar radiation [as received at the surface of the earth]. All bear witness to rapid oscillations of intensity, and it can be affirmed without exaggeration that the needle of the actinometer is continually in motion. These oscillations are translated on the curves into sinuosities from which none are exempt, but which in calm, warm, and moist weather are so close together that they become confounded, giving a smooched appearance to certain parts of the curve through a considerable breadth.[2]

In summer, at Montpellier, the part of the curve which corresponds to the rising of the sun is, in general, clear and regular; but soon the oscillations begin and attain, toward 9^{h} in the forenoon, such an amplitude that the curve becomes blurred in consequence of the rapidity of the oscillations, then it is strongly depressed, and in the afternoon the radiation, for an equal altitude of the sun, is smaller than in the morning.[3]

In winter, above all after severe frosts, the curve is relatively regular and approaches the theoretic symmetry. It is especially in great cold and with

[1] *Astrophysical Journal*, **34**, 371, 1911.

[2] *Op. cit.*, p. 543.

[3] *Ibid.*, p. 545.

high winds that the curves present the greatest regularity and the feeblest oscillations. [Thus the fluctuations must be attributed to the interception of solar radiation by ascending columns of moist air which are produced by convection and evaporation from a moist and heated soil. Strong wind mingles the contents of the convective striae indiscriminately, and cold diminishes their vapor-content.] Most of the curves show a depression which is frequently quite marked toward noon [or at the time of day when convection is most powerful]. The radiation then increases again to diminish afterward until sunset.[1]

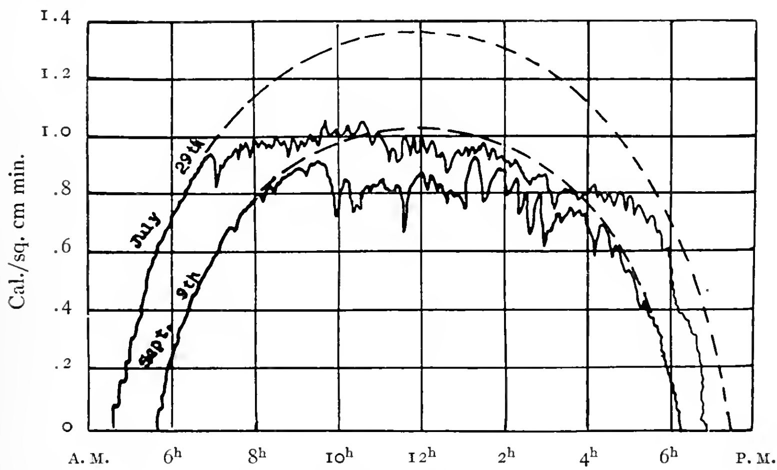


FIG. 1.—Actionograph curves (Crova)

On only 1.9 per cent of the days in 1886 were approximately symmetrical diurnal curves of insolation obtained, and still fewer in 1885 and 1887. Thus convection increases the depth through which an especially absorbent ingredient of the atmosphere is dispersed, and to such an extent that the effective absorbent mass is notably greater than the theoretical ϵ in the middle of the day, at the same time that the penetration of the rays suffers an exceptional diminution; that is to say, p is exceptionally small and ϵ larger than is assumed, or, since p is a fraction and ϵ greater than unity, the quantity p^ϵ is diminished to a still greater extent.

Fig. 1 shows two curves which were obtained by Crova with his registering actinometer on July 29 and September 9, 1886. The July curve gives an indication of the large midday loss of

[1] *Op. cit.*, p. 545.

insolation due to the temporary increase of convection of moisture-bearing air-columns. This loss is to be distinguished from the equably distributed loss which still affects the restored curve of insolation, which is represented in the figure by the dotted line and which corresponds to a radiation equivalent of 1.4 calories at the maximum. The distortion of this curve begins as early as 7^h A.M. The curve of September 9 is not affected until after 9^h A.M., and its average departure from the restored curve is much less, although the average transmissibility of the air was not as large as it frequently is.

In addition to the variations of atmospheric transmission through changes in the distribution of its moisture by convection, there is a further change in the molecular constitution of some of the atmospheric ingredients by which especial obstruction to the passage of solar radiation is induced in the middle of the day when the sun is highest. The effect is very much greater in summer than in winter, and there is reason to believe that it is caused by an ionization of the upper air by the sun's ultra-violet rays by which chemical changes ensue, particularly the production of hydrols; but a portion of the diurnal change comes simply from the more extensive dispersion of aqueous vapor and dust into the upper air during the middle of the day. The great fact of this midday variation of atmospheric quality is so obtrusive that it cannot be ignored, and the Bouguer formula must be modified in order to cover the actual facts. This M. Crova has already done in a series of admirable analyses which have not received the recognition they deserve.

M. Crova's method of reduction is not a rational one, but results from the application of a mathematician's devices to the study of the form of the curve of observation. He says:

> I can then consider the curve of calories, given in function of the thicknesses, as composed of an infinity of logarithmic arcs in which A is constant, while p varies in a continuous manner from one to the other, increasing with ϵ and serving as the envelope of the arcs. Under this hypothesis, it will be easy to measure the coefficient of transmission corresponding to a determined value of ϵ, by constructing a tangent at the point whose abscissa is ϵ and measuring the value of the subtangent.[1]

[1] *Ann. de chim. et de phys.*, Ser. 5, **11**, 496, 1877.

The coefficient of transmission thus calculated represents the ratio of intensity of the radiation transmitted through a thickness equal to unity to the intensity of the radiation before this transmission, admitting that this absorption changes only the quantity and not the composition of the incident beam.[1]

This is of course not true except for absolutely homogeneous rays which are never measured even by the spectrobolometer, since the structure of a Fraunhofer line is itself most complex; but with certain restrictions Crova's assumption may be approximately true even for the still more complex total radiation, provided we consider an absorbent layer of limited thickness within which the quality of the air is sufficiently constant, and from which p may be derived for a unit layer of like composition. The permissible depth of this assumed layer is itself variable, if a reasonable approach to accuracy is to be maintained, and further corrections are necessary before the process can be applied with any certainty to the circumstances of a complex atmosphere.

If p were really constant throughout the radiation-curve, we should have transmission $=R/A=p^\epsilon$, or the transmission by unit layer anywhere in the atmosphere should be $T=p$, instead of which we observe variable values of T according to the previous history of the radiation, or the level which it has reached in the atmosphere, such as $T=p_1$, $T=p_2$, or in general $T=p_x$.

With both p and ϵ variable, the variation of the solar radiation in a thin layer of air ($\Delta\epsilon$) is

$$R_1-R_2=Ap^\epsilon \log p\Delta\epsilon+p^{\epsilon-1}\epsilon\Delta p \; .$$

This is not a practicable formula and Crova has therefore sought some simple approximation which will be applicable. The original parabolic or hyperbolic function is thrown into a form

$$R=A/(1+\frac{m}{c}\epsilon)^{1/m} \, ,$$

where m and c are numbers to be found by observation. Bouguer's equation may be derived from this as a special case.

Since m and c are very nearly identical, the ratio m/c being sometimes greater and sometimes less than unity, and since these

[1] *Op. cit.*, p. 497.

numbers have no physical significance but are merely of use in determining the subtangents of the radiation-curve, the last equation may be written

$$R=A/(1+\epsilon)^{1/m}.$$

Differentiating, we have

$$\frac{dR}{R}=-\frac{1}{m}\cdot\frac{d\epsilon}{1+\epsilon},$$

and

$$\left(\frac{dR}{d\epsilon}\right)/R=-\frac{1}{m(1+\epsilon)};$$

but the first member is the reciprocal of the subtangent of the radiation-curve, whose sign must here be taken negative because R decreases as ϵ increases; and the subtangent is also the reciprocal of the transmission T corresponding to air mass ϵ. Hence the transmissions of the sifted rays at the stage represented by ϵ is given by the equation

$$\log_{(10)} T=-\frac{M}{m(1+\epsilon)}$$

(true only for rays of a special quality which occur at this point in the path), or

$$T=e^{-\frac{1}{m(1+\epsilon)}}.$$

Hence T is a variable which can be studied from the subtangents of the radiation-curve, which can be read graphically from the $R\epsilon$ curve, giving a series of straight lines whose equations are

$$S=(Rd\epsilon)/dR=c+m\epsilon,$$

which expresses the fact that, through a considerable range of the curve, the subtangents increase by a nearly constant quantity (c) for successive increments of ϵ by unity.

Values of m may be obtained by the equation

$$m=\log\left(\frac{1+\epsilon_1}{1+\epsilon_2}\right)\div\log\left(\frac{R_2}{R_1}\right),$$

which commonly gives a series of varying numbers, so that we are no nearer arriving at a single fixed value of the solar constant

than before, unless some way of harmonizing these conflicting data can be found.

Two methods are possible: (1) If on any one day the atmospheric quality is sufficiently constant to give equable values of m throughout the day, we may at once derive the solar constant by the equation

$$A=R(1+\epsilon)^{1/m}\,.$$

This corresponds to an ideal equation

$$A_1=R^m(c+m\epsilon)\,,$$

in which the differences of the subtangents for successive differences of one unit in ϵ are all equal to the constant c. In this equation, A_1 is not the solar constant, but the latter can be obtained from it by the equation

$$A=\left(\frac{A_1}{c}\right)^{1/m}.$$

(2) Failing to obtain an ideal series, we must look for some criterion for estimating the value of the observations from which m is derived, and either reject all values which do not conform to the criterion, or apply suitable corrections, if such can be found.

The variable transmission (T) is represented by Crova by a logarithmic curve

> which has for its asymptote a straight line parallel to the axis of X at a distance equal to unity, and whose ordinate at the origin is $e^{\frac{1}{c}}$. This coefficient varies therefore from a minimum value $e^{\frac{1}{c}}$, which represents the transmission of the rays which arrive at the limits of our atmosphere, and a maximum equal to unity, which corresponds to the case where, the rays having traversed an infinitely great thickness of air, their intensity approaches zero, and the transmission is made without loss.[1]

This does not exactly express the case. The transmission must begin with unity, fall rapidly to a minimum, and then increase slowly, eventually approaching unity once more. The transmission computed by Crova's method is still only an apparent one, because it rests on the assumption that "this absorption changes only the quantity and not the composition of the incident beam."

[1] *Op. cit.*, p. 498.

Thus the value deduced for the radiation at the limit is not the solar constant, but the quantity $A-B$, where B is *lost* radiation. There is, of course, an attempt to make allowance for changing quality, but the variation of the resulting constant from 1.865 to 2.703 (see Crova's paper in the *Annales*, Ser. 6, t. 14) shows as wide a range as is given by a simple graphical estimate on rational principles. There is little advantage in the rigorous mathematical treatment of such data, unless the operations can clarify our reasoning.

Using the more exact equation for the transmission of the sifted rays which have already passed through a thickness of ϵ atmospheres at the stage of absorption corresponding to ϵ, we have

$$T=e^{-\frac{1}{c+m\epsilon}}$$

which varies from $e^{-\frac{1}{c}}$ for $\epsilon=0$, to unity for $\epsilon=\infty$. We wish to follow the variations of the quantities $\frac{1}{m}$[1] and T.

An example of an observation on a single day with nearly constant values of $1/m$ has been given us by M. Savélief, who secured the measures on December 28, 1890, at Kief in Russia. The meteorological conditions are thus described:

> According to the *Bulletin de l'observatoire physique central de Saint-Pétersbourg*, we have had before the 26th of December, in European Russia, several days of severe cold with the wind from the east, and at Kief a north-east wind under the influence of a barometric maximum over central Russia. The ground was covered with a thick layer of snow, and the atmosphere at Kief must have contained a very small quantity of water and of dust.[2]

The temperature varied between $-21°.9$ C. at 7^h A.M., $-18°.6$ at 1^h P.M., and $-19°.9$ at 9^h P.M. Barometer $=766$ mm. Pressure of aqueous vapor $=0.8$ mm. Wind from 2 m./sec. to calm. Sky "of a pure blue without any visible cloud from morning until evening."

From the mean value of $1/m=0.647$, m is 1.55 which multiplied

[1] M. Crova sometimes uses this symbol and sometimes places $p=1/m$, thus using p in two different senses. To avoid confusion, I have kept to a single significance for each symbol.

[2] *Ann. de chim. et de phys.*, Ser. 6, **25**, 570, 1892.

into ϵ, and with the observed subtangents (S), gives the remaining figures in the following table. Column (4) contains the values of $c=S-m\epsilon$; (5) the transmission of the sifted rays, $T=e^{-\frac{1}{c+m\epsilon}}$; (6) the penetration of the total solar radiation, or p, per unit mass, computed from the equation $p=\sqrt[\epsilon]{R/A}$, taking $A=3.5$; (7) the remainders $=p-T$; (8) the final penetration of the rays, p^{ϵ}.

(1) ϵ	(2) $m\epsilon$	(3) S (obs.)	(4) c	(5) T	(6) p	(7) $p-T$	(8) p^{ϵ}
0	0	(2.29)	2.29	0.647	1.000	+0.353	1.000
0.5	0.775	2.87	2.10	.706	0.575	− .131	0.759
1	1.55	3.48	1.93	.750	.637	− .113	.637
2	3.10	4.73	1.63	.810	.699	− .111	.489
3	4.65	6.08	1.43	.848	.740	− .108	.406
4	6.20	7.54	1.34	.876	.770	− .106	.351
5	7.75	9.13	1.38	.896	.792	− .104	.311

The curve from which the subtangents of the above table have been read is obtained by Crova's method, that is to say, the extension to the axis of Y must conform to the requirement that the equation of T is to be logarithmic. The agreement with theory within the range of the observations, which cover six units of air mass, is very nearly perfect. T increases with ϵ continuously, but p diminishes at first and then increases. The apparent transmission of the sifted rays obtained by the theory can only agree with p at a single air mass between $\epsilon=0$ and $\epsilon=0.5$. It is of course impossible that the transmission should be $T=0.647$ for $\epsilon=0$, as the equation for T literally requires, and the only rational interpretation of the result is that the value of T suddenly drops from unity at $\epsilon=0$, to a minimum which is equal to $1/m$ at some very small air mass, beyond which the curve of T is in substantial agreement with the adopted logarithmic equation. This interpretation is represented in the curve of T which I have drawn in Fig. 2. This mode of viewing the matter could also be represented by changing the form of the equation, but it would no longer be suitable for computation.

The curve of R for Savélief's observations gives $R=2.23$ cal./sq. cm min. for $\epsilon=1$. This large value need not surprise us, for if it were possible to have the moisture removed from the atmos-

phere above a sea-level tropical station to the extent represented by a temperature of 20° Centigrade below zero, and with the concomitant absence of moisture implied by a pure cloudless sky with this surface temperature to have also an air free from dust, such as existed over the Russian snow fields, the vertical midday sun would shine with tremendous power.

Under less favorable conditions, Crova's method usually gives

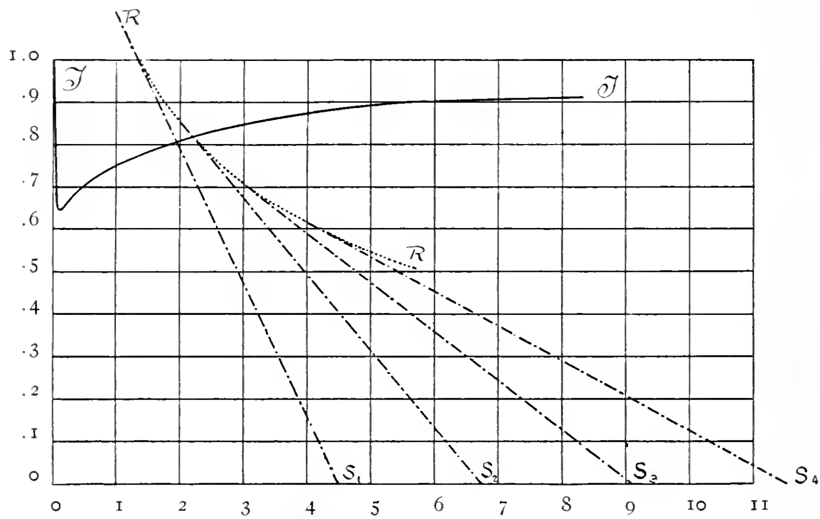


FIG. 2

larger values of $1/m$ than 0.647 and smaller values of A than 3.5, but as M. Savélief says: "It may be admitted that in this case a portion of those radiations most absorbable by the atmosphere has been entirely arrested, and consequently those which remain correspond to a higher coefficient of transmission."[1] This explanation of the small apparent values of the solar constant (A) which are frequently obtained with unsuitable atmospheric conditions, but which really relate to the quantity $(A-B)$,[2] also accords with the wide variation in the results found by different observers, using reliable instruments, but hampered by divergent atmospheric states.

[1] *Op. cit.*, p. 574.

[2] See my paper "The Solar Constant" (*U. S. Weather Bureau Publication No. 254*, pp. 26–28, 1901), where this subject is illustrated.

M. Savélief's final conclusion, just quoted, appears to me to be correct. The radiation which has vanished may belong to very different regions of the spectrum. Thus a large amount of aqueous vapor with little dust may cut off infra-red rays to an exceptional extent, while a dry and dusty air may unduly deplete the radiation of short wave-length, and the distribution of the original radiation in the different regions being quite unlike, the resultant effects will be proportionally different. Variations of other atmospheric ingredients will affect their own special regions of the spectrum. Hence p may vary widely with little corresponding change in the total intensity of the transmitted radiation, or an apparent p, deduced by incorrect methods, may remain steady, while the computed A, which is really $A-B$, varies considerably.

When the cause of error due to reflection from bright clouds has been eliminated, it is correct to select, as M. Savélief does, the highest records of the actinograph, rather than mean values, for computation. The figures which he gives for the month of July, 1.26 and 1.46 cal. at 6^h and 7^h A.M., are extraordinarily large for the season and hour, and they appear to have been the only summer readings which could match his remarkable winter one, while no other instrument than an instantaneous one could catch these exceptional moments of maximum atmospheric transmission. We are, indeed, left a little in doubt whether such casual momentaneous results are genuine, or whether they may possibly be instrumental errors; but this doubt does not affect the consistent measures of an entire perfect day.

The opening words of this author: "It is principally during the winter that the recording actinometer gives the most interesting results in the way of a determination of the solar constant and of atmospheric transmission," are fully justified. The square-shouldered form of the diurnal curve of solar radiation, which is observed on high mountains, requires small values of $1/m$. The usual form of the curve at sea-level agrees better with values of $1/m$ greater than 0.6, if the missing midday portion of the curve is supplied as shown in the broken lines of Fig. 1. Such restorations are assuredly necessary, but to attempt them as a preliminary to reducing the observations for the solar constant would be a very

problematical procedure. I have therefore sought another mode and have compared the values of $1/m$ in Savélief's series of December 28, 1890, with the mean of two unusually extensive series by Mr. H. H. Kimball on November 29 and December 1, 1909, which were continued until the air mass was over 26 atmospheres.[1] The values of $1/m$, computed from these measures, are given in the next table, after which follow the values of $1/m$ from Kimball's mean results for five years, including measurements made in every month. These are larger than the selected results.

VALUES OF $1/m$

	Atmospheres									
	6–7	7–8	8–9	9–10	10–11	11–12	12–13	13–14	14–15	15–16
November 29........	0.828	0.830	0.930	1.14	0.899	0.980	0.988	1.03	1.15	1.17
December 1.........	.678	.779	.863	0.971	.989	1.23	1.25	1.32	1.46	1.56
Mean............	0.753	0.804	0.897	1.056	0.944	1.105	1.119	1.175	1.305	1.365
	16–17	17–18	18–19	19–20	20–21	21–22	22–23	23–24	24–25	25–26
November 29........	1.45	1.28	1.39	1.41	1.58	1.68	1.78	1.75	1.81	1.87
December 1.........	1.57	1.69	1.80	1.89	1.97	2.02	2.03	2.16	2.27	2.39
Mean............	1.510	1.485	1.595	1.650	1.775	1.850	1.905	1.955	2.040	2.130

1905–1910

Atmospheres	1–2	2–3	3–4	4–5	5–6	6–7	7–8	8–9	9–10	10–11
Mean $1/m$..........	0.515	0.574	0.633	0.692	0.751	0.810	0.869	0.928	0.987	1.046

The value of $1/m$ for 9–10 atmospheres in the first part of this table is excessively large, and I have substituted for it the mean of the three pairs on either side, or $1/3\ (0.921+0.955+0.936)=0.937$.

Comparing results from the measures of Savélief and Kimball between 4 and 10 atmospheres for a mean of 7 atmospheres, I have smoothed the Kimball values a little at 4–5 and 5–6 atmospheres, taking the mean difference of $1/m$ for one atmosphere

$$(6\text{–}16 \text{ atm.}),\ (1.365-0.753)/10=0.061,$$

$$(16\text{–}26 \text{ atm.}),\ (2.130-1.510)/10=0.062.$$

[1] *Bulletin of the Mount Weather Observatory*, **3**, Part 2, pp. 90–91, 1910.

This gives a smoother curve with practically identical mean values.

Atmospheres	Savélief	Kimball
9–10	0.669	0.937
8– 9	.693	.897
7– 8	.657	.804
6– 7	.612	.753
5– 6	.605	.692
4– 5	.646	.631
Mean for 7 atmospheres	$1/m=0.647$	$1/m=0.786$

Applying the mean values of $1/m$ to the radiations for the middle of the series ($\epsilon=7$) in these limited sections of the entire curve, I obtain by the equation.

$$A=R(1+\epsilon)^{1/m};$$

from observations by Savélief,

$$A=(8)^{0.647}\times 0.94=3.606;$$

from observations by Kimball,

$$A=(8)^{0.786}\times 0.72=3.698$$

in calories per sq. cm per minute.

The form of the equation for A requires that the exponent of $1+\epsilon$ shall be a fraction, and the ideal day is one in which the value of $1/m$ remains constant as in the measurement by Savélief; but the above comparison shows that Crova's method is also capable of giving consistent values for the solar constant from the results of much less auspicious days, provided *the criterion that $1/m$ must in no case exceed unity* is applied, and with the further proviso that *midday observations which are affected by the usual midday depression must be omitted.*

Kimball's own reduction of his observation of December 1, 1909, by the erroneous method which takes no account of the diurnal change of atmospheric quality, gave $A=2.017$. The observation by Savélief is the most perfect on record and bears internal evidence of the purity of the atmosphere through which

he obtained his readings. I have therefore chosen this measurement as a standard by which less favorable ones may be judged. The application of this test shows that at about 5 atmospheres Crova's exponent ($1/m$) usually becomes larger than the value (0.647) which Savélief found for it, and that under ordinary conditions $1/m$ is apt to exceed unity at about 10 atmospheres, and to approach or exceed 2 near sunset, which would give impossible values of A. If, however, we assume $A=3.5$ and calculate $1/m$,

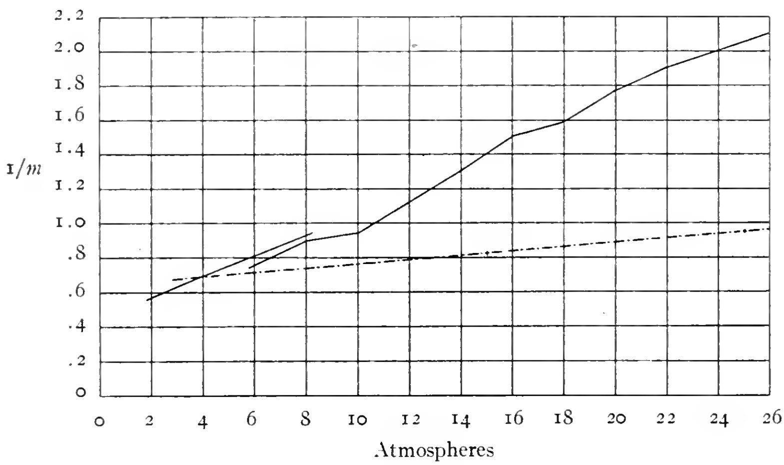


FIG. 3

for example, for $\epsilon=15$, $R=0.355$, and $\epsilon=25$, $R=0.158$, we get $1/m=0.825$ and 0.951, which are permissible values. Fig. 3 shows the relation between the apparent and the real values of this exponent.

Observations by photometry in the green of the solar spectrum, carried to a very low altitude of the sun, show that visible rays in this part of the spectrum continue to diminish indefinitely. These observations by Kimball, and some which I have made in the infrared spectrum indicate, however, that there are remnants of infrared radiation for which the air is almost perfectly transparent. When the transmitted beam approaches this condition of a readily transmissible remnant, our formulae all fail, except in the rare

condition of a very pure and dry atmosphere such as Savélief had. In this case $1/m$ probably increases very slowly; but a constant $1/m$ would give progressively diminishing values of A for increasing air mass. The logarithmic law gives a constant value of A, but an erroneous one, and no single, simple treatment will cover the whole range of varying conditions without some progressive modification on rational principles. It seems to me that the above proves conclusively that empirical mathematical formulae do not apply unless the conditions are exceptionally simple.

I might, of course, take the agreement of Kimball's December observation with that of Savélief (both being computed for the range 4–10 atmospheres by Crova's formulae) as indicating that it is possible to reach an accurate value of the solar constant by confining the computation within the range for which the formulae are applicable; but it seems a little arbitrary to fix the limits in this way. Still I do think that something can be said in favor of such a procedure, because the measurement of atmospheric action ought to be made with the sun far enough from the meridian to insure that chance fluctuations will not be larger than the thing to be measured. The records of the Crova actinograph show that chance fluctuations have their greatest magnitude in the middle of the day. The rule that only maximum points shall be considered is inapplicable to the midday portion of summer curves, since even the maxima fall far short of a true restored curve; but the portion of the diurnal curve between the limits of four and ten atmospheres conforms tolerably well to the conditions needed for a determination of its slope and general form, and, as a rule, it would seem to be the best part of the curve to select for computation.

The foregoing treatment of Kimball's observation is a rational one. It simply insists that we must not take an arbitrary formula and apply it literally and recklessly to observations which are affected by sources of error not contemplated in the scheme of reduction, but that it is preferable to follow a procedure which recognizes the existence of demonstrable sources of error, tries to detect them, and applies corrections for them.

My determination of the solar constant from the sounding-balloon measurement of M. Violle, described in my paper in the preceding article,[1] is in good agreement with Savélief's observation. By the treatment recommended here, Kimball's observations are also brought into accord, not only without doing violence to the data, but by the application of precautions which are obviously

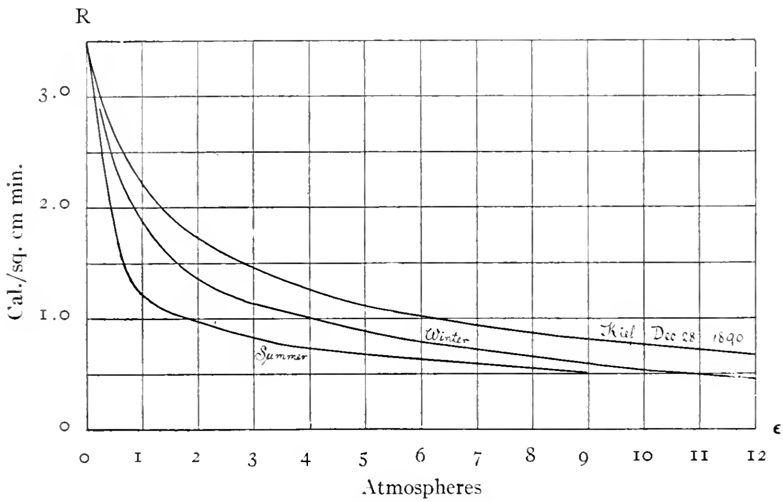


FIG. 4.—From observations of Kimball at Washington, D.C., and of Savélief at Kief, Russia.

required. If any use is to be made of solar observations secured under imperfect conditions (and the vast majority of solar actinometric observations is in this case), some such mode of selection and reduction as the above is an absolute necessity.

It will conduce to a recognition of the general facts of the atmospheric depletion of solar radiation, if we exhibit a series of radiation-curves for summer and winter, beginning with the unrivaled Kief measurement which is shown in the upper curve. The middle curve represents such a course as would be followed in less extreme winter weather, and the lowest gives a selection of measures by Kimball in June at Washington, D.C. (Fig. 4). With a general

[1] "A High-Level Measurement of Solar Radiation," *Astrophysical Journal*, **37**, 25, 1913.

parallelism from four atmospheres onward, the curves show considerable difference of form for air masses less than two atmospheres. The bend of the summer curve in this part of its course is much sharper and the ascent from one atmosphere to the radiation at the atmospheric limit is very steep at this season.

Westwood Astrophysical Observatory
Westwood, Mass.
August 15, 1912

STANDARD WAVE-LENGTHS IN THE ARC SPECTRUM OF IRON, REDUCED TO THE INTERNATIONAL UNIT

II. FROM λ 5328 TO λ 6495

By F. GOOS

In a former paper I have published the wave-lengths of the iron spectrum from λ 4282 to λ 5324.[1] The present paper contains the results of a continuation of that investigation carried out in the same manner. In order that the exposure time should not be too long, from λ 5371 on, the fifth order instead of the sixth was used. The exposure time at λ 5500 amounted to 7 minutes; at λ 5800, 12 minutes; at λ 6100, 17 minutes; and at λ 6400, 20 minutes, upon Wratten & Wainright Panchromatic-A plates.

In the yellow-red part of the spectrum in which the *Fe* lines are few and rather poor, some *Ni* lines have been added. In order to produce the *Ni* spectrum, a nickel tip was melted upon the under electrode of the ordinary iron arc, before the exposure was made, by holding a piece of pure nickel wire in the arc flame. In this way there existed, to a certain extent, both a *Fe* and a *Ni* electrode. While the photograph was being made for a third of the exposure, *Ni* was made the negative pole; then, by a reversal of the current, *Fe* was made the negative pole, during the remaining two-thirds of the time, so that the *Ni* lines should have the proper strength in comparison with the *Fe* lines. In the red part of the spectrum the ordinary iron arc was again used, but the reversal of the current was still continued, and the exposure was made half of the time with the negative pole below and the other half with the negative pole above. In this way the spectral lines maintained throughout their whole lengths a uniformly sharp and fine appearance, while otherwise they might easily appear wedge-shaped, corresponding to the greater intensity of the arc in the vicinity of the negative pole. The strength of the current, as before, amounted to about 7 amperes at 110 or 220 volts.

[1] *Zeitschrift f. wiss. Phot.*, 11, 1, 1912; *Astrophysical Journal*, **35**, 221, 1912.

Each plate covers a range of approximately 300 Å., with about nine normals of the second order. The thirty-three plates measured cover the following regions of the spectrum. The first seven have been repeated from the earlier investigation for the sake of reference.

Plate	From λ	To λ	Plate	From λ	To λ	Plate	From λ	To λ
31.........	5110	5371	42.........	5498	5763	53.........	6003	6318
32.........	5167	5406	43.........	5498	5805	54.........	6027	6335
33.........	5167	5435	44.........	5570	5893	55.........	6065	6394
34.........	5192	5435	45.........	5616	5935	56.........	6138	6431
35.........	5233	5498	46.........	5659	5953	57.........	6138	6431
36.........	5267	5507	47.........	5761	6027	58.........	6192	6495
37.........	5302	5570	48.........	5761	6065	59.........	6231	6546
38.........	5371	5659	49.........	5805	6138	60.........	6265	6546
39.........	5371	5659	50.........	5858	6192	61.........	6318	6593
40.........	5406	5659	51.........	5935	6231	62.........	6394	6593
41.........	5498	5763	52.........	5953	6265	63.........	6394	6593

In addition to the normals of the second order, established by the international resolution, some other lines, especially *Ni* lines in the yellow-red part of the spectrum, must be introduced, lines which have been measured by the interference method by only one or two observers. In the table of wave-lengths at the end of this paper, these lines are indicated by an asterisk in the fifth column. The normal λ 5455.6 must be rejected because a *Fe* line only 0.18 Å. distant almost blends with it, and seriously reduces the accuracy of its measurement. The agreement of the normals is in general good. It is only in the region from λ 5761 to λ 5953, and especially in the case of the *Ni* normals, that great discrepancies occur, but these are not so great as Kayser found.[1] The following table

I.N.	Kayser	Goos	Kayser—I.N.	Goos—I.N.	Element
I.Å.			I.Å.	I.Å.	
5371.495	.490	.495	−0.005	0.000	*Fe*
5615.661	.667	.663	+ 6	+ 2	*Fe*
5658.836	.846	.835	+ 10	− 1	*Fe*
5760.843	....	.850	....	+ 7	*Ni*
5805.211	.212	.204	+ 1	− 7	*Ni*
5857.760	.753	.761	− 7	+ 1	*Ni*
5892.882	.897	.888	+ 15	+ 6	*Ni*
5934.683	.668	.673	− 15	− 10	*Fe*
5952.739	.739	.744	0	+ 5	*Fe*
6003.039	.029	.041	− 10	+ 2	*Fe*
6430.859	.848	.860	− 11	+ 1	*Fe*

[1] *Zeitschrift f. wiss. Phot.*, **9**, 173, 1911; *Astrophysical Journal*, **32**, 217, 1910.

contains under the heading "I.N." those normals, in the case of which large differences have been found either by Kayser or by me. The greatest departure in my determinations is 0.010 Å. for the line λ 5935, whereas Kayser finds here a difference of 0.015 Å. It looks as if, in the adjustment of the normals, I had divided this difference between the two lines λ 5935 and λ 5953, for both Kayser and I find the sum of the differences of the wave-lengths of the two lines to be the same. I have not confirmed his very large discrepancies for the lines λ 5659, λ 6003, and λ 6430.

Though the results for the normals of the second order obtained by different observers show marked differences, this tendency is even more conspicuous in the new normals of the third order as shown by a comparison of my measures with those of Kayser, especially in the region from λ 5400 to λ 6200. Although these differences frequently appear in groups with the same sign, yet they become here so great—up to 0.04 Å.—that they cannot possibly be explained by the systematic difference between Kayser's measures and mine, which was mentioned in my former paper. Unfortunately, in the case of these normals of the third order, the tendency is much more common for different observers to obtain different values for the wave-lengths, in spite of similar methods of observation and in spite of the most careful reference to the normal lines.

Exner and Haschek[1] find similar results in working up their wealth of wave-length material, and enter into full particulars about it, citing also the work of other investigators on this subject. The differences between my values and those of Kayser are given in the table under the heading "$\lambda - K$." The systematic differences mentioned above are naturally still contained in these values and cannot at present be separated from them. They will hardly reach a maximum value of 0.01 Å.

As to the agreement of the measures among themselves, on account of the many broad, hazy, and weakened lines in the yellow-red part of the spectrum, the mean error of the determination of one line is, on the average, from the whole series, somewhat greater, namely $\epsilon = \pm 0.0027$ Å. (167 lines, each in general meas-

[1] *Die Spektren der Elemente bei normalem Druck*, 1, 27, 1911.

ured about seven times) as compared with $\epsilon = \pm 0.0022$ Å. of the earlier investigation.

In order to investigate the cause of the great difference between Kaysers measures and mine, I also determined the wave-lengths of certain lines by means of a Fabry-Perot interferometer. The apparatus itself is sufficiently well known, since Fabry and Buisson, and Eversheim and Pfund have used it in their determination of the normals of the second order. The distance between the half-silvered plates in my investigation amounted to 5 mm, and since it could be referred directly to the normals of the second order a determination of the phase displacement was not necessary.

The following table contains some measures of wave-lengths which belong to the region covered by my previous paper. Under "G_1" the values are the means of three interference rings upon a plate. The column headed "G" gives the values which I obtained with the plane grating. As may be seen the results obtained

G	G_1	G	G_1
I.Å.	I.Å.	I.Å.	I.Å.
4282.408	.407	4422.574	.574
94.130	.128	27.314	.315
4315.089	.085	30.623	.626
37.052	.052	42.349	.345
52.741	.741	59.127	.124
69.778	.777	61.660	.657
75.934	.935	66.555	.557
		94.573	.572

by the two methods agree to within a few thousandths of an Ångström.

Upon further interference investigations in the region from λ 5300 to λ 5500, in which there are great discrepancies between Kayser's values and mine, I found that many of the critical lines showed either very weak or absolutely no interference phenomena, and that only a part of the lines could be measured in this way. In all of the photographs which I had made up to this time either with the plane grating or with the interference apparatus, I had used a relatively short arc, about 3 to 4 mm, and a screen with a 2 mm opening had been placed in front of the slit to cut off the glowing electrodes, so that only the middle part of this short arc was effective.

It occurred to me to change the manner of producing the arc in order to get a sharper interference, and a decided improvement appeared when, with the same strength of current, the arc was made larger, and again only the middle part of about 2 mm of this long arc used. It is evident also that by changing the length of the arc, the structure of many of the iron lines is also changed, and that with the shorter arc the light of the lines in question is less effective for producing interference, and the line also becomes broader. Whether we shall find a displacemnt of the line or not depends upon the nature of the broadening, whether it is symmetrical or unsymmetrical.

I have now measured a small number of lines with the interferometer with an arc about 10 mm long, of which only the middle part was used. The corresponding plates with a short arc could not be measured, because the interference phenomena either were too faint or were wholly lacking. I had for comparison only my own measures, made with the plane grating and an arc about 3 mm long. The results are collected in the following table. Kayser's values and the international normals of the second order are also given.

Goos, Plane Grating 3 mm Arc	Kayser	Goos, Interference 10 mm Arc	International Normals
I.Å.			
5324.195	.197	.196	.196
41.035	.027	.034	
64.851	.862	.870	
71.495	.490	.498	.495
5410.878	.902	.915	
15.170	.184	.203	
24.033	.050	.066	
34.526	.527	.529	.527
97.522	.521	.518	.522

The lines 5365, 5410, 5415, and 5424 are in Kayser's list somewhat strongly displaced toward the red with the 10 mm arc as compared with the values obtained with the 3 mm arc. It is probably safe to assume that Kayser's plates were made with an arc whose length was somewhere between 3 and 10 mm. It may be mentioned that the international normals of the second order apparently show no variability, but we cannot, for this reason,

definitely consider them as constant, for, in every case, these normals have been used as reference lines, and any uniform displacement of these four lines would not appear. Nevertheless a great variability is not probable since the appearance of their interference rings hardly changes with the change in length of the arc. And, even with a very short arc, the interference is easily visible. The normals of the second order are, as we know, especially fine, sharp lines easily capable of producing the interference phenomena. It is of great practical significance that, in this part of the spectrum, the intensity of the arc diminishes rapidly with the length. The middle part of a 10 mm arc requires from 10 to 20 fold longer exposure time than the 3 mm arc. A further confirmation of the displacement of the lines is furnished by a plate which I was able to make with Kayser's concave grating, while on a visit in Bonn. I used the spectrum of the second order, in which 1 Å. = 1 mm, and made two exposures of the region about λ 5600. The first was made with a short arc, 3 to 4 mm; the second with an arc from 8 to 9 mm. In both cases the strength of current remained the same, between 7 and 8 amperes, and a screen was placed in front of the slit so that each time only 2 mm of the middle of the arc was used. The following table contains the results from the above-mentioned plate, and also Kayser's values, my plane grating measures, and the international normals of the second order which were used as reference lines.

Goos, Plane Grating 3 mm Arc	Goos, Concave Grating 3 to 4 mm Arc	Kayser	Goos, Concave Grating 8 to 9 mm Arc	International Normal
I.Å.				
5554.862	.872	.878	.893	....
63.614	.604	.608	.608	....
65.672	.689	.685	.704	....
69.633	.632	.630	.632	.633
72.854	.852	.858	.856	....
76.105	.100	.102	.104	....
86.770	.773	.774	.772	.772
98.270	.288	.292	.307	....
5602.958	.961	.962	.964	....
15.663	.659	.667	.660	.661
24.553	.559	.562	.558	....
38.276	.279	.289	.272	....
58.835	.837	.846	.836	.836

Here again there is a marked variability with the change in length of the arc especially conspicuous in the case of the lines 5555, 5566, 5598. Similar line displacements may naturally be expected if the radiation is investigated away from the middle of the arc, in the immediate vicinity of the glowing electrodes, or if the strength of the current and voltage are changed, or if iron rods of different sizes are used, etc. Fabry and Buisson[1] have recently published a thorough investigation of the great differences in appearance of the different parts of the arc in the various regions of the spectrum. From their paper may be derived many clues leading to the investigation of this question perhaps the most important of all the questions involved in the determination of normal wave-lengths.

From the results thus far obtained it seems to me to be conclusively shown that, for the establishment of a normal system of wave-lengths an exact definition of the light-source is absolutely necessary. In the program of the International Union for Solar Research[2] it is merely specified that the arc shall burn with a strength of current from 5 to 10 amperes, but that is not enough, at least not in the spectral region from λ 5000 to λ 6500.

It is to be hoped that, in the case of the normals of the second order, the lines have been selected with such care that they will not be affected by any variability in the nature or length of the arc. Henceforth, however, we must assume that the great differences in the wave-length determinations of the three observers who have used interference methods, and the differences between their values and those obtained by gratings must be reconsidered, and, in the case of the lines under consideration, new determinations of their wave-lengths should be made, after an exact definition of the light source has been made.

This brings us to the question of the definition of the light-source and indeed as to whether it is possible to find a simple definition. In the yellow-red part of the spectrum, which is the most difficult for exact measurement, one may obtain relatively sharp lines, by using the middle part of a long arc, but the intensity

[1] *Journal de physique* (4), **9**, 929, 1910.

[2] *Trans. Intern. Union for Solar Research*, **1**, 238, 1906.

λ	Intensity	No. of Plates	Mean Error	International Normal	λ−I.N.	λ−K	Remarks
I.Å.			I.Å.		I.Å.	I.Å.	
5328.040	4	7	±0.001			0.000	
28.535	3	7	3			+ 3	
32.912	2	7	2			+ 8	
39.949	3	7	1			+ 2	
41.035	3	7	3			+ 8	*Fe, Mn*
53.390	1	6	4			0	
64.851	3 *u*	7	3			− 11	
65.404	1	6	3			0	
71.495	4	9	1	.495	0.000	+ 5	
79.588	1	5	3			+ 5	
83.353	4	8	2			− 7	
89.476	2	3	6			− 6	
93.186	2	8	2			0	
97.133	4	8	2				
5400.504	2 *ur*	8	3			− 6	
05.780	4	9	1	.780	0	+ 2	
10.878	3	8	2			− 24	
15.170	4 *u*	8	4			− 14	
24.033	4 *u*	8	3			− 17	
29.700	4	8	1			− 1	
34.526	3	8	1	.527	− 1	− 1	
45.024	2	6	2			− 13	
46.924	3	6	3			+ 6	
62.952	2 *u*	5	6			− 12	
63.253	3 *u*	6	3			− 19	
66.430	1	3	4			+ 6	
73.910	2	6	6			0	
76.300	1	5	2			+ 6	
76.586	2	4	2			+ 5	
97.522	3	9	1	.522	0	+ 1	
5501.473	3	8	1			+ 4	
06.784	3	8	1	.784	0	+ 1	
22.467	1	5	5				
25.558	2	6	4			− 1	
35.419	2	7	1	.418*	+ 1	− 1	
43.178	2	7	3			− 6	
43.945	2	7	5			+ 1	
54.862	3	7	2			− 16	
63.614	3	7	2			+ 6	
65.672	3	7	4			− 13	
69.633	5	8	1	.633	0	+ 3	
72.854	6	7	2			− 4	
76.105	4	7	3			+ 3	
86.770	7	7	1	.772	− 2	− 4	
98.270	3	7	3			− 22	
5602.958	4 *uv*	7	2			− 4	
15.663	8	8	1	.661	+ 2	− 4	
24.553	4	7	4			− 9	
33.973	2	7	3				
38.276	3	7	4			− 13	
41.445	2	7	6				
49.696	1	6	5				*Ni*
58.835	5 *uv*	8	1	.836	− 1	− 11	
62.528	4	6	1			− 9	

λ	Intensity	No. of Plates	Mean Error	International Normal	λ−I.N.	λ−K	Remarks
I.Å.			I.Å.		I.Å.	I.Å.	
5678.983	2	6	±0.003				
82.180	3	6	4			−0.042	*Ni*, Kayser gives intensity 1 *u*
91.499	1	6	5				
93.618	1	6	2				
5701.543	3	6	3			− 18	
15.082	3	6	4			− 29	*Ni*
17.839	2	6	4			− 24	
31.759	2	6	5			− 15	
41.859	1	6	7				
48.346	1	4	4			− 35	*Ni*
53.136	3	6	2			− 6	
54.660	3	6	3			− 27	*Ni*
60.850	2	8	2	.843*	+0.007		*Ni*
63.010	4	8	1	.013	− 3	− 4	
75.103	2	6	4				
82.142	3	6	1			− 14	*Cu*, close pair Fabry and Perot 5782.090, 5782.159. Kayser gives intensity 1
91.059	2	6	5			+ 15	
98.204	1	5	4				*Fe*, *Ni*
5805.204	2 *u*	7	3	.211*	− 7	− 8	*Ni*
06.722	1 *u*	6	6				
09.260	1	6	2				
16.342	3	6	2				
31.630	2 *u*	6	7				*Ni*
47.006	1	6	3			+ 10	*Ni*
56.070	1	6	4				
57.761	3	7	1	.760*	+ 1	+ 8	*Ni*
59.633	3 *u*	7	4				
62.338	4	7	4			+ 16	
83.850	3	7	6			+ 28	
92.888	4	7	2	.882*	+ 6	− 9	*Ni*
5905.694	2 *ur*	6	5			+ 33	
14.144	4 *u*	6	3			+ 2	
16.237	1	6	2			− 4	
30.156	4	6	3			+ 4	
34.673	3	7	3	.683*	− 10	+ 5	
52.744	2	7	2	.739*	+ 5	+ 5	
56.692	1	5	4			− 3	
75.352	2	6	5			− 2	
76.796	2	6	5			− 4	
83.703	3 *uv*	6	5			− 6	
84.788	3	6	2			− 7	
87.040	3	6	5			− 5	
6003.041	3	7	1	.039*	+ 2	+ 12	
07.956	1 *u*	5	5			+ 3	Kayser gives intensity 3
08.587	3	7	2			+ 7	
13.519	3	7	2			+ 3	*Mn*
16.674	3	7	2			+ 8	*Mn*
20.139	3 *uv*	7	3			− 28	

λ	Intensity	No. of Plates	Mean Error	International Normal	λ−I.N.	λ−K	Remarks
I.Å.			I.Å.		I.Å.	I.Å.	
6021.829	3	7	±0.001			+0.008	*Mn*
24.051	4	7	3			+ 4	
27.057	3	8	1	.059	−0.002	− 5	
42.084	1	7	1			− 8	
55.980	2	7	4			− 12	
65.492	4	8	1	.492	0	+ 3	
78.466	2	7	2			− 10	
89.570	1	3	7			− 11	
6102.177	2 *u*	7	3			− 5	
03.167	1 *u*	7	3			− 25	
27.919	1	6	3			− 8	
36.622	5	7	1			− 6	
37.000	1	6	4			− 12	
37.701	5	9	1	.701	0	− 2	
41.760	2	8	3				
47.846	1	8	5			+ 2	
51.633	1	7	4			0	
57.736	2	8	2			− 4	Kayser gives intensity 4
65.370	1	8	3			− 17	
73.347	2	8	2			− 3	
80.218	1	7	2			− 12	Kayser gives intensity 3
88.089	1	7	3				
91.569	5	9	1	.568	+ 1	+ 1	
6200.324	2	8	3			− 6	Kayser gives intensity 5
13.447	3	8	2			+ 9	
15.152	1	8	4			− 1	
19.294	3	8	1			+ 5	
27.266	1	8	5				
30.732	5	9	1	.734	− 2	− 2	
32.673	3	8	1			+ 4	
40.662	1	8	4			− 4	
46.351	5	8	2			+ 6	
52.566	5	8	2			− 1	
54.271	3	8	2			+ 2	
56.279	2	8	2			+ 2	
65.146	3	9	1	.145	+ 1	+ 1	
70.238	2	8	2			− 7	
80.626	2	8	4			− 5	
90.956	2 *u*	8	3				
97.803	3	8	2			− 1	
6301.531	5	8	1			+ 3	
02.525	3	8	1			+ 3	
15.322	2	8	1			− 1	
15.803	1	8	4			− 19	
18.028	4	9	1	.028	0	− 3	
22.697	3	8	1			− 1	Misprint in Kayser
35.341	4	8	1	.341	0	+ 2	
36.855	5	7	2			+ 5	
44.165	2	7	1			+ 1	
55.049	3	7	2			+ 4	
58.675	2	7	2			− 18	
62.882	1	7	4				
80.753	2	7	1			− 1	

λ	Intensity	No. of Plates	Mean Error	International Normal	λ−I.N.	λ−K	Remarks
I.Å.			I.Å.		I.Å.	I.Å.	
6393.611	6	9	±0.001	.612	−0.001	−0.001	
6400.036	8	8	2			0	
08.058	4	8	2			+ 13	
11.685	6	8	1			+ 12	Kayser gives intensity 4
21.366	5	8	2			+ 12	Kayser gives intensity 3
30.860	5	8	1	.859	+ 1	+ 12	
62.743	3	6	2			+ 9	
69.237	2 *u*	5	3				
75.642	1	6	2				
81.885	2	6	3			− 11	
91.372	2	6	3				
94.992	7	6	1	.993	− 1	− 2	

of the light there is so weak that the necessary exposure time would be altogether too long. On the other hand, the short arc and the region near the negative pole have the advantage that the light which they emit is extremely intense, against which is the disadvantage that the lines which they produce are broader and weaker and poorly adapted to interference observations. The advantage of greater intensity, in this photographically inactive part of the spectrum, seems to me more important than the sharpness of the lines. On the other hand, one should avoid the vicinity of the negative pole for the blue and violet on account of the many reversals. These, to be sure, admit of very accurate settings in their measurement, but the reversal is often unsymmetrically situated in the emission line. As far as my observations extend into the violet, down to λ 4000, the wave-lengths are approximately constant. The few lines which are doubtful could easily be omitted since there are plenty of other lines left, in contrast to the red part of the spectrum, where the lines in some places are few and rather poor. In some cases, if a long slit is to be used, as for example, in connection with a large concave grating, a long arc will be preferred, so that the slit may be well illuminated. If a short arc is used, it will be necessary to enlarge it by means of a condensing lens. It might also be advantageous to try a weaker current of perhaps 2 or 3 amperes, and smaller iron rods of 4 or 5 mm diameter. Moreover, the insertion of a piece of ground glass in front

of the slit might be considered, so that all parts of the arc might be effective, but then, in the case of great dispersion, or in the red region, the exposure time would be much too long. Thus it seems to me that it will be very difficult to find a perfectly satisfactory definition of the light source. It should also be emphasized that the nickel arc is not a satisfactory substitute in the region between λ 5600 and λ 5900, where there are so few iron lines. The nickel lines are nearly all hazy and weakened and show displacements up to several hundredths of an Ångström.

At the present time, therefore, there seems to be no prospect of the establishment in the near future of a normal system of wave-lengths, accurate to a few thousandths of an Ångström over the whole spectrum. Further investigation into the phenomena of the electric arc is what is most needed now in this realm of research.

HAMBURG, PHYSIKALISCHES STAATSLABORATORIUM
July 1912

SPECTRA OF LOW POTENTIAL DISCHARGES IN AIR AND HYDROGEN

By GORDON S. FULCHER

The light emitted as a direct result of the bombardment of nitrogen, hydrogen, and oxygen molecules by cathode rays has been investigated in the case of rays with velocities corresponding to from 500 to 5000 volts difference of potential.[1] An attempt has since been made to determine the effect of slower rays, with the following results.

In the first apparatus made, a Wehnelt cathode was placed parallel to a perforated anode at a fixed distance of 1 mm. By applying a field, the electrons set free at the cathode could be projected through the hole in the anode so as to form a beam of cathode rays of a definite velocity in the chamber behind the anode. However, no visible beam was obtained except when conditions were such as to permit a discharge to pass directly between the electrodes. Evidently ionization by collision was necessary to produce a sufficient number of rays. The apparatus was therefore changed so as to permit a variation of the distance between the electrodes (see Fig. 1). It was then discovered that when they were about 5 mm apart, with a pressure of the order of 0.1 mm *Hg*, a discharge could be obtained with a very low difference of potential by using a sufficiently hot cathode. The minimum observed for hydrogen was 20 volts, for air 27 volts, for oxygen 45 volts, with a discharge current of about 0.04 ampere. The gases were dried by passing over phosphorus pentoxide, and the impurities given off by the hot cathode were washed away by maintaining a continuous stream of gas through the tube with the aid of a capillary and a Gaede pump; but otherwise no special precautions were taken to purify the gases. The spectrograms show slight traces of mercury lines, and the *CO* bands were present when oxygen was used. These minimum potentials were best obtained just after inserting a new, salted, platinum cathode, but

[1] Fulcher, *Astrophysical Journal*, **34**, 388, 1911.

could not be maintained for more than half an hour without renewing the cathode, as the discharge potential always rose steadily while the discharge was running.

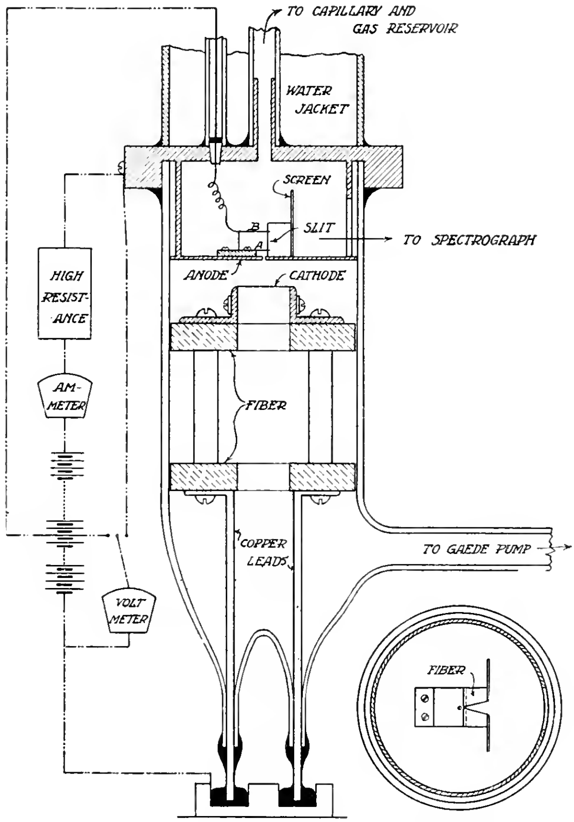


FIG. 1.—Diagram of apparatus

AIR

In the case of air, at the higher voltages (50 volts), distinct bluish columns could be seen rising from various points on the salted cathode to the anode through the reddish glow which filled the discharge chamber. Their bases were distinctly separated from the cathode by a short dark space (0.5 mm). As the tem-

perature of the cathode was increased, suddenly the discharge potential dropped, the bluish columns vanished and left only the reddish glow, separated, however, by a wider dark space from the cathode. Photographs recording the changes in the spectrum accompanying the decrease in the discharge potential were made with a Hilger constant deviation spectrograph. Three are shown enlarged four times in Fig. 2, Plate II. Note the striking decrease in the relative intensity of the negative nitrogen bands with respect to the positive bands. Under the most favorable circumstances they disappear entirely, as is shown by No. 193. It was found important to analyze only the light produced between the two electrodes, as the discharge potential is no measure of the potential differences which may exist between the cathode and other parts of the tube; at least that is the probable explanation of the fact that spectrograms of the light from the region behind the cathode showed the negative bands faintly even when the discharge potential was less than 30 volts. Cramer "spectrum" plates were used, but no variations in the relative intensity of the first (red) and second groups of nitrogen positive bands were observed.

Now to return to the problem of the production of light by slow cathode rays. The light produced by the rays projected through the perforated anode proved too faint for spectroscopic analysis; so a pair of plates, A and B (Fig. 1), were placed behind the anode and insulated from it in order that a retarding potential might be applied and a larger number of slow rays might be obtained than if the potential of the discharge were no greater than that corresponding to the desired low velocity of the rays. A slit with fiber jaws was placed close to the beam. However, it was found difficult to get and maintain a beam sufficiently bright for spectrographic analysis. The most interesting spectrogram obtained is shown in Fig. 2, No. 202. The positions of Plates A and B are indicated. It is clearly seen that as the rays are retarded, the intensity of the negative nitrogen bands decreases very greatly, while the positive bands become stronger, not only relatively but absolutely, in spite of the dispersion of the rays, until, halfway between A and B, the positive band λ 4269 is more intense than the negative band λ 4278. During the exposure the plates were

maintained at a potential 20 volts above that of the cathode, and the discharge potential was about 80 volts. However, it may well be that because of the presence of large numbers of free electrons and ions the lines of force passed through the hole in Plate A and the retarding potential was not as great as the above figures would indicate; so that the spectrum of light produced halfway between A and B may correspond to a greater energy of the rays than would be produced by a fall through 20 volts. To decrease the difference between the actual and the measured retarding potential differences, the apparatus was modified as shown in Fig. 4. The two brass

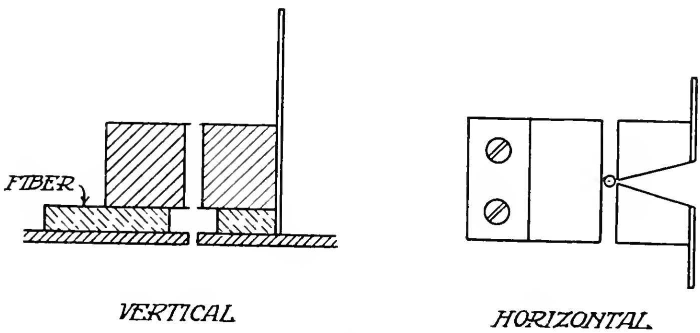


FIG. 4

jaws of the slit, together with a brass block, formed a narrow chamber with parallel walls 1 mm apart into which the rays passed through a small round hole. However, the light-intensity was reduced to such an extent that no satisfactory spectrogram was obtained.

The light analyzed in this experiment came from the path of the cathode rays; hence it was chiefly due directly to the impact of the cathode rays on the gas molecules. At the pressures used, the mean free path was several millimeters; hence, in comparison with the ionization produced directly by the cathode rays, any secondary reactions, such as the recombination of ions formed by the rays, would be scattered through a larger volume around the beam, a smaller proportion of the light produced would pass through the spectrograph, and the effect would not be localized but would be more uniformly distributed along the slit. Therefore we may

conclude that the variations in the spectrum obtained are due to the variations in the energy of the cathode rays; that the light is emitted as a direct result of the ionization produced by the rays; that slow cathode rays may excite the nitrogen molecules ionized by them to emit both positive and negative band spectra, whose relative intensity depends on the energy of the ionizing rays. But we found above that the spectrum of the light from between the electrodes in the discharge chamber showed variations with discharge potential similar to the variations shown by the spectrum produced by the cathode rays. It seems probable, then, that the light in the discharge chamber at these low voltages is chiefly the direct result of ionization by cathode rays; that the slowest rays (30 volts) excite the emission of only the positive nitrogen bands; but that the relative intensity of the negative bands increases very rapidly with the energy of the exciting rays so that for 50 volts they are the stronger and for higher discharge potentials greatly exceed in intensity the positive bands. Both series of bands seem to be emitted by the nitrogen molecules when positively ionized by cathode rays. Perhaps the faster rays can penetrate farther into the molecules and arouse a simpler and more fundamental type of vibration than the slower rays are able to do; or the positive bands may be emitted by molecular aggregates dissociated by the faster rays.

Gehrcke and Seeliger[1] recently described a very pretty experiment in which they shot a cathode beam from a Wehnelt cathode obliquely into a retarding electric field sufficiently strong to cause the rays to describe a parabolic path. Under those conditions they observed marked differences in color between different parts of the beam. In air the beam changed from bluish to reddish at a point where the energy of the rays corresponded to about 46 volts, and became non-luminous where the energy corresponded to less than 29 volts. The above results show that the change from bluish to reddish means a decrease in the relative intensity of the negative nitrogen bands, but that the change is not discontinuous but takes place continuously, though rapidly, between 50 and 30 volts. Their value for the minimum energy necessary to produce

[1] *Verhandlungen der Deutschen physikalischen Gesellschaft*, **14**, 335, 1912.

luminescence agrees well with the minimum observed discharge potential reported above, 27 volts. The changes in hydrogen described below also agree with their results except that the minimum observed potential was 20 volts instead of 29 volts, the value reported by them. The lower value may be due to the presence of a trace of mercury vapor in my discharge tube, as they found that the minimum energy for mercury vapor was only 10 volts. Yet spectrograms taken when the discharge potential did not rise above 26 volts show the mercury lines no stronger than the main compound lines of hydrogen.

The relations found between the nitrogen band spectra and the energy of the exciting rays should prove useful in the investigation of phenomena in the positive column and other parts of a discharge tube. According to Sir J. J. Thomson's[1] theory of striations, there are regularly spaced, self-perpetuating variations in the density of free electrons corresponding to the variations in luminosity observed; the distance apart of the striations is so adjusted that the electrons set free in one striation, and accelerated by the field due to the distribution of free electrons present, acquire before reaching the next just sufficient energy to cause ionization and luminescence. The positive column spectrum in air is known to consist chiefly of the positive nitrogen bands. Perhaps if the faster cathode rays from the cathode and other parts of the tube were excluded, the negative bands would vanish and we should know that we were dealing merely with cathode rays having velocities corresponding to a fall through about 30 volts, and thus confirm Thomson's theory. As soon as we know which electrical molecular changes result in the emission of each distinct spectrum associated with an element, we shall have a powerful tool for the solution of the remaining problems relating to the discharge of electricity through gases.

HYDROGEN

The changes in the appearance of the discharge in hydrogen as the discharge potential decreased were so similar to those observed in air that no further description need be given. Some

[1] *Phil. Mag.*, **18**, 441, 1909.

of the spectrograms of the light from between the two electrodes are shown enlarged four times in Fig. 3, Plate II. Between 60 volts and 40 volts the main series lines decreased greatly in intensity with reference to the lines of the compound spectrum. The ratio of the intensities of the series lines on plates Nos. 208 and 183 is several hundred as determined by taking short exposures with the Geissler tube, while the lines λ 6327, λ 6225, λ 5538, etc., are equally intense on both. The relative intensity of the two spectra did not change appreciably when the discharge potential was decreased from 40 to 23 volts. On no spectrogram obtained are the series lines entirely absent. Perhaps water is more easily dissociated into hydrogen atoms by cathode rays than hydrogen molecules are, and the residual series lines may be due to a trace of water present; or the weak lines may actually belong to the compound spectrum, a possibility suggested by the fact that their relative intensity with respect to the compound lines was found to be about the same on several spectrograms obtained under varying conditions.

There are also very interesting changes in the compound line spectrum itself to be observed by comparing No. 183 with No. 208. The spectrum for the low voltages is evidently simpler; the strongest lines seem to be arranged with some regularity, suggesting bands. In regard to the existence of bands in the compound spectrum, Dufour[1] states that some of the weakest lines have equal frequency differences, an arrangement resembling that of bands, so that the compound spectrum consists of a complicated line spectrum superposed on a faint band spectrum; but he gives no numerical data. Otherwise there seems to have been no mention of the existence of numerical regularities in the second spectrum of hydrogen. A year ago in glancing over Kayser's reprint of Watson's measurements,[2] I was struck with the recurrence of strong lines in the red at intervals of about 100 Å, and an investigation showed that the differences in wave-length between certain strong lines in the red and yellow are so regular as to prove that they together form groups resembling bands. It was very interesting to find later that these "band" lines are the strongest lines

[1] *Annal. chim. phys.* (8), **9**, 416, 1906.

[2] Kayser, *Handbuch der Spectroscopie*, V. Band, 494–502, 1910.

in the low-potential discharge spectrum of hydrogen. Some of the lines were investigated by Dufour[1] and none were found to show a Zeeman effect.

The following table contains the "band" lines and the differences in wave-length between successive members of the series.

TABLE I

MAIN BAND LINES OF HYDROGEN

	λ	Difference	λ	Difference	λ	Difference
S_1.......	....	......	6441.83(0)	101.08	6340.75(2)	102.18
S_2.......	....	......	6433.80(0)	101.10	6332.70(1)	102.22
S_3.......	6527.63(1)	99.27	6428.36(2)	101.11	6327.25(8)	102.23
S_1–S_3....	....		13.47		13.50	
S_2–S_3....	....		5.44		5.45	
S_1.......	6238.57(6)	103.01	6135.56(6)	103.42	6032.14(5)	
S_2.......	6230.48(2)	102.95	6127.53(2)	103.53	6024.00(4)	Head
S_3.......	6225.02(10)	103.01	6122.01(10)	103.48	6018.53(9)	
S_1–S_3....	13.55		13.55		13.61	
S_2–S_3....	5.46		5.52		5.47	
S_1.......	5671.10(0)	118.50	5552.60(0)	117.54	5435.06(1)	117.00
S_2.......	5661.97(0)	118.26	5543.71(0)	117.51	5426.20(1)	117.10
S_3.......	5655.98(1)	118.31	5537.67(5)	117.57	5420.10(4)	116.75
S_1–S_3....	15.12		14.93		14.96	
S_2–S_3....	5.99		6.04		6.10	
S_1.......	5318.06(0)	......	....	......	....	......
S_2.......	5309.10(0)	......	....	......	....	......
S_3.......	5303.35(2)	......	....	......	....	......
S_1–S_3....	14.96		....		....	
S_2–S_3....	5.75		....		....	

Mean S_1–S_3...............	13.54 (first band)	15.00 (second band)
Mean S_2–S_3...............	5.47 (first band)	6.04 (second band)
Ratio.....................	0.404 (first band)	0.403 (second band)

The wave-lengths are those given by Watson;[2] the agreement between the differences of S_1, S_2, and S_3 shows the accuracy of his measurements. The wave-lengths determined by Hasselberg[3]

[1] *Op. cit.*, pp. 413–416.

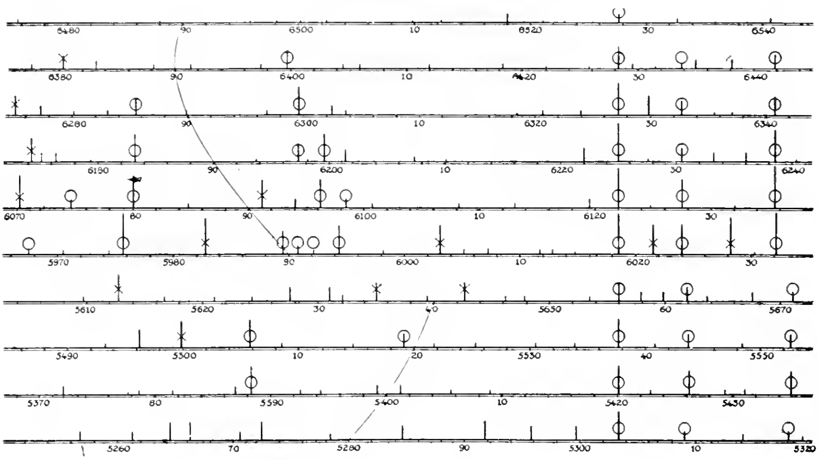
[2] *Proc. Roy. Soc.*, A, **82**, 189–204, 1909.

[3] *Mém. acad. St. Petersb.* (7), **31**, No. 14, 1883. Reprinted in Kayser's *Handbuch*, and in Dufour's paper in *Annal. chim. phys.* (*loc. cit.*).

show much greater variations. The intensities given by Watson are added in brackets.

The two bands seem to be related since the ratio of S_2–S_3 to S_1–S_3 is the same for both. There are other lines which seem to belong with these groups or bands:

S_4—λλ 6303.68(0), 6201.38(2), 6098.45(3), 5994.30(4);
S_5—λλ 6199.58(4), 6096.21(4), 5992.14(0);
S_6—λλ 6299.60(5), 6197.32(2), 6094.03(1), 5989.47(3);
S_7—λλ 6285.56(4), 6183.19(4), 6080.03(9), 5975.68(8).



Lines of group one. Lines of group two.

FIG. 5

The regularities are shown graphically in Fig. 5 where the relative positions of the band lines and of all others in their neighborhood whose wave-lengths have been determined by Watson,[1] Porlezza,[2] and Norzi[3] are indicated. The heights of the lines show the relative intensities given by Porlezza and Norzi. The two groups referred to below (Table II) are indicated.

To show more in detail the differences in the compound spectrum for high and low discharge potentials and the relation of these variations to the Zeeman-effect observations of Dufour, the

[1] Watson, *loc. cit.*

[2] Porlezza, *Atti accad. Lincei*, **20** (2), 178, 1911.

[3] Porlezza and Norzi, *ibid.*, **20** (1), 822, 1911.

following table is given. Four plates, two taken with high discharge potential and two with low, were measured, and the intensities of the various lines on the four plates were estimated independently. The agreement was in general very close. The averages for each pair are given in the columns headed "Low" and "High."

TABLE II

RELATIVE INTENSITIES OF LINES IN THE HIGH AND LOW POTENTIAL SPECTRA OF HYDROGEN

Line	Low	High	Line	Low	High	Line	Low	High
Group one—			*5989.47	2	3	*Group two—*		
6441.83	½	½	* 75.68	10	10	6380.30	0	1
28.36	3	3	* 50.15	5	5	6275.10	0	3
6399.71	4	3	* 25.09	3	2	6174.28	0	4
62.67	2½	2	* 16.76(?)	1	5	61.81	½	4
40.75	2½	2	09.71	1	1½	†6091.16	0	5
27.25	9	9	5872.12	5	5	†6070.21	4	6
6299.60	6	5	69.47			*(67.96)		
85.56	3	2	* 36.28	7	10	*(6032.14)	7	12
71.55	1	1	* 22.99	5	6	† 28.21		
* 38.57	5	5	*5786.00(?)	7	10	† 03.08	½	4
30.48	1	1	* 75.28	6	8	†5982.79	½	6
* 25.02	11	11	* 37.06	6	9	† 63.70	2	5
*6199.58	7	10	5671.10	1½	2	† 59.98		
* 97.32			61.97(?)	1	4	† 38.87	3	9
* 83.19	5	4	55.98	3	5	† 31.62	3	9
* 35.56	8	11	5600.65	3	3	* 16.76(?)	1	5
* 27.53	3	4	5597.80			†5779.22	½	3
* 22.01	12	12	52.60	3	2	59.76	2	7
*6096.21	6	6	43.71	1½	2	13.59	1	4
* 80.03	7	8	37.67	10	8	03.50	1	8
†(70.21)	4	6	19.12	1	1	5689.00	3	10
* 67.96			05.79	4	4	88.00		
* 41.27	1	1½	5435.06	2	3	84.33	2	10
* 32.14	7	12	26.20	3	4	42.65	0	3
†(28.21)			20.10	7	7	35.05	½	4
* 18.53	16	14	5388.35	4	4	12.77	½	5
*5994.30	2	2½	03.35	4	4	5499.84	1	3

* No Zeeman effect (Dufour). † Show Zeeman effect (Dufour).

The lines fall into two groups, the first containing all those which are equally intense in both spectra, the second those which are relatively very much weaker in the low potential spectrum. To the first group belong all the lines between λ 5700 and λ 6300 which Dufour found do not show a Zeeman effect, and also all the band lines listed above. To the second group belong all the lines which do show a Zeeman effect. The only exceptions seem to be

λ 5916 and λ 5786. For the latter Dufour first reported no transverse Zeeman effect (collimator perpendicular to magnetic field), but later[1] said that it shows a normal longitudinal Zeeman effect—perhaps a misprint, as the statements made are inconsistent. The lines may be double, only one showing the Zeeman effect, but both lines should be tested again. Then the band line λ 5662 seems too weak in the low potential spectrum. In general, however, it seems to be true that the lines in the hydrogen compound spectrum between λ 5300 and λ 6500 which show the Zeeman effect, like the series lines, are much weaker in the low potential spectrum; while the strongest lines which appear in the low potential spectrum do not show the Zeeman effect and from the regularity of their arrangement evidently belong to a band spectrum of hydrogen.

So far we have spoken only of the red and yellow regions of the hydrogen spectrum. Dufour has also investigated the lines in the blue region[2] and has found between λ 4300 and λ 4700 about 30 lines showing no Zeeman effect, an equal number showing a normal effect to a greater or less extent, and a few abnormal lines. As an examination of the spectrograms Nos. 208 and 185 (Fig. 3) will show, there is little change in the relative intensity of the lines in this region when the discharge potential is varied between extreme limits, and the few changes which do occur do not seem related to the Zeeman effect at all. The following lines seem relatively weaker in the low potential spectrogram:

λλ 4551.14, *4543.87, †4538.51, *4514.51, †4474.42.

I have searched in vain for any recurring wave-length differences which would group any of the lines together. The results of Dufour serve to emphasize the complexity of this spectrum, but if they are extended may perhaps furnish the clue necessary to unravel it.

In oxygen, with a discharge potential of 75 volts, the spectrum consisted chiefly of the negative bands in the red and yellow, but the spark and series lines were also present. On lowering the potential to 45 volts, the negative bands disappeared and were

[1] Dufour, *Journal de physique* (4), **8**, 258, 1909.

[2] *Ibid.*

replaced by CO bands, but the series lines $\lambda\lambda$ 6156 and 4368 were still present.

The nitrogen, hydrogen, and oxygen bands are probably emitted by the various molecules as a result of ionization by cathode rays.

I regret that some points in this paper have not been investigated more thoroughly, but I present these results, such as they are, since other work will prevent me from carrying the investigation further at present.

MADISON, WIS.
October 22, 1912

NOTICE

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention is given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

Articles written in any language may be accepted for publication, but unless a wish to the contrary is expressed by the author, they usually will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right unless the author requests that the reverse procedure be followed.

Accuracy in the proof is gained by having manuscripts typewritten, provided the author carefully examines the sheets and eliminates any errors introduced by the stenographer. It is suggested that the author should retain a carbon or tissue copy of the manuscript, as it is generally necessary to keep the original manuscript at the editorial office until the article is printed.

All drawings should be carefully made with India ink on stiff paper, usually each on a separate sheet, on about double the scale of the engraving desired. Lettering of diagrams will be done in type around the margins of the cut where feasible. Otherwise printed letters should be put in lightly with pencil, to be later impressed with type at the editorial office, or should be pasted on the drawing where required.

Where an unusual number of illustrations may be required for an article, special arrangements are made whereby the expense is shared by the author or by the institution he represents.

Authors will please carefully follow the style of this JOURNAL in regard to footnotes and references to journals and society publications.

Authors are particularly requested to employ uniformly the metric units of length and mass; the English equivalents may be added if desired.

If a request is sent *with the manuscript*, one hundred reprint copies of each paper, bound in covers, will be furnished free of charge to the author. Additional copies may be obtained at cost price. No reprints can be sent unless a request for them is received before the JOURNAL goes to press.

The editors do not hold themselves responsible for opinions expressed by contributors.

The ASTROPHYSICAL JOURNAL is published during each month except February and August. The annual subscription price is $5.00; postage on foreign subscriptions, 62 cents additional. Business communications should be addressed to the *University of Chicago Press, Chicago, Ill.*

All papers for publication and correspondence relating to contributions should be addressed to *Editors of the* ASTROPHYSICAL JOURNAL, the *University of Chicago, Chicago, Illinois, U.S.A.*

13.

THE
ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME XXXVII — MARCH 1913 — NUMBER 2

THE SOLAR ROTATION IN 1911

By J. S. PLASKETT and RALPH E. DeLURY

1. The determination of the rotation of the sun by the Doppler displacement of the spectral lines at the limb was early placed on the program of work of the Dominion Observatory; was indeed one of the investigations planned for the equipment for solar research whose nucleus was the 20-inch coelostat used in the Canadian Eclipse Expedition to Labrador in 1905. But actual observations have been delayed by various causes, the principal ones being delay in the construction of a shelter for the coelostat telescope[1] and the difficulty of securing a suitable plane grating for the spectrograph,[2] the one now in use being the fourth that has been tried for the purpose. Consequently it was not until the spring of 1910 that any suitable plates were obtained and even these were of a somewhat experimental and tentative character.

2. The whole plan of investigation was placed upon a much more definite basis at the Mt. Wilson meeting of the International Union for Co-operation in Solar Research in 1910. The regions of spectrum to be investigated were allotted to the different members of the Rotation Committee, a general region to be observed by all was selected (center at λ 4250), and the various questions to be

[1] *Report of Chief Astronomer*, 1908, p. 67; 1909, p. 149; 1910, p. 68; also *Trans. Roy. Soc. Can.*, 1911, Sec. III, p. 113.

[2] *Ibid.*

determined were laid down. It may be useful to summarize here the principal points.

A. The region to be observed at the Dominion Observatory is in the yellow green, λ 5500–λ 5700.

B. The general region to be observed by all is from λ 4220 to λ 4280 in the violet.

C. The latitudes to be observed in the special region are 0°, 15°, 30°, 45°, 60°, 75°, and if possible 80° and 85°. The latitudes to be observed in the general region are 0°, 30°, and 60°.

D. Fifteen or twenty lines are to be measured in the special regions, these to be selected to include as many elements as possible, especially those of high or low atomic weight; about ten lines, selected by the secretary of the committee after consultation, are to be measured in the general region.

3. The principal objects of a study of the sun's rotation by the spectroscopic method are:

a) The accurate determination of the velocity of rotation at various latitudes and the derivation of a formula representing the variation of velocity with latitude.

b) A definite conclusion in regard to the existence of variations in the rate of rotation.

c) The investigation of the rate of rotation, as shown by the lines of different elements and of the arc and enhanced lines of the same element, to determine whether either the absolute rate of rotation or the law of variation with latitude differs for different elements.

d) The detection of possible systematic proper motions or drifts in the sun's reversing layer.

4. Although the instrumental equipment used in this investigation has already been described,[1] it is desirable for the sake of completeness to outline briefly the salient features.

The coelostat telescope has a coelostat mirror of 20 inches aperture, a secondary plane mirror of 20 inches aperture, and a concave mirror of 18 inches aperture and 80 feet focus. It is situated directly north of the west wing of the observatory and the beam of sunlight

[1] *Report of Chief Astronomer*, 1909, pp. 207, 251; also *Trans. Roy. Soc. Can.*, 1911, Sec. III, p. 11.

which is reflected south from the coelostat mirror and north from the secondary mirror is then reflected due south, with an angle of depression of 3°5, passing below the secondary mirror and forming an image of the sun about 227 mm in diameter just outside of the observatory basement. The coelostat is covered by a louvred house rolling back on a track and the path of the beam is inclosed by a louvred shelter leading by a short tunnel into the basement. The definition of the image is usually good, better than that given by the equatorial telescope, and does not seem to be injuriously affected by the passage through the tunnel.

The spectrograph, which is of the Littrow form, has a combined collimator and camera objective of 6 inches aperture and 23 feet focus. The grating now in use, a 6-inch plane, was ruled by Anderson at Johns Hopkins; it has a ruled surface about $3\frac{7}{8}\times5$ inches, 15,000 lines to the inch. Owing to astigmatism only about 2 inches of the length of the ruling is used and it is limited to 3 inches in width to insure a sufficient margin of safety in covering it uniformly by the 5-inch diameter circle of illumination. The spectrograph is of a trussed box form which can be rotated about the optical axis and set to any desired position angle, allowing the reflecting device, used for bringing the light from the limbs of the sun, on the slit, to be set to any required latitude. This device consists of two right-angled prisms, movable radially, which reflect the light from opposite limbs inwardly, and two other right-angled prisms over the slit which further reflect it through the slit. One of the latter is ground to a point, the sides subtending an angle of 30°, while a notch of the same shape and size is ground in the other. This arrangement of the prisms produces on the plate a spectrum from the east limb about 0.9 mm wide situated midway between two spectra from the west limb of the same width, the separation between adjacent spectra being about 0.3 mm. This arrangement is preferable to that in which only two spectra are used, one from the east and one from the west limb, as the correct measurement of the displacement does not depend upon the exact orientation of the measuring wire. All of these prisms are provided with adjusting screws to enable the beams of light coming from the opposite limbs to be exactly collimated, thus insuring uniform and similar

illumination of the grating surface. In front of the prisms is an adjustable guide plate, with small adjustable windows for the admission of the light, which serves to protect the prisms and is devised to enable the desired latitudes and positions on the sun's disk to be easily obtained.

5. A large amount of experimental work to determine the best instrumental and optical conditions, the precautions necessary for accurate observations, the most suitable plates and developers for the two regions of the spectrum to be observed, was carried through in 1909 and 1910 and some preliminary rotation plates were made in 1910, a few of which were measured.[1] After the Solar Union meeting in 1910 and in accordance with the plan outlined in Sections 2 and 3, three series of plates were made in 1911, two in the special region at λ 5600 and one in the general region at λ 4250. The diameter of the solar image averaged about 227 mm; in the first series at λ 5600 the distance from the limb varied from 3.0 to 4.5 mm; in the second series also at λ 5600 it was nearly 10 mm; and in the third series at λ 4250 it was about 6.5 mm. The distance was varied in order to see if any difference in the rotational value was obtained and if much change in the definition occurred as the distance from the limb was increased. As will be seen later, the difference, if any, is slight both in the velocity and in the definition. Owing to the considerably larger corrections required to reduce the measured to the actual values of the rotation as the distance from the limb increases, it is not deemed desirable to make the spectra, in future, from points at a greater distance than 5 mm from the limb.

PRECAUTIONS

6. In all these plates particular care was taken to guard against every known cause of instrumental and other error tending to introduce spurious displacements of the lines, and the experience of one of the writers in stellar radial velocity determinations was of great value in this similar work. Temperature changes and flexure, the chief difficulties in stellar spectroscopy, are not, however, of much moment here, for, owing to the short and simultaneous expo-

[1] *Trans. Roy. Soc. Can.*, 1911, Sec. III, p. 120; and *Report of Chief Astronomer*, 1910–11.

sures on opposite limbs, temperature changes will have no appreciable effect and there can be no flexure when the spectrograph is stationary during the exposure. It may not be amiss to give here the four essential precautions for accurate observations, which have been carefully followed:

a) The emulsion on the photographic plate must be exactly in the focus of the spectrum.

b) The illumination of the grating from the opposite limbs of the sun must be similar and uniform.

c) The solar definition must be good, the image steady, and the sky free from haze.

d) Care must be taken that the reflecting prisms receive light from the desired latitudes.

7. Precautions *a* and *b*, conditions inside the spectrograph, to which may be added the avoidance of undue heating of the slit jaws, are very necessary to prevent systematic displacements of the lines as a whole introducing corresponding errors in the velocity values. If either *a* or *b* is exactly fulfilled, an approximate realization of the other should be sufficient, but, as it is practically impossible either to get or keep the plate at the exact focus or to have absolutely equal and uniform illumination of the lens and grating from the opposite limbs, the only safe procedure is to fulfil both conditions as closely as possible. Consequently the plate focus was determined frequently both by the definition test and, as a check, by the Hartmann method of extra-focal exposures. It was found that the field both in the λ 5600 and in the λ 4250 region was curved, concave to the lens, about 2.5 mm longer at the center than at the ends of a plate 30 cm long and inclined about 1°, in opposite directions for the two regions, to the normal to the axis. The illumination of lens and grating was tested before and after each plate, which consisted usually of seven spectra, one of each of the six latitudes from 0° to 75° and one of the pole. This was done by opening the slit wide enough to allow a visible image of the illuminated concave mirror to be projected on the diaphragmed front surface of the collimating objectives. If this image was not central for both systems of prisms it was easily made so by the adjusting screws provided. It was found frequently that a slight

change in position of the overlapping images occurred during the time the seven exposures were made but never sufficient (since the image is considerably larger than the used portion of the grating) to prevent uniform illumination. This change of adjustment of the prisms must be due to heating produced by the sun's rays, and, to minimize this effect, the heating of the slit jaws, and the distortion of the coelostat, secondary, and concave mirrors, the coelostat mirror, and consequently the whole system, is kept shaded by a blind except during the actual exposures, which occupy from 30 to 60 seconds each.

8. Precautions *c* and *d*, conditions external to the spectrograph, were always carefully looked after. The solar definition during the summer months, on the clear and bright days which only were employed, is usually fairly good and, as undue heating of the mirrors was prevented by keeping them shaded for suitable intervals between the exposures, the definition did not much deteriorate. It is essential that there be fair definition to insure that the light reaching the slit may be confined to a small region around the desired portion on the sun's disk. Great care was taken in the relative adjustment of guide plate and prisms so that when the image was kept central and the spectrograph rotated to the desired and previously calculated position angle from the E.-W. line (determined by the drift of the solar image when the coelostat clock was stopped) the positions of the points on the disk from which the light was taken were accurately known. This is rendered much easier and more certain by the large size of the image (about 227 mm), and consequently it is improbable that any errors can have arisen either in this regard or due to poor definition. The only effect of the latter would be to introduce a small amount of light at slightly higher and lower latitudes or at greater and less distances from the limb and the effects thereby produced would practically compensate one another. The necessity of observing only when the sky is free from haze will be evident when it is realized that the effect of the superposed sky spectrum, which is a blend of the spectrum from the whole disk of the sun, is to diminish the displacement and give too low a value of the velocity. DeLury made some experiments on this effect and found a measurable influence

on the equatorial displacement only when the ratio of intensity of sky spectrum to limb spectrum reached about 1:20. As on a clear day this ratio is 1:100 or less, it is evident that no error can thereby be introduced.

OBSERVATIONAL DATA

9. The plates were made by the authors jointly, since the making of the focus and illumination tests and the guiding of the sun's image carefully can be much more easily and satisfactorily done by two than one. As stated above, in the λ 5600 region, rotation spectra of each of the six latitudes to be observed, 0°, 15°, 30°, 45°, 60°, 75°, with one of the pole, 90°, for check purposes, were made on each plate, but in the higher latitudes 80° and 85° three of each with one of the pole were made on each plate. In the λ 4250 region two spectra each of the latitudes 0°, 30°, 60°, and one at 90° were made on each plate. If any of the plates showed a greater displacement in the spectrum at the pole than about 0.03 km, they were rejected for fear that some instrumental displacement had occurred and that possibly the other latitudes were affected.

10. The plates of Series I and III were measured by Plaskett on a Repsold measuring engine with an eyepiece micrometer, while those of Series II were measured by DeLury on a Toepfer measuring machine with 300 mm screw. The lines measured in Series I and III at λ 5600 and in Series II at λ 4250 are given with intensities, velocity constants, etc., in the following tables. Four settings are made on a line in the center strip and two each on the outside

TABLE I

LINES IN λ 5600 REGION

No.	Wave-Length	Element	Intensity	Velocity Constant	No.	Wave-Length	Element	Intensity	Velocity Constant
1...	5506.095	*Mn*	1	19.336	11..	5598.524	*Fe*	1	18.801
2...	5514.563	*Ti*	2	19.289	12..	5601.505	*Ca*	3	18.788
3...	5514.753	*Ti*	2	19.287	13..	5624.769	*Fe*	3	18.653
4...	5528.641	*Mg*	8	19.207	14..	5638.488	*Fe*	3	18.575
5...	5544.157	*Fe*	2	19.118	15..	5658.097	*Y*	2	18.461
6...	5560.434	*Fe*	2	19.024	16..	5682.869	*Na*	5	18.320
7...	5562.933	*Fe*	2	19.010	17..	5684.710	*Si*	3	18.309
8...	5578.946	*Ni*	1	18.919	18..	5686.757	*Fe*	3	18.297
9...	5582.198	*Ca*	4	18.899	19..	5688.436	*Na*	6	18.288
10...	5590.343	*Ca*	3	18.852					

strips and, after measurement of all the lines, the plate is reversed on the machine and remeasured. This diminishes the danger of systematic errors and also, as the lines are viewed in the opposite direction in the two cases and the number of settings doubled, the accidental errors.

TABLE II

LINES IN λ 4250 REGION

No.	Wave-Length	Element	Intensity	Velocity Constant	No.	Wave-Length	Element	Intensity	Velocity Constant
1....	4196.699	*La*	2	26.906	9..	4257.815	*Mn*	2	26.400
2....	4197.257	*C*	2	26.902	10..	4258.477	*Fe*	2	26.394
3....	4216.136	*C*	1	26.745	11..	4266.081	*Mn*	2	26.331
4....	4220.509	*Fe*	3	26.710	12..	4268.915	*Fe*	2	26.296
5....	4225.619	*Fe*	3	26.666	13..	4276.836	*Zr*	2	26.243
6....	4232.887	*Fe*	2	26.606	14..	4290.377	*Ti*	2	26.133
7....	4241.285	*Fe-Zr*	2	26.502	15..	4291.630	*Fe*	2	26.122
8....	4246.996	*Sc*	5	26.490					

The lines in the yellow-green region were selected to include as many elements as possible among the limited number of measurable lines in the region. Some, such as the lines of *Mn*, *Ti*, *Si*, are not of very good quality for measurement but were included in order to give evidence in regard to question *c*, Section 3, above. In the violet region from No. 4 to No. 13, inclusive, are the ten lines selected to be measured by all the observatories co-operating in this work and the other five are lines which Adams and Lasby[1] found gave systematically higher or lower values of the rotation than the general reversing layer. The column "Velocity Constant" gives the half-value of the multiplier required to reduce the millimeter displacement to kilometers per second and will evidently give the observed velocity of the sun's limb. These multipliers are readily determined, in the well-known way, when the linear dispersion at the region is known. As the grating gives practically a normal spectrum over the narrow limits used, it is sufficient to determine this dispersion, which is about 0.70 Å. per millimeter at λ 5600 and 0.75 Å. at λ 4250, for five or six lines over the region used. When these values and the multipliers are plotted on cross-

[1] Adams and Lasby, *An Investigation of the Rotation Period of the Sun by Spectroscopic Methods*, p. 119.

section paper they are found to lie within the errors of observation on a straight line, and the constants for all the lines measured can be at once read off.

REDUCTION OF MEASURES

11. The observed or measured velocities are the radial components of the actual velocities at certain points on the sun's disk whose latitudes can be readily computed, and it is hence necessary to know the angle of inclination between the radius vector and the direction of motion at the point in order to apply the necessary corrections, the further correction for the motion of the earth in its orbit being made to obtain the sidereal rate. In the early observations, by Dunér and Halm, of the rotation of the sun by the spectroscopic method, the measurements were made at the limb and the computations and corrections were straightforward. When, however, as in Adams' work and our own, the observed points are some distance within the limb, the matter is not quite so simple. Adams' method of reduction depends upon projecting the observed points radially to the limb and obtaining the corrections by Dunér's methods and tables, but this assumes the rotation of the sun to be that of a solid body, which is of course not the case. A further correction is therefore necessary for the difference in angular velocity at the observed and computed points. Nearly all of Adams' plates were made with the observed points close to the limb and this final correction is in the majority of cases inappreciable and only reaches in a few plates, around latitudes 45° and 60°, 0.01 km per second. Nevertheless, as it is always in the same direction, it should be applied. This is especially the case in our own observations where the distance from the sun's limb is frequently much greater and where the value of the correction may reach 0.03 km per second. Two methods have been followed here in reducing the observed to the actual velocity. The first method consists in applying a correction to Adams' method for the change in angular velocity, thus obtaining the sidereal rate at the radially projected point on the limb. The second method determines the correction to be applied to reduce the measured to the sidereal velocity at the actually observed points on the disk. This entails

the determination of the heliographic latitude and longitude and of the angles between the direction of motion and the radius vector at the observed points, the division of each measured velocity into two parts proportional to the angular velocities at the two observed points, and the sidereal corrections. By these two correction methods, three reduced values (at three different latitudes) of each measured velocity displacement are obtained, but of course this does not increase the information obtained from the observations. A comparison of the residuals from the two correction methods (Table VII) shows that so far as accuracy is concerned it is immaterial which is employed.

SUMMARY OF MEASURES

12. It is impossible within the limits of this paper to give the separate measures for each spectrum or even the measured and reduced velocities for each separately, so only a summary of the mean values at each observed latitude is given in the following table. In Series I these mean values are obtained from the measures of 19 spectra at the latitudes 0° to 75° inclusive and 18 spectra at the latitudes 80° and 85°. These were observed on various dates between June 15 and August 1, 1911. In Series II the mean values are obtained from the measures of 16 plates at each latitude, observed between August 10 and September 11, 1911. In Series III at each of the three latitudes 0°, 30°, 60°, 24 plates were measured which were observed between October 3 and October 9, 1911. In Series I the 19 lines given in the preceding tables were measured on 14 of the plates. On the remaining 5 and on the 18 at 80° and 85° latitudes, 8 of the best defined lines only were measured. This number was reduced to diminish the labor of measurement, as the measures of the 14 plates were sufficient to show, as will be seen later, that any differences of rotational value for different elements were accidental in character. Furthermore, even with the reduced number of lines, the probable error of a plate as determined from the internal agreement among the lines was less than half the probable error obtained from the agreement between the plates. In Series II, however, owing to the higher probable error of measurement all the lines were measured

throughout and in Series III also on account of the systematic difference previously found for several of the lines by Adams.

In these summaries ϕ and V are the latitudes and reduced velocities at the points radially projected through the observed points to the limb (first correction method), and ϕ_1, ϕ_2, V_1, V_2 the latitudes and reduced velocities at the actually observed points (second correction method).

TABLE III

SUMMARY OF MEASURES

MEAN VALUES

Series I—λ 5600—Measured by Plaskett

Position Angle	Measured Velocity	ϕ	V	ϕ_1	V_1	ϕ_2	V_2	P. E. Plate	P. E. Mean
90° ...	1.824	0° 2′	2.017	1° 1′	2.018	0° 54′	2.018	±0.013	±0.003
75° ...	1.704	15 0	1.886	15 28	1.882	13 37	1.907	.027	.006
60° ...	1.493	29 58	1.652	30 5	1.652	27 56	1.698	.025	.006
45° ...	1.143	44 52	1.273	44 29	1.286	41 56	1.356	.042	.010
30° ...	0.723	59 46	0.809	58 40	0.842	55 16	0.935	.042	.010
15° ...	.363	74 28	.417	71 58	.487	66 44	.628	.026	.006
10° ...	.224	79 53	.247	75 13	.365	72 43	.426	.027	.006
5° ...	.113	84 47	.131	77 55	.310	74 58	.379	.020	.005
Means	......	......	......	......	......	......	......	±0.028	±0.006

Series II—λ 5600—Measured by DeLury

Position Angle	Measured Velocity	ϕ	V	ϕ_1	V_1	ϕ_2	V_2	P. E. Plate	P. E. Mean
90°....	1.659	0° 0′	1.950	2° 46′	1.949	2° 46′	1.949	±0.038	±0.009
74°.8 ..	1.561	14 59	1.834	16 39	1.810	10 57	1.884	.058	.014
59°.7 ..	1.416	29 53	1.654	30 30	1.650	24 16	1.775	.039	.010
44°.5 ..	1.070	44 58	1.251	44 14	1.274	36 56	1.457	.052	.013
29°....	0.633	59 47	0.752	57 31	0.809	48 16	1.061	.039	.010
13°....	.301	74 31	.386	69 25	.525	57 6	0.859	.041	.010
Means	......	......	......	......	......	......	......	±0.044	±0.011

Series III—λ 4250—Measured by Plaskett

Position Angle	Measured Velocity	ϕ	V	ϕ_1	V_1	ϕ_2	V_2	P. E. Plate	P. E. Mean
90°....	1.754	0° 0′	2.012	2° 10′	2.013	2° 10′	2.013	±0.018	±0.004
59°.8 ..	1.424	29 59	1.625	30 33	1.616	25 39	1.725	.020	.004
29°.5 ..	0.688	59 53	0.788	58 25	0.834	50 55	1.037	.039	.008
Means	......	......	......	......	......	......	......	±0.026	±0.005

COMPARISON OF MEASURES

13. Before discussing the velocity and the law of its variation with the latitude it is desirable to attempt an explanation of the systematic difference between the values of the velocity obtained

by Plaskett and DeLury. Although the plates used are not the same, the difference persists when the same plates are measured by the two observers, as will be seen later. In the early measures of rotation plates the fields of the measuring microscopes were left unmasked, but later, as the quantity of light getting through was very fatiguing to the eye, diaphragms were arranged to cut out part of the illumination. This was effected in the case of the Repsold measuring machine used by Plaskett by placing a thin disk with three slots cut in it in the eyepiece of the micrometer just above the focal plane. By this means only the three strips of spectra were visible, the light coming through on the outside and between the strips being occulted by the disk. In the case of the Toepfer machine used by DeLury a single slit of the width of one of the spectral strips was cut in a brass plate which was held by a movable arm attached to the rigid microscope carriage close to the plate and which could be, by a convenient screw, readily moved transversely, so that, in measuring, only one of the three strips of spectrum could be seen at a time. This latter arrangement was devised by DeLury to keep the configuration of the spectrum lines the same for each measurement because he found that his measures were affected by the configuration of the lines in the three strips;[1] and further, to keep himself in ignorance of the magnitude and direction of the displacement so that his measures could in no way be affected by prejudice. For this latter reason also he postponed his computations of the velocities until all the measurements of his series were completed. On the other hand, as only part of the line in the strip being measured can be seen distinctly at one time, as the eye has to move up and down to make the best placing of the wire, it was felt by Plaskett that as the other strips could not be seen while the setting was being made, they could not influence it in any way. Consequently the simpler expedient of a fixed diaphragm occulting only the extraneous light was deemed sufficient by him. This is corroborated by the fact that the difference between Plaskett and DeLury is practically constant at all latitudes (except the pole), although the relative displacement of the lines in the spectra varies widely. On the other hand, the measures of

[1] *Jour. Roy. Astron. Soc. Can.*, **5**, 384–407.

15 equatorial spectra by DeLury both with and without the mask gave a systematically greater value for the former of 0.012 km on the average. It is an open question, since these measures were made at different epochs, whether this difference is to be ascribed to the use of the mask or to a change in the habit of measurement. The measures were made with great care by both observers and in precisely the same way: four settings on the line in the center strip, two on each of the outside strips with the screw moving alternately forward and back, and, after all the lines were measured, repeated with the plate reversed on the carriage. Moreover, as the measures are purely differential (the displacement of one absorption line with respect to another presumably similar absorption line), the presence of this comparatively large systematic difference between the two observers is not readily explainable. Different methods of measurement and various comparisons were made in an attempt to explain or overcome the difficulty, but the difference still persisted practically unchanged in magnitude and sign throughout.

14. It is proposed[1] by DeLury, in order to obtain absolute values of the displacement, which are uncertain under present circumstances, to impress upon the spectra, in addition to the rotation displacement, an arbitrary displacement of, say, the order of a millimeter in magnitude. This would be effected by using a double or broken slit, the central section (of the width of one of the spectral strips) being displaced laterally any desired distance with respect to the body of the slit. If a rotation spectrum be made through such a slit, the displacement will be $S+r$ where r is the rotation displacement and S the displacement due to the slit. If a spectrum of the limb at the pole where there is no rotational displacement be made through this slit the displacement will be S. The measured values of these displacements will be $S+r+e$ and $S+e$ where e is the error of measurement, varying with different observers, yet which should (for each observer) have the same value *in the mean* of a large series of measures, as the two displacements are relatively of nearly the same magnitude. The true value of the rotational displacement will then be

$$S+r+e-(S+e)=r$$

[1] *Ibid.*, **5**, 405.

and in this result personal habits of measurements should be eliminated. Besides the mechanical and observational difficulties in the way of this proposal, however, there is the further one that the accidental error of measurement would be increased and the amount of measuring required doubled. Furthermore, as these spectra could not be taken under identical conditions the possibility of instrumental errors affecting the results is rather a serious one. Even with rotation spectra made directly following one another, on the same plate, and under apparently identical conditions, such errors creep in, as for example in the equator plates of Series III. In plates 860, 865, 867, 869, the differences in the displacements of successive exposures are 0.066, 0.074, 0.051, 0.051 km per second, greater differences than the one in question. Consequently, although the method will be tried later, it was not deemed desirable to delay further the publication of the values obtained but to determine if possible the probable corrections to be applied to the velocities given above.

15. For this purpose all the equator spectra of Series I and 7 of Series III were measured by DeLury and all of Series II by Plaskett to determine systematic differences at the equator. In addition, to see how this difference varied with the latitude, 5 complete plates (7 latitudes on each) of Series I were measured by DeLury and 5 complete plates of Series II by Plaskett. Two representative plates of Series I, Nos. 813 and 820, were sent to Mt. Wilson and were kindly measured by Mr. Adams and Miss Lasby in order to compare Ottawa and Mt. Wilson measures. A tabulation of these comparisons, the measures of No. 820 being given in detail, not only serves to show the differences in velocity obtained by different measurers from the same plates, which appear to be generally systematic in character, but indicates also the accidental errors of measurement to be looked for. The measures of plates 813 and 820 show the great differences in accuracy of setting, for the probable error of setting on a single line varies on the average from 0.008 by Miss Lasby to 0.019 by Adams and Plaskett and to 0.052 km per second by DeLury, equivalent in linear values to 0.0004, 0.001, and 0.003 mm.

TABLE IV

COMPARISONS OF MEASURES. PLATES AT EQUATOR

Series I

Plate	Plaskett	DeLury (Mask)	Diff. P−D
772	1.812	1.770	0.042
777	1.840	1.814	.026
779	1.850	1.839	.011
782	1.854	1.832	.022
784	1.818	1.740	.078
787	1.848	1.786	.062
789	1.776	1.770	.006
796	1.841	1.805	.036
804	1.833	1.748	.085
813	1.858	1.845	.013
814	1.801	1.794	.007
817	1.839	1.823	.016
819	1.809	1.784	.025
820	1.792*	1.750*	.042
821	1.806	1.744	.062
822	1.800	1.734	.066
826	1.800	1.709	.091
827	1.815	1.713	.102
831	1.840	1.823	.017
Means	1.823	1.780	0.043

Series II

Plate	Plaskett	DeLury (No Mask)	DeLury (Mask)	Diff.	Diff. P−D (Mask)
833	1.770	.....	1.715	.....	0.055
834	1.807	1.765	1.771	6	.036
836	.....	1.549	1.565	16	.....
837	1.660	1.608	1.615	7	.045
838	1.694	1.651	1.664	7	.030
839	1.703	1.651	1.682	31	.021
842	1.731	1.644	1.651	7	.080
843	1.786	1.732	1.763	31	.023
844	1.709	1.690	1.671	−19	.038
845	1.695*	1.611*	1.620	9	.075
846	1.669	1.639	1.626	−13	.043
847	1.694	1.574	1.605	31	.089
848	1.705	1.632	1.643	11	.062
849	1.638	1.580	1.619	39	.019
851	1.772	1.691	1.705	14	.067
852	1.679	1.607	1.616	9	.063
Means	1.711	1.642	1.658	0.012	0.050

Series III

Plate	Plaskett	DeLury	Diff. P−D
859	1.764	1.791	−0.027
860	1.733	1.721	.012
861	1.780	1.776	.004
865	1.710	1.674	.036
866	1.774	1.768	.006
867	1.766	1.716	.050
869	1.719	1.752	−.033
Means	1.749	1.742	+0.007

* Mean values.

TABLE V

COMPARISONS OF MEASURES. PLATES WITH ALL LATITUDES

SERIES I

Plate	Observer	0° Meas-ures	0° Diff.	15° Meas-ures	15° Diff.	30° Meas-ures	30° Diff.	45° Meas-ures	45° Diff.	60° Meas-ures	60° Diff.	75° Meas-ures	75° Diff.	90° Meas-ures	90° Diff.
813	P	1.858		1.645		1.468		1.193		0.643		0.351		−0.001	
813	D	1.827	+31	1.627	+18	1.456	+ 12	1 170	+23	.635	+ 8	.331	+20	+ .034	−35
814	P	1.801		1.670		1.455		1.171		.692		.370		+ .029	
814	D	1.805	− 4	1.599	+71	1.402	+ 53	1.117	+54	.670	+22	.343	+27	+ .063	−34
817	P	1.839		1.651		1.520		1.195		.771		.373		− .006	
817	D	1.828	+11	1.578	+73	1.458	+ 62	1.138	+57	.743	+28	.362	+11	+ .017	−23
819	P	1.809		1.722		1.440		1.134		.689		.304		− .044	
819	D	1.787	+22	1.767	−45	1.452	− 12	1.118	+16	.678	+11	.318	−14	+ .081	+37
820	P	1.799		1.702		1.504		1.125		.652		.415		+ .001	
820	D	1.757	+42	1.675	+27	1.404	+100	1.055	+70	.676	−24	.345	+70	+ .006	− 7
Mean diffs. P–D			+20		+29		+ 43		+44		+ 9		+23		−12

SERIES II

Plate	Observer	0° Meas-ures	0° Diff.	15° Meas-ures	15° Diff.	30° Meas-ures	30° Diff.	45° Meas-ures	45° Diff.	60° Meas-ures	60° Diff.	75° Meas-ures	75° Diff.	90° Meas-ures	90° Diff.
839	P	1.683		1.501		1.372		1.156		0.696		0.421		+0.056	
839	D	1.682	+ 1	1.529	−28	1.320	+ 52	1.126	+30	.681	+15	.419	+ 2	+ .015	+41
842	P	1.717		1.519		1.499		1.132		.723		.247		− .007	
842	D	1.651	+66	1.454	+65	1.482	+ 17	1.065	+67	.682	+41	.246	+ 1	− .028	+21
843	P	1.786		1.593		1.398		1.087		.624		.315		+ .025	
843	D	1.763	+23	1.580	+13	1.369	+ 29	1.050	+37	.595	+29	.270	+45	+ .043	−18
844	P	1.709		1.595		1.425		1.059		.630		.261		+ .001	
844	D	1.671	+38	1.568	+27	1.371	+ 54	1.064	− 5	.588	+42	.246	+15	− .020	+21
845	P	1.679		1.601		1.410		1.048		.619		.230		+ .023	
845	D	1.633	+46	1.574	+27	1.411	− 1	1.017	+31	.569	+50	.213	+17	+ .011	+12
Mean diffs. P–D			+35		+21		+ 30		+32		+35		+16		+15
Mean diffs. Series I and II.			+27		+25		+ 36		+38		+22		+20		+ 1

TABLE VI

COMPARISON OF OTTAWA AND MT. WILSON MEASURES

Plate 813

Latitude	Mean Measured Velocities			Probable Error Single Line		
	Plaskett	DeLury	Lasby	Plaskett	DeLury	Lasby
0°.........	1.858	1.827	1.854	±0.010	±0.043	±0.007
15..........	1.645	1.627	1.710	.021	.068	.010
30..........	1.468	1.456	1.479	.015	.046	.012
45..........	1.193	1.176	1.130	.019	.060	.009
60..........	0.643	0.635	0.644	.023	.059	.005
75..........	.351	.331	.334	.025	.050	.006
Means..	....	....	....	±0.019	0.054	0.008

Plate 820

No. of Line	0°						15°		
	Plaskett 1	Plaskett 2	DeLury 1	DeLury 2	Lasby	Adams	Plaskett	DeLury	Lasby
1.....	1.852	1.823	1.818	1.869	1.850	1.797	1.725	1.784	1.758
2.....	1.849	1.814	1.721	1.622	1.847	1.821	1.729	1.635	1.746
3.....	1.790	1.854	1.711	1.836	1.851	1.876	1.739	1.790	1.778
4.....	1.753	1.783	1.823	1.823	1.854	1.817	1.687	1.508	1.761
5.....	1.824	1.847	1.812	1.699	1.841	1.785	1.746	1.719	1.790
6.....	1.826	1.776	1.729	1.725	1.868	1.779	1.688	1.655	1.751
7.....	1.783	1.744	1.808	1.762	1.861	1.806	1.696	1.692	1.759
8.....	1.810	1.765	1.733	1.757	1.861	1.774	1.696	1.697	1.767
9.....	1.795	1.782	1.671	1.682	1.855	1.800	1.696	1.695	1.742
10.....	1.780	1.770	1.727	1.854	1.854	1.770	1.696	1.642	1.758
11.....	1.764	1.754	1.628	1.693	1.855	1.776	1.713	1.676	1.765
12.....	1.830	1.780	1.707	1.637	1.871	1.793	1.712	1.621	1.762
13.....	1.787	1.749	1.746	1.805	1.839	1.786	1.721	1.635	1.755
14.....	1.823	1.805	1.804	1.779	1.851	1.808	1.699	1.745	1.765
15.....	1.806	1.824	1.787	1.818	1.848	1.855	1.705	1.759	1.749
16.....	1.756	1.746	1.663	1.709	1.841	1.767	1.650	1.606	1.763
17.....	1.778	1.781	1.677	1.788	1.835	1.767	1.653	1.613	1.762
18.....	1.760	1.751	1.735	1.745	1.856	1.785	1.706	1.769	1.743
19.....	1.808	1.749	1.827	1.721	1.829	1.802	1.685	1.576	1.746
Means..	1.799	1.784	1.744	1.757	1.851	1.798	1.702	1.675	1.758
P.E. line	±0.020	0.023	0.043	0.047	0.007	0.019	0.017	0.052	0.008

16. The comparison of plates at the equator shows a systematic difference for measures of the same plates of 0.046 km per second. When the five complete plates of Series I and II are compared it is found that in these plates the average difference at the equator

is smaller about 0.027 and that this remains unchanged practically for all latitudes except the pole. This shows that the difference is evidently not due to any effect of the magnitude of the displacement of the lines of one strip with respect to the other, else it should vary with the latter, which changes from 0.1 mm at equator to about 0.017 mm at 75°. It may be said therefore that Plaskett measures the displacements from 0.03 to 0.05 km per second higher than DeLury in the region at λ 5600. The peculiar nature of the difference P−D at the pole should not pass without comment. The mean value of this difference is −0.001. Although it is of the same sign as the other differences in the Series II plates, it is of the opposite sign in Series I and is hence not systematic as at the other latitudes, and it might therefore be regarded as evidence that the magnitude or sense of the displacement influences the measures of one or both of the observers. Owing to the method of measurement used by DeLury, he would seem to be less likely to be influenced in this way. When we compare the measures in the λ 4250 region we find that the difference found in the λ 5600 region nearly vanishes, being only 0.007 km, scarcely large enough, considering the few plates measured by DeLury, to be deemed systematic. The spectra in the λ 4250 region are much more easily measurable than at λ 5600. Not only is the grain of the plate finer but the lines themselves are much more uniform in character and better defined. Consequently it seems likely that the large difference between the two measures in the λ 5600 region depends in some way upon the character of the lines for measurement. Although the probable error of measurement of a single line, given for plates 813 and 820 above, for Plaskett is only about a third of that for DeLury, ±0.019 and 0.054 km per second, and hence the former's measures should be considered of greater weight, yet that does not settle the question of the correct value of the velocity. Possibly some information may be obtained from the Mt. Wilson measures.

17. Mr. Adams and Miss Lasby have had greater experience than anyone else in the measurement of photographic rotation spectra and their measurements should be given great weight. Yet

when we come to make comparisons, Table VI, plates 813 and 820, we find practically the same difficulties and the same differences as between the writers. For example, in plate 820 at the equator we have Miss Lasby's value 1.851, Mr. Adams' 1.798, Plaskett's 1.799 and 1.784, DeLury's 1.757 and 1.744. Indeed, in several cases Miss Lasby's value is as much higher than Plaskett's as his is than DeLury's. On the other hand, in plate 813, 45°, it is lower than both and in plate 813, 60° and 75°, all three are practically the same. When we compare these differences with the probable error of measurement of the plates, less than one-quarter of the probable errors of single lines, varying from 0.002 to 0.015, we are forced to the conclusion that they are systematic and personal in nature, but are at a loss to account for their cause.

It is unfortunate that Mr. Adams was unable to measure more than the one spectrum but the close agreement of his result with Plaskett's and the generally higher values of Miss Lasby and lower of DeLury would naturally, from the law of averages, lead to the acceptance of Adams' and Plaskett's measures as probably being nearest to the true values. If such a conclusion be accepted, then it would be necessary to apply a positive correction to DeLury's measures in the λ 5600 region, which, when all the comparisons are taken into account, should be about 0.040 km at the equator and possibly slightly less at the higher latitudes. A further evidence that this is probably the proper course is given by the practical agreement of Plaskett's and DeLury's measures in Series III at λ 4250. As the velocities of rotation obtained by Plaskett from the measures of Series I, II, and III are all practically the same, while those obtained by DeLury are about 3 per cent lower for Series I and II but the same for Series III, the inference is that, in the poorer quality lines in the yellow green, some personal effect causes the difference and that this disappears when the lines become better defined as is the case in the violet. On the other hand, if there be no systematic differences in the measuring of the line displacements by DeLury at the two regions λ 4250 and λ 5600 this would imply a difference in the rates of rotation as determined from lines of different wave-length, a thing which, though in itself not impossible, is perhaps not very probable.

ABSOLUTE VALUE OF VELOCITY. VARIATION OF VELOCITY WITH LATITUDE

18. The above discussion and comparison of measures have shown that it is hardly possible to state exactly the absolute velocity of the rotation of the sun and, furthermore, if, as seems likely, earlier determinations were affected in the same way, they are also uncertain to the same extent, that of the "personal equation" of measurement.

In order to place the preceding summaries of measures in a more convenient form for discussion and comparison, the following tables containing the observed mean linear velocities at the mean latitudes have been compiled. From these linear velocities the observed angular velocities have been directly computed, while the other columns will be explained below.

TABLE VII

Summary

Series I

Latitude	Linear Velocities			Angular Velocities		
	Observed	Computed	Residual (O.−C.)	Observed	Computed	Residual (O.−C.)
0° 2′.....	2.017	2.014	+0.003	14°.32	14°.40	−0°.08
0 54......	2.018	2.014	+ .004	14.33	14.40	− .07
1 1......	2.018	2.014	+ .004	14.33	14.40	− .07
13 37......	1.907	1.928	− .021	13.93	14.17	+ .24
15 0......	1.886	1.910	− .024	13.86	14.12	+ .26
15 28......	1.882	1.905	− .023	13.86	14.10	+ .24
27 56......	1.698	1.679	+ .019	13.64	13.50	+ .14
29 58......	1.652	1.632	+ .020	13.54	13.38	+ .16
30 5......	1.652	1.630	+ .022	13.55	13.37	+ .18
41 56......	1.356	1.328	+ .028	12.94	12.58	+ .36
44 29......	1.286	1.258	+ .028	12.80	12.40	+ .40
44 52......	1.273	1.246	+ .027	12.75	12.38	+ .37
55 16......	0.935	0.951	− .016	11.65	11.66	− .01
58 40......	.842	.854	− .012	11.50	11.44	+ .06
59 46......	.809	.823	− .014	11.41	11.37	+ .04
66 44......	.628	.625	+ .003	11.29	10.97	+ .32
71 58......	.487	.481	+ .006	11.17	10.73	+ .44
72 43......	.426	.460	− .034	10.18	10.70	− .52
74 28......	.417	.413	+ .004	11.06	10.66	+ .40
74 58......	.379	.399	− .020	10.37	10.62	− .25
75 13......	.365	.392	− .027	10.16	10.61	− .45
77 55......	.310	.320	− .010	10.51	10.53	− .02
79 53......	.247	.267	− .020	9.98	10.46	− .48
84 47......	.131	.137	− .006	10.23	10.37	− .14

TABLE VII—*Continued*

Series II

Latitude	Linear Velocities Observed	Linear Velocities Computed	Linear Velocities Residual (O.−C.)	Angular Velocities Observed	Angular Velocities Computed	Angular Velocities Residual (O.−C.)
0° 0′.....	1.950	1.971	−0.021	13°84	14°04	−0°20
2 45......	1.949	1.969	− .020	13.85	14.03	− .18
10 56......	1.884	1.917	− .033	13.62	13.89	− .27
14 59......	1.834	1.870	− .036	13.48	13.78	− .30
16 40......	1.810	1.847	− .037	13.41	13.72	− .31
24 16......	1.775	1.716	+ .059	13.82	13.37	+ .45
29 53......	1.654	1.596	+ .058	13.54	13.04	+ .50
30 30......	1.650	1.583	+ .067	13.60	13.01	+ .59
36 56......	1.457	1.424	+ .033	12.94	12.60	+ .34
44 14......	1.274	1.230	+ .044	12.62	12.10	+ .52
44 48......	1.251	1.215	+ .036	12.52	12.06	+ .46
48 23......	1.061	1.115	− .054	11.08	11.81	− .73
57 6......	0.859	0.871	− .012	11.23	11.23	0
57 32......	.809	.859	− .050	10.70	11.20	− .50
59 47......	.752	.796	− .044	10.61	11.06	− .55
69 26......	.525	.532	− .007	10.61	10.54	+ .07
74 31......	.386	.397	− .011	10.37	10.33	+ .04

Series III

Latitude	Linear Velocities Observed	Linear Velocities Computed	Linear Velocities Residual (O.−C.)	Angular Velocities Observed	Angular Velocities Computed	Angular Velocities Residual (O.−C.)
0° 0′.....	2.012	2.020	−0.008	14°28	14°33	−0°05
2 10......	2.013	2.018	− .005	14.30	14.33	− .03
25 39......	1.725	1.720	+ .005	13.59	13.54	+ .05
29 59......	1.625	1.619	+ .006	13.32	13.27	+ .05
30 33......	1.616	1.605	+ .011	13.32	13.23	+ .09
50 55......	1.037	1.046	− .009	11.68	11.78	− .10
58 25......	0.834	0.830	+ .004	11.30	11.26	+ .04
59 53......	.788	.789	− .001	11.15	11.16	− .01

19. From these mean values, about one-third of which are due to Method I of reduction and two-thirds to Method II, the law of variation of latitude has to be obtained. Many different forms containing both sine and cosine terms of the latitude in different powers were tried and, although some gave close agreement, none on the whole were as good as the simple Faye formulae

$$V=(a+b\cos^2\phi)\cos\phi$$
$$\xi=a'+b'\cos^2\phi.$$

Using the method of least squares to determine the constants, the following formulae were obtained:

$$\text{Series I} \begin{cases} V=(1.504+0.509 \cos^2 \phi) \cos \phi \\ \xi=10°34+4°06 \cos^2 \phi \end{cases}$$

$$\text{Series II} \begin{cases} V=(1.448+0.523 \cos^2 \phi) \cos \phi \\ \xi=10°04+4°00 \cos^2 \phi \end{cases}$$

$$\text{Series III} \begin{cases} V=(1.421+0.599 \cos^2 \phi) \cos \phi \\ \xi=10°10+4°23 \cos^2 \phi \end{cases}$$

From these formulae the values in columns headed "Computed" and "Residual" in the preceding tables, Table VII, were obtained. The residuals in Series I and III are satisfactorily small and show no tendency to systematic arrangement of sign. In Series II, however, they are considerably larger and systematically grouped as to sign, indicating the necessity of an additional term in the Faye formula.

If the observations of Series I and III are grouped together we get formulae which represent the observations in both series nearly as well as the separate formulae. The difference between the formulae for Series I and III above is probably due to the small number of latitudes observed (only three) in Series III, in which case a small deviation of one of the values would make a large change in the coefficients. The formulae from both series

$$\text{Series I and III (combined)} \begin{cases} V=(1.483+0.532 \cos^2 \phi) \cos \phi \\ \xi=10°32+4°05 \cos^2 \phi \end{cases}$$

may therefore be considered as the formulae obtained from Plaskett's measurements. Series II is not included in this on account of the systematic difference and because another term would be necessary to obtain reasonable agreement between the observed and computed values. However, if we compare the coefficients from Series II with those from Series I and III combined, we find them practically the same except for the difference in the first terms, which is in line with what has been found by comparison of the measures. Moreover this difference, when the necessary allowance is made for the difference of the coefficients of the second terms, is 0.044 km or 0°33, which is not far from the assumed 0.040 km.

20. For convenience of comparison the previously obtained formulae are tabulated beside those just given and we at once notice a remarkable similarity between the Ottawa and Mt. Wilson coefficients.

TABLE VIII

FORMULAE FOR SOLAR ROTATION

Observer	Linear Velocities	Angular Velocities
Dunér	....	$10°60+4°21 \cos^2 \phi$
Halm	....	$12.03+2.50 \cos^2 \phi$
Adams (1906–1907)	$(1.575+0.480 \cos^2 \phi) \cos \phi$	
Adams (1908)	$(1.507+0.546 \cos^2 \phi) \cos \phi$	$10.57+4.04 \cos^2 \phi$
Adams (mean)	$(1.550+0.501 \cos^2 \phi) \cos \phi$	$11.04+3.50 \cos^2 \phi$
Plaskett (1911)	$(1.483+0.532 \cos^2 \phi) \cos \phi$	$10.32+4.05 \cos^2 \phi$
DeLury (1911)	$(1.448+0.523 \cos^2 \phi) \cos \phi$	$10.04+4.00 \cos^2 \phi$

This is especially the case with the 1908 Mt. Wilson determination and the mean formulae from Series I and III, where, in the angular form, the difference is only in the constant term. In the linear form also they are quite similar and their agreement in both forms is so marked as compared with the widely different coefficients obtained from the 1906–1907 Mt. Wilson observations as to confirm the presence of some systematic error in the latter, suspected by Adams, and to indicate the substantial accuracy of the law of variations obtained.

The daily angular value of the rotational velocity has been computed from the empirical formulae given in the preceding table for the latitudes from the equator to the pole by intervals of 5°. A column containing the results of Storey and Wilson[1] at Edinburgh is added and a column for the velocities of sun-spots, the means from three formulae given in Adams'[2] work. Further, the linear velocities from Adams' 1908 and Plaskett's formulae have been computed and are given in the last two columns (Table IX).

The agreement of Dunér's, Adams' 1908, and the Ottawa values except for small and nearly constant angular differences is quite striking and gives good grounds for the belief that the law of variation with latitude is represented to a high degree of accuracy by a Faye formula with coefficients approximately the same as those given in these three formulae.

21. In regard to the absolute value of the rotational velocity the question cannot be regarded as by any means settled. Considering the velocity values at the higher latitudes, we find that

[1] *M.N.*, **62**, p. 674.

[2] Adams and Lasby, p. 118.

TABLE IX

VELOCITIES OF ROTATION

Latitude	Daily Angular Velocities								Linear Velocities	
	Sun-Spots	Dunér	Halm	1906-1907 Adams	1908 Adams	1908-1910 Storey and Wilson	1911 Plaskett	1911 DeLury	1908 Adams	1911 Plaskett
0°	14°.40	14°.81	14°.53	14°.63	14°.61	14°.81	14°.37	14°.05	2.053	2.015
5	14.38	14.78	14.50	14.59	14.58	14.72	14.34	14.02	2.041	2.003
10	14.31	14.68	14.46	14.50	14.49	14.59	14.25	13.93	2.007	1.968
15	14.20	14.53	14.37	14.37	14.34	14.46	14.10	13.78	1.948	1.912
20	14.06	14.32	14.24	14.17	14.13	14.32	13.89	13.58	1.869	1.835
25	13.89	14.06	14.09	13.94	13.89	14.15	13.65	13.34	1.772	1.740
30	13.69	13.76	13.90	13.67	13.60	13.97	13.36	13.05	1.659	1.630
35	13.47	13.42	13.70	13.39	13.28	13.74	13.04	12.73	1.535	1.508
40	.....	13.07	13.50	13.09	12.94	13.52	12.70	12.40	1.400	1.375
45	.....	12.70	13.28	12.81	12.58	13.26	12.34	12.05	1.259	1.237
50	.....	12.34	13.07	12.54	12.24	13.01	11.99	11.70	1.113	1.093
55	.....	11.99	12.86	12.30	11.91	12.71	11.65	11.37	0.967	0.950
60	.....	11.65	12.66	12.11	11.58	12.43	11.33	11.05	.821	.807
65	.....	11.35	12.48	11.97	11.29	12.04	11.04	10.76	.677	.666
70	.....	11.09	12.32	11.91	11.05	11.64	10.80	10.52	.538	.528
75	.....	10.88	12.20	11.91	10.84	11.24	10.59	10.32	.399	.392
80	.....	10.74	12.11	12.00	10.69	.....	10.44	10.17	.265	.260
85	.....	10.63	12.05	12.17	10.60	.....	10.35	10.08	.130	.128
90	.....	10.60	12.03	12.43	10.57	.....	10.32	10.05	0	0

Halm and Adams get nearly the same values, Dunér and Storey and Wilson are about 1 per cent higher, Plaskett about 2 per cent lower, and DeLury about 4 per cent lower. But at the higher latitudes Dunér and Adams (1908) agree, Plaskett is 2 per cent lower as before, DeLury about 5 per cent lower, Storey and Wilson are 5 per cent higher, while Halm and Adams (1906–1907) are some 15 or 20 per cent higher. At the equator Plaskett's values are in practical agreement with the motion of sun-spots. As it is generally considered that the reversing layer and sun-spots are at the same level from the practical identity of their spectra, this, so far as it goes, gives weight to the lower value of 14°.4 at the equator. On the other hand, as the latitude increases the sun-spot velocities agree better with the higher values of the reversing layer such as those of Halm and of Adams' 1906–1907 observations.

22. These differences in values may be due to one or more of three causes: (*a*) a variation in the rate of rotation of the sun; (*b*) instrumental errors; (*c*) personal errors of measurement.

a) *Variation in rate of rotation of sun.*—The question of a change in the rotational velocity of the sun, which was raised by Halm,[1] was quite fully discussed by Adams,[2] who reached the conclusion that the evidence to date was against variation. The later values by Storey and Wilson and those obtained here of which the former is higher and the latter lower than Adams' results, would indicate a variation in the rate of rotation were it not for the possibility of small instrumental and the probability of personal measurement errors (Sections 15–17). As it is, until the latter are eliminated, it will be impossible to make any definite statement in regard to either the variation or constancy of the rate. Certainly the possibility of a variation must, until further evidence is available, be taken into account in considering the differences obtained.

b) *Instrumental errors.*—So far as instrumental errors are concerned, although every known precaution was taken to avoid them, it is possible that some small systematic effects may be present in these results. The only means of detecting such an error would be by the comparison of spectra made at the same epoch by different instruments and methods and measured by the same observer but such is not easy to arrange. The differences in value for successive plates taken under, so far as known, identical conditions (previously referred to in Section 16) is most likely due to some sort of instrumental error unless rapid changes in local motions in the reversing layer are responsible. Although these differences are apparently quite accidental, they may nevertheless contain a small systematic deviation.

c) *Personal errors of measurement.*—It has been shown (Sections 15–17) that it is possible, even probable, for such differences as those in question to be obtained on measurement of the same plate by different observers and it seems useless to consider other sources of error until it is possible to eliminate this. Although the difference between Plaskett and DeLury is fairly well determined at λ 5600 as, at present, about 0.04 km per second, sufficient plates in common have not yet been measured to determine the difference between Miss Lasby, by whom most of the Mt. Wilson plates were measured, and the writers. Her measures appear to be somewhat higher on the whole (Section 17) than Plaskett's and the same

[1] *A.N.*, **173**, p. 294.

[2] Adams and Lasby, p. 115.

tendency was shown even more markedly during a visit of the latter to Mt. Wilson in 1910, where comparisons of the measured displacements of several lines on rotation plates at the equator showed that Miss Lasby's measures were always 2 or 3 per cent higher than Plaskett's. If there is this difference, then the actual velocity displacements on the Mt. Wilson and Ottawa plates are approximately the same and it only remains to determine whose measurement is the most nearly correct. At present, however, we shall have to be satisfied with recognizing the presence of personal differences of measurement, as accounting for part at any rate of the differences in velocity obtained.

23. In view of these actual differences of velocity obtained by the different observers and after the discussion of the probable causes of these differences, we can only state that the velocity of the solar rotation as determined from Plaskett's measurements is represented by the formulae

$$V=(1.483+0.532\cos^2\phi)\cos\phi$$
$$\xi=10^\circ.32+4^\circ.05\cos^2\phi$$

and that the angular form differs from Adams' 1908 formula practically only in the constant term and is also in good agreement with Dunér's, and that hence it probably represents very closely the relative velocities at the different latitudes, although the absolute values may be uncertain by, say, 2 per cent.

PROBABLE ERRORS

24. As Adams[1] has already compared his errors of measurement with those of Dunér and Halm, showing the marked advantage of the photographic method, it will suffice here to give the Ottawa values and compare them with Adams'.

The mean probable error of measurement of the velocity from a single line determined by the use of *all* the lines on *all* the plates is

Series I $= \pm 0.024$ km per sec.
Series II $= \pm 0.056$ km per sec.
Series III $= \pm 0.015$ km per sec.

The probable errors in Series I vary for the different plates from 0.010 to 0.040 and in Series III from 0.006 to 0.023. As the

[1] Adams and Lasby, p. 117.

number of lines measured on each plate in the two series has been 19 and 15, respectively, the probable error of an average plate as determined from the internal agreement of the measures is

Series I = ±0.0055 km per sec.
Series II = ±0.012 km per sec.
Series III = ±0.0038 km per sec.

The average probable error of a plate determined from comparisons of the velocities of all plates at the same latitudes and for all the latitudes is

Series I = ±0.028 km per sec.
Series II = ±0.044 km per sec.
Series III = ±0.026 km per sec.

or 5, 4, and 7 times the probable error as determined from the internal agreement of the lines.

These somewhat anomalous results are not unusual, as about the same ratio of probable errors is obtained in stellar radial velocity work and in many other astrophotographic methods, but the cause of this comparatively high ratio cannot be satisfactorily explained. One can imagine that changing instrumental conditions might cause differences in displacement in plates taken on different dates but where, as in the example previously cited, differences of from 0.05 to 0.07 km were found on exposures taken one immediately after the other on the same plate on the same region of the sun and under, so far as known, identical conditions, no explanation, except the not very likely one of rapidly changing proper motions on the sun, can be assigned.

25. In comparing these probable errors with those of Mt. Wilson, only Series III, which is in the same region, λ 4250, as the Mt. Wilson plates, must be considered, for, as the relative probable errors indicate, the lines are of much better quality for measurement than at λ 5600. When the probable errors (in kilometers) are reduced to linear measure they become more than twice as great at λ 5600 as at λ 4250. The probable errors for a single line obtained at Mt. Wilson are

P.E. = ±0.015 km per sec. (1906–1907)
P.E. = ±0.009 km per sec. (1908).

The Ottawa value, as above stated, is ±0.015. It must not be forgotten, however, that the Mt. Wilson values are from one or two plates, the Ottawa from the mean of all the plates; that on the Mt. Wilson plates the lines giving, systematically, velocities differing from the mean were excluded, on the Ottawa plates these and all lines were included; and lastly, that the Mt. Wilson linear dispersion was in 1906–1907, 10 per cent and in 1908, 30 per cent greater than that at Ottawa. Hence it is evident that the probable error of measurement is about the same at the two places. Although the probable error of a plate determined from the agreement among the plates is not given, it is readily computed and for the equator (1908) is ±0.011 km per second as compared with ±0.018 here. This is considerably smaller but yet about five times that obtained from agreement among the lines.

26. It is evident from the ratios of the probable errors that a great many more lines than necessary for the actual determination of the rotation have been measured and that it would be preferable to measure four or five times as many plates with only one-fourth or one-fifth the number of lines, and that even then the probable error obtained from comparison of the plates would be twice that deduced from the internal agreement of the lines. However, in this investigation a larger number of lines was measured for the purpose of determining whether different elements and different lines of the same element give different velocities of rotation.

SYSTEMATIC DIFFERENCES OF VELOCITY FOR DIFFERENT ELEMENTS

27. Considerable attention has been devoted to this phase of the investigation, which is of importance not only because of its interest in the theory of the sun but also because it was one of the questions proposed by the Rotation Committee and because Adams has found some small systematic differences for different elements and his results should be confirmed.

As previously mentioned, in the λ 5600 region the lines were chosen particularly with this point in view and include as large a number of elements as is possible among the limited number available for measurement. Similarly in the λ 4250 region, besides the 10 lines selected for measurement by the committee, 5 other

lines, embracing those found by Adams to give systematic deviations, were included.

28. The following table contains the mean residuals in meters per second obtained from Plaskett's measures of about 14 plates and DeLury's measures of 16 plates at λ 5600. The first three columns contain the wave-length, source, and intensity of the lines measured. The first column under each observer contains the number of measures on which the residuals in the next two columns depend; the second columns the average residual without regard to sign, and the last columns the mean residual when the sign is taken into account, or the algebraic residual.

TABLE X

MEAN RESIDUALS

λ 5600—Series I and II

Wave-Length	Element	Intensity	Plaskett, Series I			DeLury, Series II		
			Number of Measures	Mean (Numerical)	Mean (Algebraic)	Number of Measures	Mean (Numerical	Mean (Algebraic)
5506.095...	*Mn*	1	97	32	+ 3	112	85	+20
5514.563...	*Ti*	2	81	36	+ 1	112	82	+ 3
5514.753...	*Ti*	2	97	41	+ 9	112	71	− 3
5528.641...	*Mg*	8	97	27	− 9	112	83	− 6
5544.157...	*Fe*	2	97	28	− 5	112	66	− 6
5560.434...	*Fe*	2	97	32	+ 6	112	71	− 2
5562.933...	*Fe*	2	97	25	− 1	112	59	+11
5578.946...	*Ni*	1	97	31	+ 3	112	66	+18
5582.198...	*Ca*	4	97	25	− 3	112	48	+ 6
5590.343...	*Ca*	3	97	24	− 1	112	65	+ 6
5598.524...	*Fe*	1	81	26	− 1	112	65	− 2
5601.505...	*Ca*	3	96	22	+ 1	112	57	− 2
5624.769...	*Fe, V*	4	97	28	+ 9	112	59	−10
5638.488...	*Fe*	3	97	28	+ 5	112	59	+ 1
5658.097...	*Y*	2	97	25	− 1	112	69	+10
5682.869...	*Na*	5	97	28	−14	112	65	−18
5684.710...	*Si*	3	97	27	+ 1	112	69	−21
5686.757...	*Fe* ·	3	81	28	0	112	63	+ 5
5688.436...	*Na*	6	97	25	− 1	112	62	− 2

The trend and magnitude of the mean residuals in Plaskett's measures for the different latitudes, which are, for lack of space, not given here, and the ratio of the mean algebraic to the mean numerical residual, which is except in one case less than one-third, do not indicate any systematic differences for the different lines.

If any lines or elements gave a different velocity than the mean reversing layer, then the mean residuals for the different latitudes for these lines should be of the same sign, should diminish as the latitude increased, and should vanish at the pole. We find on the contrary that none of the lines fulfil this condition but that the residuals bear the appearance of being quite accidental in character. Even in the case of the *Na* line 5682.869, which gives a strong negative residual, we find no decrease with higher latitudes and the mean residual for the pole is much higher than the average, showing that the difference is probably due to something in the line. Again if this sodium line did give a lower value of the velocity, the other sodium line, the last on the list, should also give a negative residual, whereas we see its residuals are entirely accidental. The same condition of affairs is shown by the tabulated residuals from DeLury's measures of Series II in which the mean algebraic is always less than one-fourth the mean numerical residual, although the numbers are higher owing to his higher probable error of measurement.

These considerations form sufficient grounds for the statement that in the region around λ 5600 none of these lines or elements give velocities differing from that of the general reversing layer to a greater extent than can readily be accounted for by accidental errors of measurement.

29. The same thing appears to be the case in the λ 4250 region. The following table contains the mean residuals in meters per second from the 15 lines measured on 24 plates at the equator, at 30° and at 60° latitude; and the final mean numerical and algebraic residuals for the whole 72 measures of each line (Table XI).

Again it will be noticed that in the final values no mean algebraic is one-third as large as the mean numerical residual, even though in three cases the algebraic mean for one of the latitudes is nearly as large as the corresponding numerical mean. The mean residuals obtained from Adams' 1908 values are given in the last column and those lines which Adams claimed gave lower or higher values than the general reversing layer are indicated by the letters L and H in the preceding column. It will at once be seen that the results obtained from the 72 plates measured by Plaskett do not

agree with those of Adams, but on the contrary are generally of the opposite sign. It seems to us therefore that the only safe conclusion to be drawn from the evidence at hand is that any differences found in both Adams' and Plaskett's values are not real differences of velocity but are, if not wholly accidental, rather some personal

TABLE XI

MEAN RESIDUALS—λ 4250

Wave-Length	Element	Intensity	Mean (Numerical)				Mean (Algebraic)					
			0°	30°	60°	All	0°	30°	60°	All		Adams
4196.699...	*La*	2	20	21	26	22	+ 9	+ 3	+8	+7	L	−14
4197.257...	*C*	2	17	22	25	21	+ 2	− 2	+8	+3	L	−11
4216.136...	*C*	2	15	26	22	21	+ 3	+ 4	0	+2	L	−11
4220.509...	*Fe*	3	15	16	17	16	+ 6	− 8	−8	−3	..	+ 4
4225.619...	*Fe*	3	17	24	21	27	− 2	+18	0	+5	..	....
4232.887...	*Fe*	2	13	15	15	14	− 4	− 2	+1	−2	..	+ 2
4241.285...	*Fe, Zr*	2	19	17	14	17	+ 7	0	−6	0	..	....
4246.996...	*Sc*	5	15	19	21	18	− 8	+ 6	+9	+2	..	....
4257.815...	*Mn*	2	15	19	15	16	+ 1	0	0	0	H	+ 9
4258.477...	*Fe*	2	17	17	20	18	−16	− 2	−3	−7	..	+ 5
4266.081...	*Mn*	2	19	19	11	16	+ 2	− 2	−6	−2	..	+ 7
4268.915...	*Fe*	2	16	15	20	17	− 4	− 7	−5	−5	..	+ 7
4276.836...	*Zr*	2	12	20	17	16	0	−14	0	−5	..	+ 2
4290.377...	*Ti*	2	14	16	17	16	− 1	− 1	−2	−1	L	− 4
4291.630...	*Fe*	2	12	22	18	17	+ 2	− 3	+3	+1	..	+ 6

effect in the measurement due possibly to the character of the line. The measures of arbitrary displacements[1] also support the conclusion that such differences as occur are due to errors of measurement. It is unfortunate that no plates containing H_α and *Ca* λ 4227 were obtained here in order to compare the rotational values obtained from these lines with the general reversing layer as was done by Adams, but it seems likely that personal differences at least as high as those occurring in the general reversing layer would be present in the measures of these broad and difficult lines.

SUMMARY

30. The principal conclusions reached from this investigation may be briefly summarized as follows:

[1] *Trans. Roy. Soc. Can.*, 1911, Sec. III, p. 116.

a) The Ottawa values of the solar rotation may be represented by the formulae

$$\left.\begin{aligned} V &= (1.483+0.532 \cos^2 \phi) \cos \phi \\ \xi &= 10^\circ\!.32+4^\circ\!.05 \cos^2 \phi \end{aligned}\right\} \text{Plaskett}$$

$$\left.\begin{aligned} V &= (1.448+0.523 \cos^2 \phi) \cos \phi \\ \xi &= 10^\circ\!.04+4^\circ\!.00 \cos^2 \phi \end{aligned}\right\} \text{DeLury}$$

which are in remarkably good agreement with Dunér's and Adams' 1908 results except for a small and nearly constant angular difference and which probably represent very closely the law of variation with latitude.

b) The absolute velocity of the solar rotation seems to be uncertain by the small difference above referred to, of the order of 2 or 3 per cent, which is apparently due to personal differences in the habit of measurement of the rotational displacements on the plates.

c) The tabulation and discussion of about 3,000 residuals from different lines and elements in the regions measured show that no systematic difference of velocity for different elements is present in the Ottawa plates. The frequently opposite signs of the mean residuals at Ottawa and Mt. Wilson from the same lines (those found at the latter place to give systematically higher or lower velocities) would point to the conclusion that the deviations previously found might have been either accidental or, more probably, personal and due to the character of the lines.

It gives us much pleasure to record here our appreciation of the interest the director, Dr. W. F. King, has taken in this work, of the help he has afforded, and of his willingness to meet the many needs in the matter of apparatus arising in the course of the work.

DOMINION OBSERVATORY
OTTAWA
September 24, 1912

105

A PHOTOGRAPHIC CHART OF THE MILKY WAY AND THE SPIRAL THEORY OF THE GALACTIC SYSTEM

By C. EASTON

A more reliable and more detailed representation of the apparent distribution of the stars than can possibly be furnished by naked-eye observations of the Milky Way is badly needed. I am well aware that the accompanying chart supplies this want only to a very limited extent. First of all, the aid of drawings and descriptions, resulting from visual observations only, cannot yet be dispensed with entirely in estimating the comparative brightness of distant parts of the Milky Way, no complete, perfectly homogeneous series of photographs embracing the whole zone being available; photographs and naked-eye drawings of the Milky Way, however, must picture somewhat different portions of the stellar world. Nevertheless the advantages of the camera above the human eye, for most purposes connected with the study of star-distribution, appealed so strongly to my mind that I preferred the somewhat vexatious labor of combining the available photographs of the Milky Way in a general chart, rather than trying to discriminate between the often conflicting representations given by different observers of the galactic zone, although I have myself spent some years upon a naked-eye study of the Milky Way. As an independent evidence, and for certain well-limited purposes, the drawings, and particularly the "isophotal drawings"[1] of the Milky Way, retain of course their value.

The advantages of photographs are, first, trustworthiness, second, accuracy, third, wealth of detail. On the other hand, it is a drawback that a limited portion only of each photograph—the central parts—can safely be used, and that extensive nebulosities, mingled with the stars, may impair the general traits of their distribution. In combining the photographs, the draughtsman must be careful to avoid those difficulties as best he can.

[1] Houzeau, *Uranométrie générale*, 1878; Easton, "Distribution de la lumière galactique," *Proc. Amsterdam Acad. Sci.*, 1903.

For some years I have been occupied in compiling materials[1] and training myself for the particular work of rendering an interpretation from photographs in a manner similar to that in which the visual observer interprets the corresponding parts of the sky. The result of this work was embodied in four sheets (not yet published) on the original scale furnished by Marth's galactic co-ordinates of the stars, about 17×10 inches.[2] In the accompanying chart, Plate III, constructed after these drawings, Marth's data have also been used. It shows the Milky Way between $+20°$ and $-20°$ galactic latitude. The circular projection renders a certain amount of distortion in the higher galactic latitudes inevitable, but this drawback is not serious since this is the only possible way to get an uninterrupted view of the whole galactic zone in the two hemispheres.

The photographs furnished each a certain amount of detail and the connecting of these details presented no great difficulties, overlapping photographs, often by different authors, being avail-

[1] A detailed list cannot be given here. I made use of the following photographs chiefly: Professor Barnard's "Lick photographs," published for the greater part in successive numbers of the *Astrophysical Journal*, *Popular Astronomy*, and *Knowledge*, since 1894; those of Professor Max Wolf, in *Knowledge*, since 1891, partly in *Die Milchstrasse*, Leipzig, 1908; Russell's photographs of the Southern Milky Way, Sydney, 1890; Pickering's "Harvard Series," covering the whole sky (prints of a number of these plates were prepared for me, in order to bring out the "cloud-forms," by Dr. Neuhauser, in Amsterdam); Bailey's "Southern Milky Way," *Harvard Annals*, **60**, 8, and **72**, 3; some photographs by Wilson, Mrs. Maunder, and others. Through the courtesy of Professor Kapteyn I was able to use a number of Professor Barnard's and Professor Pickering's photographs as early as 1906. I have also to thank the Leyden and Utrecht observatories for lending materials, and I am under great obligations to Professor Max Wolf, who kindly supplied me with a number of glass plates, among them two large-scale photographs especially taken for this purpose. Numerous photographs are scattered in different astronomical publications; the beautiful reproductions in the *Researches on the Evolution of the Stellar System*, **2**, by Professor T. J. J. See, deserving a special mention. Professor Barnard has just now completed a volume containing his "Lick photographs," his volume of "Mount Wilson plates," covering the entire galactic zone up to $-20°$ S.P.D., published by the Carnegie Institution of Washington, being on the way to completion. In most cases, of course, I preferred the use of glass plates to prints.

For a complete catalogue of photographic charts of the sky, up to the year 1908, see *Harvard Annals*, **60**, 9.

[2] These indispensable data were given by Mr. A. Marth in *Monthly Notices*, **53**, 78, 384, 420.

able in almost every case. In this manner the general picture of the Milky Way resulted with a minimum of bias on the part of the draughtsman. In comparing the brightness of widely separated regions it remained, however, necessary to resort to visual estimates, e.g., for the patches in *Sagittarius*, *Aquila*, and *Cygnus*.[1] Although I proceeded very critically, I must confess that this is the weakest part of the work, but it proved unavoidable, even the best photographic series not being sufficiently homogeneous to allow a direct comparison of such distant parts.

The contrast between the bright and dark patches and lanes has intentionally been somewhat exaggerated in drawing. The galactic equator, shown as a circle, was inserted after Marth's position of the galactic North Pole ($12^h\ 40^m + 30°$, 1880). I may, however, remark that the old determinations of the galactic plane, based on insufficient data, have but little value; the photographic chart furnishes a sound basis for a redetermination of the medial line of the galaxy.

The stars were inserted on the scale of their *visual* magnitudes.

Although great care has been taken to make the accompanying chart as complete and as reliable as possible, it will hardly be necessary to state that the author did not aim primarily at completeness and exactness of detail—for which the original photographs can be consulted—but to furnish a general representation of the galactic zone for orientation purposes, in the same way that small-scale maps are used in geography. We owe it to the great skill and the untiring perseverance of Barnard, Wolf, Pickering, Russell, Bailey, and other astronomers, that it is now possible to construct such a "photographic chart of the Milky Way."

There is a question as to whether we are justified in regarding this representation of the stellar distribution in the galaxy as something definite, in its *principal* features. I think we are.

It seems almost incredible that more sensitive plates and longer exposures, showing one or more magnitudes beyond the limits

[1] For the Southern Hemisphere: J. Herschel's description (*Outlines*, § 787); the *Uranometria Argentina;* Houzeau's "isophotal chart" in the *Uran. générale*, and T. W. Backhouse's valuable description in *Publ. of West Hendon House Observatory*, **2**.

attained in the best photographs now available, will give a very different picture of the Milky Way. Theoretical considerations and actual counts of stars[1] point to the probability that the bulk of the small stars, at least of those within the range of the existing instruments, belong to the already known agglomerations, and that they will not fill up the vacancies between these bright parts. The remarkable correspondence between the visual and the photographic Milky Way *in their most characteristic features* is also very significant in this respect. Compare, for instance, my description of the *Aquila-Sagitta* region of the Milky Way (*La Voie lactée*, 1893, pp. 38 f.) with Professor Barnard's photograph (*Astrophysical Journal*, **2**, 35, 1895). Besides, in comparing photographs taken with different lenses and different exposures, we find that not only the main features of the galactic picture are the same, but that very minute details are preserved throughout. A comparison of two photographs of the η *Argus* region may serve as an example. The one is by Russell, 1890 (*op. cit.*, Plate 1) with a lens of 6 in. aperture and 32 in. focus (Dallmeyer), exposure 3 hours; the other by Bailey, 1909 (*op. cit.*, Plate 4) with a Cooke lens of $1\frac{1}{2}$ in. aperture and 13 in. focus, exposure 13 hours. The latter photograph contains perhaps three times as many stars as the first, but virtually all the features, here so vividly depicted, are already indicated in Russell's photograph.

If further researches disclose outlying parts of the system, the present picture will yet hold good for the inner parts, considered as a whole.

I have insisted on this point, because it is evident that the photographic chart would be of little value if we could expect that better instruments and longer exposures would give us quite a different picture of the Milky Way. Happily the photographs of the galaxy, at least of that part of the system now accessible to our researches, may be likened to a sketch of a drawing or map, whose broad outlines remain unaltered as more detail is filled in; rather than to a misty landscape, which presents continually changing

[1] Newcomb, *The Stars*, chap. xx, concl. 4; Easton, *Astronomische Nachrichten*, Nos. 3270, 3803, *Astrophysical Journal*, **1**, 216, 1895.

views as the lifting fog discloses more and more distant houses and groves.

This picture, as we have it now before us, speaks for itself. It leaves next to nothing of the traditional simplicity of the galactic zone, as it is still described in textbooks: "a broad and ample road," vague and rather uniform, split in two over half its circumference. On the contrary, what strikes us most in our photographic chart is the sharp definition of many features, especially near the axis of the Milky Way, and the truly perplexing intricacy of structure, presented by the greater part of the zone. We even have a certain difficulty in recognizing the "two branches." It is true that a winding channel, composed of irregular vacancies and narrow lanes, frequently spanned by bridges of light, runs from about α *Cygni* along the galactic axis to κ *Circini*, where it is connected by a scarcely interrupted, serpentine lane with the "Coal Sack" in *Crux*.[1] This "great rift" (if we keep the name) can be traced on the other side as far as κ *Cassiopeiae* and even μ *Persei;* the remaining portion of the galactic zone presenting a somewhat different character.

Other striking features revealed by this chart are the long "lateral offsets," which often seem connected with groups of bright stars (as Boeddicker has already remarked), forming extensive lateral systems in *Taurus* and *Orion*, in *Cepheus*, and particularly in *Scorpius*. They were of course known beforehand, but the relations of these highly significant formations to the remainder of the zone were never so well exhibited. I would especially draw attention to the streams north and south of the galactic equator, converging to the bright central parts in *Auriga* and *Taurus*, a dark lane running between them, from the *Hyades* to β *Tauri* and θ and β *Aurigae* right across the Milky Way; then to the bright masses and streams encircling the "Northern Coal Sack," between α *Cygni* and α *Cephei*, and returning to the principal branch in *Cassiopeia;* also to a complicated structure, partly described by John Herschel (*Outlines*, § 789), in the constellations *Lupus*, *Scorpius*, and *Ophiuchus*. It is obvious that the extraordinary chasms

[1] Russell draws attention to these lanes, but does not state the connection of the rift with the "Coal Sack."

and bright and dark streams, discovered by Professor Barnard between *52 Ophiuchi* and *Antares*, form but a part of a much more extensive system of interlacing branches—perhaps there are *two* systems present, seen in perspective and projected one against the other—covering the whole region to the north of the galactic axis between η *Serpentis* and β *Centauri*, and certainly related to the principal (southern) branch of the Milky Way, though separated from it by rifts and cavities. It can hardly be by mere chance that most of the streams radiating from the gigantic assemblage of suns about σ and ν *Scorpii* are directed to the very brilliant patches of milky light in *Scutum* and *Sagittarius*. It is obvious that all these formations are interdependent and not merely the outcome of a chance projection; the unity in this apparent complexity is highly significant. This whole region seems the counterpart of the great appendage in the northern Milky Way between β *Cygni* and γ *Ophiuchi*.

Space forbids drawing attention to many other characteristic features, revealed by the photographic chart; the reader may easily discover them for himself. I only wish to enumerate some general divisions of the galactic zone which seem indicated by the characteristics of different parts.

1. The brilliant parts β to γ and a to π *Cygni*, and π *Cygni* to κ *Cassiopeiae*, with the northern extensions over *Cygnus*, *Cepheus*, and *Cassiopeia*, perhaps as far as χ *Persei* and δ *Aurigae*.—The region β *Cygni* to γ *Ophiuchi* seems loosely attached to these parts . . . . *Alpha Region*

2. The southern parts between χ *Persei* and ϵ *Cygni* and as far as γ *Sagittae* . . . . *Beta Region*

3. The varied region in *Sagitta*, *Aquila*, *Scutum*, *Sagittarius*, *Ara*, *Norma*, as far as β *Circini* or a *Centauri*.—The series of bright patches at δ *Sagittae* and δ and ν *Aquilae* seems to hold an intermediate position between the "Alpha" and "Gamma" regions . . . . *Gamma Region*

4. The northern parts between η and ζ *Serpentis*, covering *58*, ρ *Ophiuchi* and δ *Scorpii*, and stretching as far as ρ *Lupi* and a *Centauri* . . . . *Delta Region*

The characteristics of these four regions are easily recognized.

The remaining portion of the Milky Way, between *Crux* and *Perseus*, has many distinguishing characteristics: its light is relatively faint, it has no central rift, but extensive lateral offsets. It can be subdivided in three regions:

5. Roughly between β *Circini* and ζ *Puppis* (225°–285° galactic longitude) we come to a region with extensive ramifications: the somewhat fan-shaped divisions originating about η *Argus*. Already in *Centaurus* and *Crux*, the bright part, narrow and perforated in these constellations, "twists around," so to speak; between *Crux* and η *Argus* and farther on, the light decreases gradually, not from south to north, as in *Sagittarius* and *Norma*, but from north to south. . . . . *Epsilon Region*

6. From 225° galactic longitude (about *a* and *b Mali*) onward, the curved wisps on the north side take another direction, and the southern extensions cease. Toward *Canis Major*, there is nothing left of a central rift, and the central parts, broad and faint, are almost structureless; but in *Puppis* and *Monoceros* we find a maze of wisps and narrow lanes, typical for this region, and noticeable even to the naked eye. . . . . *Zeta Region*

7. The remaining part (165°–110° galactic longitude) is characterized by a rather brilliant though much furrowed central region, with loose and extensive streamers emanating from it, as ribs from a spine; there is a lack of continuity between this region and the outer parts of Alpha Region. . . . . *Eta Region*

In the photographic chart of the Milky Way we now possess solid ground from which to test the spiral theory of the galactic system, enunciated in the *Astrophysical Journal*, **12**, 136, September 1900. Since the appearance of that paper, the spiral form, till then something exceptional in the known stellar world, has revealed itself, in the late Professor Keeler's words, "as the usual or normal accompaniment of contraction in cosmical masses," and the theory of Moulton and Chamberlin testifies that it may lead to important theoretical developments. The researches of Kapteyn, Kobold, Dyson, Eddington, Schwarzschild, and others render a closer inquiry into the fitness of the spiral theory as a working hypothesis highly desirable, as it has lately been shown[1] that the Milky

[1] J. C. Kapteyn, *Proc. Amsterdam Acad. Sci.*, January 1912.

Way is also of fundamental importance in relation to the star-streams.[1]

I am well aware that the great problem of the Milky Way can never be solved in this way, and that we may aim only at a plausible interpretation of known facts, and at a working hypothesis. I may be allowed to repeat what I wrote in the *Astrophysical Journal* twelve years ago: the figure in the center of our plate does not pretend to give even an approximate representation of the galactic system, but only to indicate in a general way how the stellar accumulations might be arranged so as to produce the phenomenon of the Milky Way—on the supposition of a spiral galaxy.

I have now taken as typical a figure intermediate between two of the best known spiral nebulae: *M 51 Canis Venaticum* and *M 101 Ursae Majoris*. The adopted spiral must answer to the condition that the principal features of the Milky Way result freely and plausibly from its projection on the sphere, viewed from the sun (S).

We must take into account:

1. The branches of the spiral are not situated in the same plane.[2] The first branch, stretching from the central nucleus outward, at first rises somewhat above the plane of the chart, but lies below it in its farther course. The second branch lies at first below, then comes nearer to the middle plane, crosses it somewhere in the direction Se, and continues to run above it for the greater part of its course. The central condensation itself would lie somewhat to the north of (above) the middle plane of the galaxy, as adopted by Marth (cf., however, what is said on p. 107).

2. Up to a certain distance from the sun, the stellar agglomerations must seem scattered, so that they cannot produce any milky light.

3. The apparent brightness of the patches and streams of the Milky Way depends not only on the richness of the stellar condensations and strata, but also on our point of view, in S, so that star-streams running almost exactly in our visual line must show as brilliant patches.

[1] Cf. T. J. J. See, *Researches on the Evolution of the Stellar Systems*, **2**, 1910, and Sv. Arrhenius, *Nord und Süd*, January 1912.

[2] Cf. the *Cassiopeia* to *Sagittarius* region of the photographic chart with a spiral nebula seen edgewise, e.g., "*H. V., 1 Ceti*," *Lick Publications*, **8**, Pl. 2.

I have indicated in the spiral figure the divisions α, β, γ . . . . corresponding to the "galactic regions" named above.

The central nucleus of the galactic spiral would project itself between β and γ *Cygni;* the northern continuation of this brilliant part, and the patch about α *Cygni*—a stream seen foreshortened in *D*—may be reckoned as belonging to the central condensation. The feeble interspiral streams to the left of the nucleus, together with the part of the first branch nearest to *S* (and therefore mostly resolved into stars), form the northern outlying parts in *Cepheus* and *Cassiopeia;* the apparent interlacing of these streams, as in *f*, producing the galactic condensations about ξ *Cygni;* they leave between them relatively dark spots, as the "Northern Coal Sack" indicated by *a*.

In the direction *SB*, the greater part of the spiral branch swings round (at κ and ι *Cassiopeiae*), bends somewhat to the south, and pursues its course through *Lacerta* and over ϵ *Cygni* ("Beta Region"). Some outlying streams in the direction *SC* form the branching structure between γ *Cassiopeiae* amd χ *Persei*, which terminates rather abruptly at η *Persei*. Here then ends the "Alpha Region," in this direction.

The second branch proceeds from the central nucleus mainly between β *Cygni* and *Sagitta*, being also in relation with the α *Cygni* stream. Descending in galactic longitude,[1] we find that the "knots" or "nodosities" of the branch in the direction of *F* must appear isolated, and the "fringes," curved away from those "knots," must be seen foreshortened—giving rise to the brilliant patches, somewhat in the shape of crescents or shields (*Scutum!*) from *Sagitta* up to the tail of *Scorpius* ("Gamma Region"). It is worthy of remark that "shield-forms" occur elsewhere in the Milky Way, the most remarkable instance being the series of bright patches between π *Cygni* and μ *Cephei* (Professor Barnard's photographs; see *Knowledge*, March 1894, and *Astrophysical Journal*, 21, 48, 1905).

The spiral branch No. II (see chart) then approaches the sun,

[1] I will use the term "ascending" to indicate, along the galactic axis, the direction in which the galactic longitudes increase, viz., from *Aquila* to *Cassiopeia;* "descending" the opposite direction, viz., from *Aquila* to *Sagittarius*.

broadening and at the same time becoming vaguer and fainter ("Epsilon Region"). The bright stars are exceptionally numerous in those parts, as well as in the opposite direction (*SB* and *SA*), in accordance with the theory.[1]

Where shallow parts separate the "knots" in the convolutions, the lateral rifts, already present between the bright patches in *Sagittarius* and *Aquila*, become more evident. With increasing distance from the sun, beyond *Sc* (*descending*), the breadth of the central parts of the Milky Way again decreases. (The great width of the galactic zone in the regions *Scorpius* to *Sagittarius* and *Auriga* to *Taurus* is due to secondary streams, projected alongside the head-stream.)

We may look upon the "feeler" β *Cygni* to γ *Ophiuchi* as an appendage (H) of the central nucleus A; the series of patches from γ to ν *Aquilae* forming a side-branch (E) of the second convolution.

The interspiral region surrounding S is filled loosely with less condensed streams and strata of stars, whose general direction is tangential to the central nucleus, but which are crossed by others running between the great spiral branches and parallel to these. If those two main directions are everywhere present in the spiral system, as they show in the nebulae (cf. Ritchey's admirable photograph of *M 51*), they probably cause the intersecting streams in *Lupus* and elsewhere, and determine the outlines of the "blocks of light" about γ and μ *Sagittarii*, also of the "caves" near γ *Aquilae* and η *Argus*, etc. May not these two directions in the spiral strata be the expression of Kapteyn's two star-streams penetrating the whole stellar system?

The secondary streams between the branches of the spiral account readily for the aspect of the Milky Way in *Lupus*, *Scorpius*, and *Serpens* ("Delta Region"), and in the opposite direction ("Eta Region"); the bright and probably preponderating stars forming Gould's "belt," inclined at an angle of about 18° to the general plane of the galaxy. The "helium stars" are numerous in these parts. Beginning between *Cassiopeia* and *Perseus*, these

[1] Schiaparelli, "Sulla distribuzione . . . . ," *Publ. Brera*, **34**; Stratonoff, *Publ. Tachkent*, **1**; Easton, "Distribution de la lumière galactique," *Verhand. Amsterdam Acad. Sci.*, **8**, No. 3, p. 31.

strata run first below the general plane, crossing it in the neighborhood of the sun (that is why the points of intersection of Gould's belt with the galactic equator lie about at 90° and 270° of galactic longitude) to rise above the plane, in *Scorpius* and *Ophiuchus*, before uniting with the central parts of the system. Vistas as in *Sd*, between the extensive lateral streams in *Orion* and *Taurus*, are readily explained, and the faintest parts of the Milky Way as a whole, seen from *S*, must of course lie in the direction of *e* (*Perseus*) and *b* (η *Serpentis*).

We may perhaps limit our interpretation of the Milky Way to these remarks; the analogy could easily be pushed farther.[1] No geometrical postulates for the shapes of "stellar spirals" having been found as yet,[2] such an interpretation is always more or less arbitrary. To give an example: the *Scorpius* (Delta) Region might perhaps as well be placed outside as inside the great spiral convolution. My aim was to show that the spiral theory holds good in the face of the more reliable and more detailed picture of the Milky Way, furnished by the photographic method.

Two objections have been raised against this theory. The central nucleus in the nebulae seems much more important in comparison to the remainder than would be indicated by the importance of the bright patches in *Cygnus*. This is true, but in the photographs this preponderance of the nucleus diminishes as the excellence of the photograph increases; i.e., the nucleus itself is often unduly magnified, on a rather poor plate, by adhering portions of the convolutions. I have already insisted on the excessive brilliancy which certain distant portions of the spiral acquire, owing to their running almost exactly in our line of sight. For the rest, the proportion in mass between the nucleus and the branches of a spiral does not seem a serious criterion for a theory, if it is otherwise acceptable.

[1] It is noteworthy that Arrhenius (*Nord und Süd*, January 1912) arrives independently at a somewhat similar structure of the galactic spiral.

[2] Wolf, *Die Milchstrasse*, p. 19. Cf. See, *Evolution*, **2**, 39 f.; also E. v.d. Pahlen, *Astronomische Nachrichten*, No. 4503, and W. Sutherland, *Astrophysical Journal*, **34**, 3, 1911.

I can by no means share the opinion[1] that if the spiral theory is true, the many thousands of nebulae found by Keeler and Wolf must be "external galaxies," so that the confirmation or disproval of this supposition would be a test of the spiral theory. I wish only to remark that nobody will deny the existence of a whirlpool because he sees a number of small eddies in the convolutions of the great one. I think we may safely assume that the great majority of the small spiral nebulae, if not all, form part of our galactic system.

As to the part which those small nebulae play in the galactic system, I made an attempt, some years ago[2] to explain their relations to a spiral galaxy; contending in the first place that the usual idea of a quasi-symmetrical distribution of the nebulae in the two hemispheres, gathering around both galactic poles, is untenable; secondly, that (the *nubeculae* being excluded) the distribution of the nebulae shows maxima in galactic longitude 105° and 280°, about 90° from the supposed central nucleus in *Cygnus*. This peculiar distribution is readily explained if we may admit that our present observations only reveal to us, among those faint objects, those that are relatively near to us, and that the nebulae—perhaps remnants left behind by the rotatory movement of the system—chiefly fill up the space not occupied by the galactic convolutions, their distribution being complementary not to the distribution of the stars but to that of the "vast, irregular" nebulae.

On the other hand, the convolutions themselves seem to be extensively veiled by a stratum of nebulous light, which is perhaps nowhere wanting in the Milky Way proper, except in the "perforations," those singular holes and cracks discovered and studied by Barnard and Max Wolf. I see no evidence whatever of *continuous* masses of opaque matter lying between us and the stars, although such non-luminous matter must be scattered freely throughout the system. It is evident that the more or less luminous nebulosity is intimately connected with the stellar accumulations

[1] Ristenpart in Valentiner's *Handwörterbuch der Astronomie*, 4, 123; cf. Newcomb-Engelmann, *Populäre Astronomie*, 3d ed., p. 618.

[2] Easton, "The Nebulae Considered in Relation to the Galactic System," *Proc. Amsterdam Acad. Sci.*, 1904, p. 189; *Astronomische Nachrichten*, No. 3969.

and strata of the Milky Way. Those clouds, composed of stars and nebulous matter exhibit in various degrees the influence of what W. Herschel already has indicated as the "clustering power," breaking up the coils of the spiral, as we see it in the nebulae *M 101 Ursae Majoris* and *M 33 Trianguli.*

In this "clustering" of the galactic clouds, the star-clusters apparently play an important part. Certain clusters have evidently been formed at the expense of the strata in which they are imbedded (see photographs of the bright patch in *Scutum*). In many other cases, they seem to act as centers for the grouping of the galactic condensations, and it is natural to think that these clusters have far more important masses than is indicated by their size and brightness.

The study of the photographs, together with the theoretical considerations indicated above, lead to a similar inference in regard to certain stars. In the galaxy there must exist a number of stars, immensely surpassing in mass the components of the galactic star-clouds—true miniature suns even in comparison to our own luminary. Those gigantic globes, especially where they are grouped together, as in *Scorpius*, seem to act a leading part in the dismembering of the galactic convolutions.

Certain parts of the galactic zone, probably analogous to the "knots" in the great spiral nebulae, exhibit forms in which the spiral structure, though less apparent than in the small nebulae, and (I think) in the system as a whole, is not altogether wanting. We sometimes come across very curious little spirals in the galactic regions—e.g., to the south of ϵ *Carinae*—but great masses also exhibit this peculiarity. So we find a spiral structure of 150 sq. degrees between α *Crucis* and ϕ *Argus*.[1] In this case, we may note *two* central nuclei, at κ and η *Argus*, so that the term *bihelical* seems more appropriate.

A comparison of the different types of spiral nebulae (*Lick Publications*, **8**, "Keeler Memorial") leads to the conclusion that our galactic system would seem to be already in an advanced stage

[1] Speaking of the bright region in *Scutum*, Professor Barnard wrote (*Astrophysical Journal*, 1, 11, 1895): "I have often received the impression that this huge cloud of stars has been generated by some tremendous whirling motion."

of development, approaching rather to the *M 101* (*Lick*, Pl. 49), than to the *M 51* (*Lick*, Pl. 47) type.

It is natural to think that the Magellanic clouds should be analogous to the small nebula connected with Lord Rosse's spiral *M 51*. The analogy is strengthened by the resemblance in shape of the *nubeculae*, as seen in the well-known photographs of Russell, to the galactic formations about η *Argus* and elsewhere just described.[1] In any event, the actual connection with the main system, so obvious in the nebula, must be slight in the case of our galactic system. It is true that Backhouse[2] saw a wisp of galactic light uniting the Greater Magellanic Cloud with the Milky Way about π *Puppis*, and that, in some photographs, rows of stars seem to connect that *nubecula* with the galactic zone in *Carina*, but the evidence is doubtful. I must add, however, that Stratonoff's chart of the distribution of the nebulae shows a connection between the *nubeculae* and the *nebulae* in the southern hemisphere, and in my opinion the small nebulae form part of the galactic system.

AMSTERDAM
October 1912

[1] I believe that the great structural resemblance between the κ and η *Argus* region and the *Nubecula Major* has never been insisted upon. Russell's photographs Nos. 1 and 14, however, show it at a glance.

[2] Backhouse, *Publ. of West Hendon House Observatory, Sunderland*, **2**, 30.

ON THE OCCURRENCE OF THE ENHANCED LINES OF TITANIUM IN ELECTRIC FURNACE SPECTRA

By ARTHUR S. KING

In the course of an investigation now in progress on the variations in the spectrum of titanium given by different temperatures of the electric furnace, it seemed highly desirable to make a special effort to obtain the enhanced lines and fix their place, if possible, on a temperature scale. In general, enhanced lines are very difficult to produce in the furnace, as is to be expected from the fact that for most metals they appear in the arc only under special conditions, either close to the poles or in an interrupted arc which gives momentary conditions approaching those of the spark. However, the arc shows these lines for some substances much more easily than for others. Thus, the H and K lines of calcium, which are to be classed as enhanced lines, are also among the strongest arc lines. It was shown in a previous paper[1] that they may be obtained in the furnace at a stage considerably below the highest furnace temperatures, and exhibit a rapid rise in intensity with temperature. The enhanced lines of titanium also occur in the arc, some of them being fairly strong, though weaker than many of the arc lines. It has been shown in previous publications[2] that the furnace at high temperatures gives a spectrum for titanium comparing in richness with that of the arc, but the enhanced lines were notably lacking in the furnace. Their strength in the arc, however, gave promise that by forcing the furnace temperatures the enhanced lines could be made to appear. This has been accomplished, and some supplementary experiments have shown several remarkable phenomena bearing directly on the nature of enhanced lines in general which will be reported in this paper.

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 32; *Astrophysical Journal*, **28**, 389, 1908.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 28; *Astrophysical Journal*, **28**, 300, 1908; *Contributions from the Mount Wilson Solar Observatory*, No. 60; *Astrophysical Journal*, **35**, 180, 1912.

In the previous experiments very high temperatures were occasionally employed, but with only moderate dispersion, so that the continuous spectrum given by such a temperature was strong enough to conceal faint lines. An increase of scale was required, combined with brightness of the spectrum. Fortunately, a new plane grating was available, ruled by Anderson on the Rowland machine, which gave very bright spectra in the first and second orders in spite of its small area (6.3×7.2 cm). The second order of this grating was used in the Littrow spectrograph of 30 ft. (9.1 m) focal length, the scale being approximately 0.9 Å. per mm. The photographs were made on Cramer "Crown" plates for the region from λ 4220 to λ 4600, in which range titanium has about forty enhanced lines of varying intensities, but all showing a behavior in arc and spark quite typical of this class.

The tube resistance furnace was used with its parts arranged as described in previous papers, but without any jacketing material around the tube. The chamber was pumped out to a low pressure. The tubes of Acheson graphite first employed were 12.5 mm inside diameter, 18.3 mm outside diameter, and 30.5 cm long, the heated portion being 20.3 cm long. A potential of 30 volts on these tubes gave 1600 amperes, falling to about 1500. Pyrometer readings for several trials gave above 2600° C, when the instrument was sighted at the interior of the tube. This value is probably low, being measured for an open tube. Under such conditions, all of the stronger enhanced lines appeared very distinctly for exposures lasting 3 minutes. A 2-minute photograph, showing less continuous ground, is reproduced as No. 2 of Plate IV. The enhanced lines are plainly visible in the negative, but can hardly be expected to show in the reproduction. No. 1 in this plate is the spectrum for about 2400° and shows no enhanced lines.

We thus have the result that the temperatures attainable in the furnace, when operated in the regular way, will give all classes of titanium lines, and so far as the examination of the visible spectrum has proceeded, it appears that all lines of titanium appearing distinctly in the arc can be obtained in the furnace.

The next step was an attempt to obtain still higher temperature by the use of thinner tubes. Tubes with slightly thicker walls

than those previously used were turned down so as to give a wall about 2 mm thick for 5 cm at the middle of the tube, the powdered titanium carbide being placed in the middle of the thin portion. As surprising effects were soon obtained with these tubes, the experiments will be described in some detail.

A preliminary heating for 1 minute at about 1000 amperes drove off the more volatile substances, but did not melt the titanium. The tube would stand 1400 amperes at 25 volts for several minutes, giving a rich titanium spectrum with the enhanced lines visible. When 30 volts were placed on the tube, the current went above 1600 amperes (the limit of the ammeter scale), remained there a few seconds and began to drop, while the appearance of the image of the tube's interior projected on the slit showed that a violent vaporization of carbon was taking place. When below 1500 amperes the current fell rapidly, showing that the tube was burning through, with the formation of an arc. This arc often held for 5 seconds or more with the ammeter usually registering about 800 amperes, before the ends of the tube were burned so far apart that the arc broke. During this time a voltmeter connected across the furnace terminals remained close to 30 volts, though increasing 2 or 3 volts at the beginning of the arcing stage. With a gradual burning around the circumference of the tube before the formation of the arc, it is doubtful if a high momentary voltage occurred such as would result from a sudden complete break. The voltmeter would not have recorded this and tests have shown that the spectra obtained were not materially influenced thereby.

When the furnace was opened after cooling, the tube was found burned apart near the middle of the thin portion, a space of 3 to 4 mm separating the edges all around the circumference. (Contraction of the parts during cooling may have altered the original interval.) Each section of the broken tube thinned down gradually at each side of the break, beginning about 1 cm from the sharp edge, showing how the carbon had vaporized with decreasing violence at each side of the weak point where the break occurred. An interesting feature was that this vaporization took place almost entirely within the tube, the outside diameter remaining the same up to

the break. This was doubtless due to the strong radiation from the exterior of the tube, no jacket being used.

By making a large number of runs of the furnace with tubes of the same kind, it has been possible to photograph the spectrum given at various stages of the run which, from the time of turning on the 30-volt current to the final breaking of the arc, never lasted more than 30 seconds. Three plates were made for the conditions prior to the breaking of the tube, when it was wearing thin with strong vaporization of the carbon. The slit was covered as nearly as possible just before the instant of break. These plates were underexposed, but the enhanced lines, though faint, were relatively strong in reference to the arc lines as compared to photographs taken under the regular furnace conditions with a long tube of uniform thickness. This evidence indicates a slight relative strengthening of the enhanced lines with rising temperature while furnace conditions may be considered still to exist.

A photograph for which the slit was covered just after the tube burned through is shown as No. 3 of Plate IV. This registers for the most part the conditions preceding the break, but in the short interval in which the plate was exposed during the burning of the tube the enhanced lines appeared so strongly that the integrated effect gives them about the same intensity relatively to the arc lines that they usually have in the carbon arc containing titanium. Number 4 of Plate IV represents a stage more favorable to the enhanced lines. The tube broke sooner than usual and the plate was exposed after the break longer than in the case of No. 3. The enhanced lines are seen to predominate strongly in the spectrum. The relative intensities resemble those of the condensed spark, but the arc lines show the peculiar softness characteristic of furnace lines. Spectrum No. 5 is that given by the carbon arc containing titanium, from which the usual strength of the enhanced lines in the arc may be seen.

It is evident from the way the enhanced lines stand out in No. 4 of Plate IV that conditions during and after the burning through of the tube are especially favorable for the production of these lines. A consideration of what transpires during this part of the experiment shows that we have here a source different in

many respects from the furnace, arc, or spark as regularly operated, and perhaps approaching nearer to the conditions of radiation in solar and stellar atmospheres than is done in any of our laboratory sources. An important point is the high concentration of electrical energy over a very short stretch of the tube. Just before the break the transformer supplies from 40 to 50 kilowatts, a very large part of which must be used over the 5 cm of thin tube, the rest of the circuit being made up of low-resistance connections and the relatively thick tube at each side of the turned-down portion. When the tube has burned very thin at the weakest portion, the rupture must be in the nature of an explosion, in which probably the highest temperature is generated that can be obtained from carbon at the existing pressure. When the tube has burned through around its circumference, the vapor inside whose light falls upon the slit may be considered as surrounded by a wall of arc which must consume nearly all of the 24 kilowatts which the transformer usually supplied for a period of from 5 to 15 seconds. When we consider that the power ordinarily used for an arc in the laboratory is less than 500 watts, the energy consumed in the two sources is seen to be of quite a different order.

While the conditions following the break of the tube are widely different from the regular furnace conditions, the low potential employed furnishes an important difference from the arc or spark. As has been noted, the voltage across the terminals outside the furnace appeared not to rise above 33 volts. Granting the possibility of a high momentary potential which the voltmeter would not follow, experiments have shown definitely that the conditions favorable to the enhanced lines extend through the arcing period and are not limited to the instant of explosion. The most decisive test on this point was made by taking two successive photographs, the exposure for one being made as the tube burned apart, that for the other beginning about 10 seconds later. The second photograph showed all of the enhanced lines in that region, and very little besides.

The foregoing experiments had been made with an image of the interior of the tube about 2 cm in diameter projected on the spectrograph head, the light which passed through the slit (4 mm long)

being taken from the central part of the image. It now occurred to me that there might be a distinct difference between the spectra given by the vapor at the center and by that near the wall where the arc was in action. As the spectrograph shows almost no astigmatism, it was a simple matter to test this. The length of slit used was increased so as to pass across the diameter of the image from wall to wall. When a tube was used at a temperature which did not burn it through, there was no perceptible difference in strength or appearance of the lines from center to wall, so that in the regular operation of the furnace the long slit offers no advantage.

For the conditions when the tube was burned through, however, a difference of the most striking nature appeared. The non-enhanced lines were much weaker in the center of the tube than near the wall, *while the titanium enhanced lines showed almost no change in intensity across the diameter of the tube.* In other words, the enhanced lines were very strong in the middle of the tube relatively to the lines characteristic of the arc. This phenomenon was photographed on six plates with no variation in the general features.[1] The spectra in Plate V were made with the long slit as described, and are reproduced almost full scale. Each spectrum is in two sections. No. 1 is a true furnace spectrum taken with a turned-down tube but with only 25 volts, which did not burn it through in the minute required for the photograph. Nos. 2 and 3 were made by exposing at the moment of break with 30 volts on the tube. The enhanced lines are marked in No. 2. Spectrum No. 3 has less general intensity and the enhanced lines stand out in the middle of the strip more distinctly than in No. 2. The uniformity of intensity across the diameter of the tube makes the selection of the enhanced lines a matter of the greatest ease, and is striking evidence of their dependence on a different physical condition of the vapor than that favorable to the non-enhanced lines and to the carbon flutings,

[1] Note added January 1913:

Supplementary photographs have been taken extending from λ 3900 to λ 5227, in which all of the titanium enhanced lines in this range have been obtained in the broken-tube spectrum. Their intensities relative to the arc lines were found to be fully in accord with those of the lines discussed in this paper.

of which the band with heads at $\lambda\lambda$ 4382, 4371, and 4365 comes out strongly. At λ 4267 appears the spark line of carbon, strong in the center and scarcely visible at the wall of the tube, its behavior being just opposite to that of the carbon bands. This line will be discussed farther on.

The enhanced lines would thus seem to show a distinct lack of dependence upon either temperature difference or potential fall. A temperature difference as we pass from the arcing wall of the tube toward the center is to be expected, and this is borne out by the diminution in intensity of the non-enhanced lines. The carbon flutings also, whose intensity in the regular furnace closely follows the temperature, are much weaker in the center of the tube. Likewise, such potential fall as there is in this low-voltage arc must be much greater close to the wall than at a point in the center of the tube 6 mm distant from the arcing point. If a decided difference in vapor density prevails between center and wall of the tube, the enhanced lines might be less affected by this than the arc lines. It is difficult to say how great a difference of this sort there may be, but the width of the arc lines, which is governed largely by the vapor density, seems to decrease less rapidly toward the center of the tube than does their brightness, which is controlled, for this class of lines, chiefly by the temperature.

The spark line of carbon, λ 4267, shown on Plate V, appeared on all plates taken with the burned-through tube, being strong and unsymmetrically reversed in the center of the tube, and decreasing in strength toward the wall, until it was almost invisible at the edge of the image representing the point nearest to the hot carbon. As has been noted, the vaporization of the carbon was very violent under the conditions of this experiment, but the line does not appear in the furnace when the tube does not break nor in the carbon arc burning with continuous current. Crew and Spence[1] obtained it in an arc in which one terminal was a rotating disc of graphite. They describe its behavior as follows:

A very curious thing happens with this rotating graphite arc, viz., the appearance of the well-known carbon spark line at λ 4267, a *Hauptlinie* of Eder and Valenta. Few lines are so sensitive to rapid changes in E.M.F.

[1] *Astrophysical Journal*, **22**, 199, 1905.

This line is strong in the spark between amorphous carbon poles, *provided they are cold;* if, however, the carbon tips have been heated, the line disappears even from a powerful transformer spark. On the other hand, this line which does not appear in the ordinary carbon arc comes out strong in the rotating graphite arc, where the natural quickness of break which characterizes the graphite arc is accentuated by the motion of the electrode. In other words, this line disappears with great readiness from its natural source (the carbon spark) and appears, on slight provocation, in a source where it might be least expected, the graphite arc. Such capricious behavior might lead one to hesitate in assigning this line to carbon; but its presence in pure Acheson graphite leaves little room for doubt as to its identity.

It is characteristic of enhanced lines in general that they weaken in the spark when the electrodes become hot; also, that such lines in many cases are given by the rotating arc used by Crew,[1] especially when operated in an atmosphere of hydrogen. The decrease of potential gradient in the spark with hot electrodes and the increase due to the sudden break in the rotating arc have been given as the most plausible explanation of this behavior of the enhanced lines. An extended investigation may prove that this conduct of the carbon line λ 4267 is typical of those enhanced lines which do not appear at all in the arc when burning continuously, such lines being a step more difficult of production than the titanium enhanced lines. When we have a source of great energy like the burned tube which is able to produce lines of all classes, the present evidence indicates that the arc lines, the titanium enhanced lines, and such lines as λ 4267 form a sequence for which a high potential gradient is not necessary and the region of highest temperature becomes increasingly unfavorable for the successive classes. The proximity of hot carbon appears as destructive to the carbon spark line in the furnace as in the spark while as we recede from the walls the vapor rapidly becomes better able to give this line.

Another highly interesting phenomenon appears in the spectrum when the tube burns through. The ends of the long lines, given by the vapor close to the arcing walls, show nearly uniform brightness across their width, while as we approach the middle, the red side of the line becomes constantly stronger. If the line is reversed, the dissymmetry of the two sides is very marked; if unreversed,

[1] *Astrophysical Journal*, **12**, 167, 1900.

chiefly the red side remains at the middle. This difference does not show for the titanium enhanced lines, though these lines being narrow and "hard" could not be expected to show it so distinctly if it is present. The spark line of carbon is very unsymmetrically reversed (see Plate V), the red side being almost twice as strong as the violet.

The effect, as compared to the regular appearance of the furnace line, may be seen in Plate VI. Three spectra are shown, taken with the same sort of tubes and the same optical arrangements, the conditions for the third being a combination of those for the first two. No. 1 is a regular furnace spectrum exposed 1 minute. The lines show little difference in intensity from end to end and are symmetrical in structure, whether reversed or not. Traces of the enhanced lines are barely visible in the negative. No. 2 is the broken-tube spectrum taken in 15 seconds, the plate being exposed at the moment of break. The large scale shows to advantage the variable intensity of the arc lines from end to end as compared with the unchanged condition of the enhanced lines, but it is given chiefly to illustrate the difference in structure at center and ends of the arc lines. The reversed lines show almost complete symmetry when given by the vapor near the wall of the tube, while in the middle of the tube little remains of the violet side of the line. The unreversed line $\lambda 4527$ has the same structure narrowed down. No. 3 was made by an exposure of about 1 minute before the break, followed by 12 seconds while the tube was burning through and arcing. The superposition of the two states is seen in the presence of strong enhanced lines, while the arc lines are nearly in the state of No. 1, but show a weakness and dissymmetry in the middle caused by the short period corresponding to No. 2.

These photographs seem to offer unmistakable evidence of a difference in condition between center and wall of the tube which affects the structure of the line. Whether it is a real shift of the maximum or an unsymmetrical widening cannot now be decided. It is probably the latter, but in any case it is a disturbance which would greatly affect wave-length measurements and must be recognized as a condition likely to occur in any source in which the vapor approaches a state such as we have here. As would

be expected, different photographs do not show this effect in the same degree. The general condition is always the same, but the violence of the rupture of the tube and resulting arc will not be alike in two experiments. A variability of the excitation during the time of an exposure is evident from the poor definition of the lines given by the broken tube, indicating the integration of a changing state; while the regular furnace lines, if the temperature is kept constant, are excellent for measurement. This difference in the quality of the lines is plainly seen in Nos. 1 and 2 of Plate VI.

The connection of this effect with another spectroscopic phenomenon is of special interest. It is well known that the condensed spark gives lines whose wave-lengths in many cases measure greater than in the arc.[1] Whether this is a real displacement or not, measurements are affected to such an extent that lines given by a strongly condensed spark are useless for accurate standards. The conditions in the middle of the furnace tube, when broken, give lines of such relative intensities as are obtained in the laboratory only with very powerful sparks, and we have the same apparent displacement toward the red as in the spark, when referred to the end of the line which is produced by vapor apparently more nearly in the state of the arc. This difference in the case of the furnace spectrum can scarcely be ascribed to pressure, a hypothesis for the spark which has been carried to the extent of estimating the pressure of the spark from known values for pressure displacements.

These preliminary observations appear very promising in the way of furnishing supplementary data as to the conditions for producing the enhanced lines, our knowledge of which has advanced slowly owing to the more or less conflicting evidence given by the arc and spark phenomena. Among the astrophysical applications, which can be made when observations for a number of elements are available, one of the most important will probably be to the spectrum of the chromosphere. The presence of enhanced lines in the upper regions of the sun's atmosphere agrees with the furnace results in that these lines may appear in regions where the highest temperatures are not to be expected.

[1] See, among others, N. A. Kent, *Astrophysical Journal*, **22**, 182, 1905.

SUMMARY

The leading features brought out in this study may be summarized as follows:

1. The enhanced lines of titanium appear in the regular furnace spectrum for temperatures probably somewhat higher than 2600° C; but are very faint compared to the arc lines.

2. At still higher temperatures, while furnace conditions still exist, there are indications of a slight increase in the relative strength of the enhanced lines.

3. When the furnace tube burns through with the formation of a low-voltage arc, the consumption of electrical energy at the point being very large, the enhanced lines of titanium and the spark line λ 4267 of carbon appear with an intensity usually attainable only in powerful sparks.

4. By photographing with the slit across the image of the tube's interior, the relative strength of the enhanced lines is seen to be much greater in the center of the tube than near the wall, this effect being very pronounced in the case of the carbon spark line.

5. The vapor in the center of the broken tube shows a tendency to give a line farther to the red than that near the wall, this being shown in the increasing dissymmetry of the lines from the end toward the middle. The effect is in harmony with the action of the condensed spark.

Mount Wilson Solar Observatory
December 4, 1912

MEASUREMENTS OF SOLAR RADIATION[1]

By C. G. ABBOT

Readers of this *Journal* are informed in two articles by Mr. F. W. Very:[2]

(1) The Violle actinometer [is] an instrument of unsurpassed precision.

(2) The absorption of solar radiation by the earth's atmosphere not only varies with the changing composition of the atmosphere from summer to winter, but is inconstant even during a single day of cloudless sky.

(3) The records of the Crova actinograph show that the chance fluctuations have their greatest magnitude in the middle of the day.

(4) The great fact of this midday atmospheric quality is so obtrusive that it cannot be ignored, and the Bouguer formula must be modified in order to cover the actual facts.

(5) [Referring to the diurnal march of the intensity of the solar radiation:] The portion of the diurnal curve between the limits of four and ten atmospheres conforms tolerably well to the conditions needed for a determination of its slope and general form, and, as a rule, it would seem to be the best part of the curve to select for computation.

(6) [Referring to some high-level measurements which he gives in the first paper:] A smooth curve passed through these points (Fig. 1) meets the axis of X at 3.5 calories per sq. cm. per min., which is the solar constant of radiation.

(7) I obtain . . . . from observations by Savélief, ... 3.606; from observations by Kimball, ... 3.698 in calories per sq. cm. per minute.

(8) My determination of the solar constant from the sounding-balloon measurement of M. Violle, described in my paper in the preceding article, is in good agreement with Savélief's observations. By the treatment recommended here, Kimball's observations are also brought into accord, not only without doing violence to the data, but by the application of precautions which are obviously required.

If these things are so, we must infer: first, that the work of Michelson, Ångström, Callendar, Marvin, Abbot, and Aldrich in the devising, perfecting, and testing of pyrheliometers has led to no improvements over the instruments which existed forty years ago; and second, that the work of the Smithsonian Astrophysical

[1] Published by permission of the Secretary of the Smithsonian Institution.

[2] *Astrophysical Journal*, **37**, 25–47, 1913.

Observatory since 1902 has been principally misguided and worthless. I therefore find myself directly interested in these statements, although no mention of me or of my work has been made by Mr. Very directly in connection with them.

I. PYRHELIOMETRY

I quote the following extract from a recent paper by Abbot and Aldrich:[1]

Summary.—A new form of standard pyrheliometer has been devised and tested. In this new instrument, as in the water-flow pyrheliometers, the solar rays are absorbed in a deep chamber approximating to the perfect absorber or "black body." Means are provided for introducing electrically test quantities of heat.

It is shown that with Standard Water-flow Pyrheliometers Nos. 2 and 3, and the new Water-stir Pyrheliometer No. 4, test quantities of heat may be measured to within 1 per cent.

A summary is given of all definitive comparisons of the three standards just named with Secondary Silver-disk Pyrheliometers, and also the net of inter-comparisons connecting all Smithsonian secondary pyrheliometers now in use. From these data are derived the best values of the constants of all these secondary pyrheliometers. This system of pyrheliometry we call "Smithsonian Revised Pyrheliometry of 1913."

It rests on 72 comparisons on 20 different days of 3 different years with 3 standard pyrheliometers of different dimensions and 2 widely different principles of measurement, all capable of recovering and measuring within 1 per cent test quantities of heat, and all closely approximating to the "absolutely black body." The 72 comparisons, 40 at Washington, 32 at Mount Wilson, were made in 6 groups. The maximum divergence of the mean results of these groups is 1 per cent. Hence it is believed that the mean result of all the comparisons made under such diverse circumstances must be within 0.5 per cent of the truth. The probable error is 0.1 per cent. It is believed that this standard scale is reproducible by the secondary pyrheliometers with the adopted constants given to within 0.5 per cent. The divergence of this scale from that of Ångström appears to be 3.9 per cent.

Experiments as yet unpublished have shown that an increase of at least 2 per cent should be applied to the readings of the Ångström instruments, so that the agreement between the Ångström and Smithsonian scales of radiation would then become practically exact. It is also known that the ice calorimeter pyrheliometer of

[1] *Smithsonian Miscellaneous Collections*, **60**, No. 18, p. 7, 1913.

Michelson, the new electrical compensation pyrheliometer of Callendar, and the new electrical disk pyrheliometer of Marvin are in very close accord with these scales of radiation.[1]

Two of the three applications of the Violle actinometer referred to by Mr. Very occurred on the expedition to Mount Whitney of the late Dr. S. P. Langley in 1881. Referring to the *Report of the Mount Whitney Expedition*, we learn the following facts in regard to the use of the actinometers in question.

Three Violle actinometers were carried on the expedition, and of these No. 1 and No. 3 were read at Mountain Camp or on the summit of Mount Whitney.[2] The determination of the water equivalents of the sun thermometers of the instruments is given on pp. 78 to 86 of the report. From this we find that the water equivalent of No. 1 was about one-fourth gram and of No. 3 about one-half gram; that the adopted value for No. 1 differed by 4 per cent from a check value found by other means; the adopted value of No. 2 differed by 6 per cent from its check value; and for No. 3 the difference was very slight. However, the thermometer No. 3 was broken in the transit, so that the determination of its constant was made by other methods than those of the other two instruments. In each case an allowance of 8 per cent was made for the conduction of heat along the glass stem of the thermometer. Pp. 100–115 of the *Report* are given over to the discussion and determination of six corrections to be applied to the readings of the instruments. The sum total of these corrections amounted to 25 per cent and 23 per cent for high and low sun, respectively, at Lone Pine; 23 per cent and 21 per cent for high and low sun, respectively, at Mountain Camp. To these corrections Mr. Very has now added a few others, so that, including the correction for loss of heat by conduction along the stem of the thermometer, we may say that about ten different corrections are applied, making a total increase of the reading of the actinometers of about 35 per cent.

[1] See *Transactions International Solar Union*, **2**, 175, 1907; also *Proceedings Royal Society London*, Series A, **77**, 6, 1906; also *Bulletin Mount Weather Observatory*, **3**, Part 2, p. 84.

[2] The results obtained with No. 1 differed from those obtained with No. 3, and had to be reduced to the scale of No. 3. See *Report*, p. 98.

I should regret exceedingly to make any criticism of the apparatus or the work of my esteemed friend Professor Violle; but in this case I feel less hesitancy because, so far as I know, neither the magnitude of these corrections, nor in many cases even the necessity for them, has been passed upon or approved by him.

We may now inquire whether it is probable that these corrections were necessary and exact. In order to do so it will be convenient to consider the observations at highest sun made on ten different days during the expeditions of 1908, 1909, and 1910 of the Smithsonian Astrophysical Observatory to the summit of Mount Whitney. The values which are given below are the highest observations made during each of the ten days; and, *as it happens, they are also the observations made at the least zenith distance of the sun for each of the days*, a fact of interest in connection with some of Mr. Very's statements.[1]

Station: Mount Whitney, California. Altitude: 4420 meters
Observer: C. G. Abbot

Date	Hour Angle	Secant z	Pressure Aqueous Vapor	Precipitable Water*	State of Sky	Calories per cm.² per Min.
1908 Aug. 24...	2^h51^m W.	1.445	1.24mm.	.....	Cloudless	1.594
Aug. 25...	1 45 E.	1.250	0.92	.....	Cloudless	1.663
1909 Sept. 2...	3 28 W.	1.750	1.80	.....	Cloudless	1.561
Sept. 3...	0 54 E.	1.175	1.91	.....	Exceptionally good	1.691
1910 Aug. 12...	1 14 E.	1.123	2.37	1.07mm.	Cloudless	1.635
Aug. 13...	0 50 E.	1.101	2.34	0.85	Cloudless	1.622
Aug. 14...	1 28 E.	1.150	1.98	0.69	Cloudless	1.643
Aug. 15...	0 8 W.	1.080	2.00	0.65	Cloudless	1.644
Aug. 16...	0 12 E.	1.085	2(.48)	.....	Cloudless	1.617
Aug. 17...	0 2 E.	1.080	1.30	.....	Exceptionally good	1.694

* "Precipitable water" means the depth of liquid water which would have been produced if all the water-vapor in the path of the beam between the pyrheliometer and the sun had been condensed. Determined from bolographic observations by F. E. Fowle. For method see *Astrophysical Journal*, 35, 149–162, 1912.

If there was anything in the higher atmosphere which tended to produce a haze or undue diminution of the intensity of the solar radiation at Mount Whitney on September 3, 1909, and

[1] In all Smithsonian solar constant work pyrheliometer readings are made not only at high and low sun, but at many intermediate zenith distances besides, so as to enable us to form a judgment of the uniformity of the sky conditions which have prevailed.

August 3, 1910, it was certainly absolutely invisible to the eye, and non-apparent with the apparatus which I had at my command. I remember repeatedly holding up a single finger at such a distance as barely to cover the sun's disk. The sky then appeared to have a deep blue color right up to the very edge of the sun. There was no perceptible whiteness at all. The quantity of aqueous vapor present was, as the measurements show, extremely slight, so that less than 1 mm. of precipitable water intervened in the path of the solar rays between the spectro-bolometer and the outer limit of the atmosphere. I see no reason to believe that the intensity of the radiation of the sun at these times was appreciably less than it was at the time of the measurements made by Keeler and by Nanry in September 1881, to which Mr. Very refers. Hence I feel convinced that we must attribute the difference between 1.70 calories, the highest value which I found, and the practically 2 calories found by Mr. Very, to a difference in pyrheliometry.

In view of the agreement of different kinds of pyrheliometers which I have cited above, I think it is more reasonable to conclude that the measurements made during the year 1881 were too high than that the measurements made during the years 1908 to 1910 were too low. I therefore adopt the conclusion that Mr. Very's value at 2 calories should be reduced to about 1.70 calories.[1] It also follows that Langley's value of 2.22 calories per sq. cm. per minute, found on Mount Whitney, if reduced to the Smithsonian scale of pyrheliometry would be about 1.92 calories per sq. cm. per minute, which would agree exactly with the mean of all the values obtained on Mount Wilson from 1905 to 1910.[2]

We may now consider the lowest point of Mr. Very's curve.[3] From *Annals of the Astrophysical Observatory*, **2**, 93, 94, I find that the highest observations which have ever been made in Washington,

[1] This conclusion is strengthened by the fact that the well-established measurements of K. Ångström on the peak of Teneriffe in 1895 and 1896 were all below 1.64 calories as stated by him. If 3.9 per cent is added, they also reach 1.70 calories.

[2] "Report on the Mount Whitney Expedition," *Professional Papers Signal Service*, No. 15, p. 148, Table 120, columns 3, 4 and 5: also *Astrophysical Journal*, **33**, 194, 1911.

[3] *Astrophysical Journal*, **37**, 29, 1913.

at sea-level, of the radiation of the sun, when reduced to the scale of Smithsonian Revised Pyrheliometry of 1913 are as follows:

DETERMINATIONS OF SOLAR RADIATION AT WASHINGTON

Date	Hour Angle	Secant z	Pressure Aqueous Vapor	Calories per. Sq. cm. per Min.*
1902 Oct. 15	$1^h 8^m$	1.552	.....	1.400
1904 Apr. 4	0 20	1.205	.....	1.310
Oct. 21	0 36	1.568	7.29mm.	1.447
1905 Mar. 2	1 22	1.557	.....	1.387
Sept. 26	0 33	1.328	4.57	1.365
1906 Feb. 15	1 3	1.689†	2.16	1.433
May 29	1 29	1.129	9.83	1.411
1907 Feb. 15	1 6	1.700	1.45	1.418

* According to Smithsonian Revised Pyrheliometry of 1913.

† The value given in *Annals* is misprinted.

From this I conclude that the lowest point of the curve which Mr. Very has shown is well taken at 1.5 cal. for sec. $z=1.4$.

Turning now to the point which he finds at 2.86 calories for a position in the free atmosphere at an elevation of 13,700 meters, I think this is subject to strong criticism.

The circumstances of the experiment were these: A ball of blackened copper supported by a balloon of above 2 m. diameter was alternately shaded by a double screen of aluminium and exposed to sunlight. At an elevation stated 13,700 m., barometric pressure stated 118 mm., air temperature stated 208° absolute Centigrade, the copper bulb in sunlight was stated 261° absolute Centigrade. Let r be the radius of the copper ball; a_1, a_2, a_3 its average absorption coefficients for solar rays, for its own proper emission, and for the radiation of the surroundings, respectively; S the intensity of solar radiation; σ a constant adapted to fit the radiation of the ball to Stefan's formula; K the coefficient of convection; Q the average intensity of radiation from surroundings, all per sq. cm. per min.; T_1 and T_2 the average temperatures of the ball and surroundings, respectively.

$$\begin{aligned}
\text{Then solar radiation absorbed} &= \pi r^2 S a_1 \\
\text{Other radiation absorbed} &= 4\pi r^2 Q a_3 \\
\text{Radiation emitted} &= 4\pi r^2 \sigma T_1^4 a_2 \\
\text{Convection loss} &= 4\pi r^2 K (T_1 - T_2)
\end{aligned}$$

Hence

$$\pi r^2 S a_1 + 4\pi r^2 Q a_3 = 4\pi r^2 \sigma T_1^4 a_2 + 4\pi r^2 K(T_1 - T_2)$$

Or

$$S = \frac{4\sigma T_1^4 a_2 + 4K(T_1 - T_2) - 4Q a_3}{a_1}$$

If the value a_1 is omitted, we shall not probably go 5 per cent astray. The values a_2 and a_3 are decidedly more uncertain, as indicated below under σ. T_1 and T_2 were observed. We may justly ask, however, whether a single determination of them by necessarily unsupervised registering apparatus in trying circumstances is entitled to great weight.

The remainder of the quantities are all unknown.

As for σ, the determinations of constants of emission in Stefan's formula appropriate for use with the actual blackened surfaces ordinarily employed are now covering the range from 0.50 to 1.40×10^{-12} cal. per sq. cm. per second.[1] The range appears to depend on very obscure details of surfaces of the bodies investigated.

As for Q, we are not informed as to the temperatures in sunlight, radiation coefficients, solar radiation reflected, or solid angles subtended, by the balloon, registering apparatus, aluminium screen, and layers of atmosphere sending radiation to the copper ball.

As for K, no experiments in conditions comparable to these are known to me. Certainly none such are given with sufficient detail for critical study in Mr. Very's communications.

From the paper of P. Compan just cited I find from a direct experiment that a blackened copper ball of 2 cm. diameter in free air cooled at the rate of 0.00789 calories per sq. cm. per min. per 1° C. temperature excess, when the air temperature was 5° C. and the ball temperature 50°96 C. This cooling occurred at atmospheric pressure, and included all elements of cooling, air convection, air conduction, radiation, etc. If the copper ball referred to by Very cooled similarly we should have as the measure of solar radiation

$$4 \times 53 \times 0.00789 = 1.67 \text{ cal. per sq. cm. per min.}$$

The matter is, however, wholly hypothetical. So far as I can see,

[1] See P. Compan, *Annales de chim. et de phys.*, 7 sér., **26**, 571, 1902; H. Kayser, *Spectroscopie*, **2**, 78–80 and 133; C. Fery, *Soc. franc. de phys.*, Nr. 293, pp. 2–3, 1909.

there is no way of knowing the intensity of the solar radiation to within 100 per cent of itself from the experiment cited by Mr. Very. It is quite significant, I believe, that M. Violle himself has not indicated any numerical value as the result of the experiment.

I submit that a curve similar to Mr. Very's drawn through a point at 1.5 calories at sea-level, 1.7 calories at Mount Whitney level, and a wholly unknown value at 13,700 meters level may very likely be in excellent agreement with the value of 1.92 cal. per sq. cm. per min. at the outer limit of the atmosphere; and I shall continue to think so until something equal in weight to the 900 spectro-bolometric determinations already made indicates otherwise.

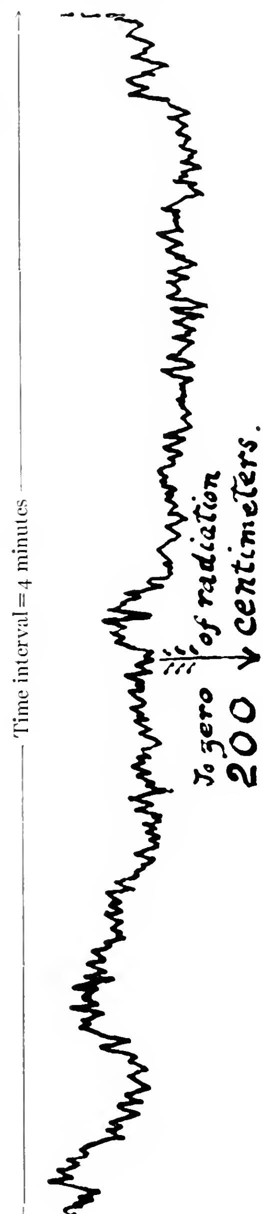


Fig. 1

II. ATMOSPHERIC TRANSMISSION

It may be freely admitted that for many stations at nearly all times, and for all stations at some times, the deeply indented actinograph curve shown by Mr. Very[1] is typical. But, as I shall show, for good stations at almost all times, and for ordinary stations some times, the case is altogether different.

Fig. 1 shows on the original scale a portion of the curve made on Mount Wilson on September 12, 1905, for the express purpose of testing the steadiness of the transmission of the atmosphere. The following notes were made at the time.

[1] *Astrophysical Journal*, **37**, 33, 1913.

Test of steadiness of sunlight.—A direct beam of sunlight was passed through the slit and both diaphragms of the spectro-bolometer, and reflected by the collimating mirror directly to the image forming mirror [without passing through the prism] and thence upon the bolometer. The beam was left out of focus so that it came from all parts of the sun. A diaphragm was provided over the image forming mirror, so as to reduce the equivalent deflection of the galvanometer to about 2 meters (200 centimeters). The spot of light reflected from the galvanometer was adjusted to fall near the middle of the recording photographic plate. The run was started at $2^h 27^m 25^s$ in good blue sky with a distant cloud gathering from the southwest. The run continued until $2^h 57^m$ when the clouds reached the sun.

From this quotation it will be seen that the sky at this time was probably by no means exceptionally good, but rather perhaps on account of the cloud we should regard it as decidedly below the average found upon Mount Wilson. Furthermore, the afternoon sky there hardly equals that of morning. The time of single swing of the galvanometer during these experiments was about 1.5 seconds and the bolometer was so fine that it responded more quickly to changes of radiation than the galvanometer could record. Under these circumstances the plate shows that *during the first 23 minutes of the run there was no change of radiation, either of short period or long, as great as 1 per cent.* The latter part of the curve shows a gradual falling-off of the radiation, due, I suppose, to the close approach of the cloud which at the end covered the sun.

The Smithsonian Astrophysical Observatory has in its possession several thousand automatic records, some of the spectro-bolometer, others of the recording water-flow pyrheliometer and other instruments, which show that the curve I have just referred to is by no means exceptional in showing an almost perfect uniformity from minute to minute in the transmission of the atmosphere not only above Mount Wilson but at Mount Whitney, Bassour, Algeria, and occasionally even at Washington. We have also several thousand pyrheliometric determinations of solar radiation made at Mount Wilson and elsewhere which indicate that on a great majority of the days observed the march of the intensity of solar radiation was perfectly regular and normal from a zenith distance of 70 or 75 degrees up to zenith distances of 20 or 30 degrees. Such a curve may be found in the writer's book on *The Sun*,[1] pp. 285 and 287.

[1] D. Appleton & Co., 1911.

The observations there referred to were begun as soon as the sun peeped above the horizon and were continued until 5 minutes before noon. Each sun exposure was 100 seconds. It is only the very last observation for the series which shows any falling-off of the intensity of solar radiation due to the approach of the haze or water-vapor which Mr. Very has described; and the falling-off which then appeared is only about 2 per cent of the highest value.

My colleague Mr. Fowle has kindly allowed me to take in advance of publication the data which are given below in regard to the quantity of water-vapor prevailing between Mount Wilson and the outer limit of the atmosphere and the changes of it which occur during the time interval covered by the ordinary bolographic observations.

During the observations of 1910 and 1911 on Mount Wilson, the mean quantity of precipitable water in the atmosphere between the station and the zenith was 6.9 mm. The number of days when the change of this quantity during the observations reached certain limits is as given in the following table:

CHANGE IN ATMOSPHERIC VAPOR DURING DAY OBSERVATIONS

Range	0 to 0.05 cm.	0.05 cm. to 0.10 cm.	0.10 cm. to 0.15 cm.	0.15 cm. to 0.20 cm.	0.20 cm. to 0.30 cm.	Above 0.30 cm.
1910..............	22 days	19 days	34 days	8 days	12 days	4 days
1911..............	14 days	21 days	21 days	8 days	9 days	7 days
Total...........	36 days	40 days	55 days	16 days	21 days	11 days

Average change, 1910, 0.13 cm.: 1911, 0.14 cm.;
Average increase, 1910, 0.11 cm.; 1911, 0.12 cm.;

From this it will appear that on a very great number of days of observation on Mount Wilson there has been no appreciable change at all in the quantity of water-vapor intervening between the observer and the sun. Further evidence will appear in the next section.

III. BOUGUER'S FORMULA

Mr. Very says:[1]

If it were possible to measure strictly homogeneous radiations, and if occasions could be found when the atmospheric quality remains unchanged between

[1] *Astrophysical Journal*, **37**, 32, 1913.

high-sun and low-sun observations, we could apply Bouguer's equation

$$R = Ap^{\epsilon}$$

in which R = solar radiation received normally on the unit of area at the surface of the earth, A = the solar constant, p = the coefficient of transmission, assumed to remain unchanged during the day, and ϵ = the air mass taken as unity for sea-level and normal pressure, so that, if B is the observed barometer reading at the place of observation and ζ the sun's zenith distance, $\epsilon = (B/760)$ sec. ζ, until zenith distances greater than 56° are reached, beyond which a small correction is required.

There seems to be so much confusion in regard to these matters that I shall quote here from pp. 14 and 15 of the *Annals of the Astrophysical Observatory of the Smithsonian Institution*, **2**.

Langley showed that the exponential formula would still approximately apply if we assume that the atmosphere is composed of thin layers concentric with the earth, each one of uniform transparency [during the two or three hours necessary to make a determination] but each differing slightly from the next adjacent one in transparency. This may be proved as follows:

PROOF OF FORMULA FOR TRANSMISSION

Imagine the atmosphere to be made up of n concentric layers so chosen in thickness as to produce separately equal barometric pressures, and let the number n be so great that the transparency of any single layer is sensibly uniform, although the layers may differ from each other in transparency by any gradual progression. The index of refraction of air is so near unity that there will be no sensible regular reflection in passing from one layer to the next, and the transmission of each layer may be expressed exponentially by Bouguer's formula, but with different coefficients of transmission for the several layers.

Thus, suppose E_0 to be the original intensity of a beam of light incident upon the outermost layer at the angle whose secant is m.

Then after passing successive layers the remaining intensities become

$$E_1 = E_0 a_1^{m_1}, \qquad E_2 = E_0 a_1^{m_1} \cdot a_2^{m_2}, \qquad E_n = E_0 a_1^{m_1} a_2^{m_2} \ . \ . \ . \ . \ a_n^{m_n}. \qquad (1)$$

The value of the secant of the angle of incidence will change slightly in passing from layer to layer from two causes: first, the curvature of the earth; second, the refraction of the beam in air. These causes produce opposite effects, the first tending to increase the angle of incidence, the second tending to diminish it as the beam approaches the earth's surface. Their combined effect is dependent on the height to which the temperature exercises absorption and on the distribution of density with the height. But it is generally supposed that the absorption of the air above 40 miles from the earth's surface is negligible, and, remembering that the atmospheric density diminishes with the height, it appears that for zenith distances less than 70° the effect of change of the secant of the angle of the incident beam from the outermost

to the innermost layer of the atmosphere will not introduce error greater than 1 per cent. Accordingly for zenith distances less than 70° we may write approximately

$$E_n = E_0(a_1 a_2 \ldots\ldots a_n)^m \qquad (2)$$

The symbols a_1, a_2 . . . . a_n denote constants (providing no change of transparency occurs during the interval of time in question), and their values are slightly less than unity. We may substitute for their product a single constant, a, itself a proper fraction, and remembering that E_n is the intensity at the earth's surface, above denoted simply by E, we have

$$E = E_0 a^m \qquad (3)$$

LIMITATIONS OF FORMULA

No mention is made in this expression of the barometric pressure, but it is easy to see that an alteration of barometric pressure would signify, under the conventions adopted in deriving the formula, a change in the number of layers, n. This would cause an alteration of the quantity a, which is the continued product of the transmission coefficients of the layers, by introducing additional multipliers a_{n+1}, a_{n+2} . . . . or by the withdrawal of some a_{n-1}, a_{n-2}. . . . . Since we have no means of determining the value of the terms so introduced or taken away, there is no means of correcting for change of barometer in the use of the expression (3) and it would, for instance, be impossible to compute, from knowledge of the values of E, E_0, a, and m for one station, what would be the value of E at some station of different barometric pressure.[1]

From this we see that the unit of air mass to be taken for each station is the air mass traversed by beams from zenith celestial objects *between the station itself and the outer limit of the atmosphere*, and that the definition of unit air mass as that lying between sea-level and the outer limit of the atmosphere, as stated by Mr. Very, would be wholly indefensible unless the optical quality of the atmosphere was uniform from sea-level to its outer limit, which Mr. Very explicitly denies.

But he is not alone in his misconception of the applications of Bouguer's formula to the earth's atmosphere. Professor F. H. Bigelow is of the opinion that the unit of air mass is entirely indeterminate.[2] Professor Bigelow is also of the opinion that there is situated in the higher atmosphere a reflecting layer to which Bouguer's formula does not apply and which according to him

[1] This formula holds only for homogeneous rays.

[2] *Bericht über die erste Tagung der Strahlungskommission des Internationalen Meteorologischen Komites*, September 2 and 3, 1912, p. 17.

doubles the value of the solar constant. I would freely admit that if there were up there a sheet of plain parallel glass, or of water, its reflection would not follow Bouguer's formula, but rather the formula of Fresnel for the reflection of light.[1] But this is of course not the case. Any reflecting layer which exists in the atmosphere, so far as is known, consists of small particles distributed through a great thickness. To such a combination the formula of Bouguer does apply.

If there were any doubt about the applicability of Bouguer's formula to such a layer, it would have been removed during the past summer. For the volcano of Mt. Katmai in Alaska on June 6, 1912, sent up such a cloud of particles of volcanic glass and other ejecta to enormous heights that the atmosphere of the whole earth was made hazy by it for months. This haze was so great in quantity as to reduce the intensity of the direct solar beam by nearly or quite 20 per cent, yet the Bouguer transmission coefficients for homogeneous rays determined by spectro-bolometric observations at Mount Wilson and in Algeria took care so exactly of this floating obstruction that the average values of the solar constants obtained during the year 1912 agreed within 1 per cent of those obtained in former years.

We have at the Astrophysical Observatory several hundred days of observations from Mount Wilson, Mount Whitney, and Bassour, Algeria, for which the departures of the bolographic observations at individual wave-lengths are no greater than those recorded at p. 66 of the *Annals of the Astrophysical Observatory*, **2**. It will be noted that in this example of the applicability of Bouguer's formula the air masses varied from nearly 5 to about 1.2, or between zenith distances of 78° and 30°. Mr. Very, however, states that the proper part of the curve to compute from lies between 75° and 85° of zenith distance (air masses four to ten). This is not the fact, for the derivation of the exponential formula which I have given above shows distinctly that the formula is applicable to the case of the atmosphere as a whole only when the increase of path by obliquity of the beam in each of the layers of which the atmosphere

[1] See E. C. Pickering on "Applications of Fresnel's Formula for the Reflection of Light," *Proc. American Academy of Arts and Sciences*, 1873.

may in imagination be composed is approximately the same. This is no longer true above 70° of zenith distance, although the departure is not very serious up to 75°; but beyond this zenith distance the departures from proportionality in the path within the different layers of the atmosphere, due to curvature and refraction, soon become very great.[1] In these circumstances Bouguer's formula no longer applies. It is for this reason that in our work we do not observe ordinarily at greater zenith distances than 75°.

Mr. Very has another criticism, for he says "this of course is not true except for absolutely homogeneous rays which are never measured by the spectro-bolometer."

Referring to Volume **2** of the *Annals of the Astrophysical Observatory*, p. 16, I quote:

Suppose a ray composed of amounts A_0 and B_0 of light of two different wave-lengths to pass through a homogeneous stratum of air, and let a and b denote the fractions of the respective kinds of light transmitted by the stratum at vertical incidence. Suppose the intensity of the beam after transmission to be observed, first when the secant of the angle of incidence is m, and again when the secant is $2m$. Let C_1 and C_2 represent the results of these observations; let c be the coefficient of vertical transmission which they yield and C_0 the intensity of the beam before transmission, as computed from the observed data.

By Bouguer's formula:

$$C_1 = C_0 c^m \text{ and } C_2 = C_0 c^{2m}$$

Hence

$$c^m = \frac{C_2}{C_1} \text{ and } C_0 = \frac{C_1^2}{C_2}$$

But the original intensity of the beam was in reality $A_0 + B_0$; its intensity observed as $C_1 = A_0 a^m + B_0 b^m$ and its intensity observed as $C_2 = A_0 a^{2m} + B_0 b^{2m}$. If there is a difference between the real and computed intensity prior to transmission, this is $A_0 + B_0 - C_0$, and substituting for C_0 we have the defect of C_0 as follows:

$$A_0 + B_0 - C_0 = A_0 + B_0 - \frac{(A_0 a^m + B_0 b^m)^2}{A_0 a^{2m} + B_0 b^{2m}} = \frac{A_0 B_0}{A_0 a^{2m} + B_0 b^{2m}} (a^m - b^m)^2 .$$

Suppose, better to fix our ideas, that we introduce numerical values into this expression, in order to see how great the error which Mr. Very implies may really be. Let $A_0 = B_0 = 0.5$, let $m = 1$, let $a = 0.9$, and let $b = 0.5$, then we find $A_0 + B_0 - C_0 = 0.093$.

Hence it appears that if the bolometer habitually fails to

[1] See *Annals Astrophysical Observatory*, **2**, 63, 64.

recognize differences of transmission between adjacent wave-lengths amounting to the difference between 0.5 and 0.9, the correction to be applied to the results on account of this failure on its part will amount to a little less than 10 per cent. Mr. Very maintains that the nature of that which diminishes the intensity of the solar beam in passing through the atmosphere is such that lines of almost complete absorption and lines of almost complete transmission habitually succeed one another so intimately that no spectroscope has ever been able to discover these hypothetical absorption lines. It is much more likely that the atmosphere in producing what is loosely termed its "general absorption" exercises a scattering which is a continuous function of the wave-length, as supposed by Lord Rayleigh, rather than that it presents a curious absorption phenomena like that indicated by Mr. Very. If so, the lack of homogeneity in the rays investigated by the bolometer will make a defect in our solar constant observations which is wholly negligible.

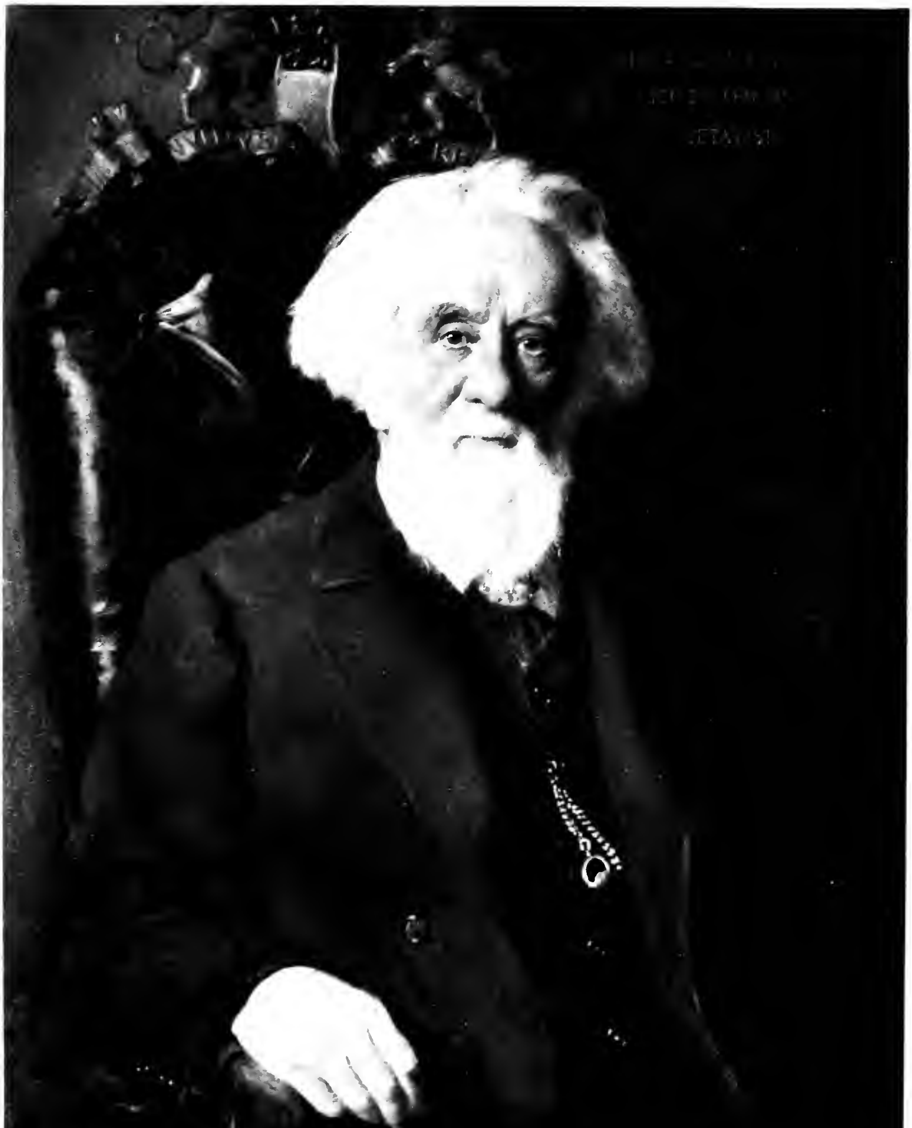
CONCLUSION

Mr. Very presents evidence to show that pyrheliometric measurements have been made within the atmosphere exceeding our value of the "solar constant." I have controverted this evidence.

He has admitted that our method of determining the "solar constant" would be sound if the atmosphere remained of uniform transparency while we observe, and if the bolometer could measure homogeneous rays, but he denies that these conditions are well enough fulfilled. I have shown how probable it is that they are.

Mr. Very finds that at least one day has occurred perfect enough to suit him. It seems remarkable that out of 900 days (some yielding sea-level values equal to that quoted by Mr. Very) in which we have observed with the spectro-bolometer, not any approached so near this paragon condition *as to appreciably raise our "solar constant" values*, computed, as Mr. Very admits, by the right method, and lacking only good sky and homogeneous rays for success.

Smithsonian Astrophysical Observatory
Washington
February 3, 1913



145

THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME XXXVII | APRIL 1913 | NUMBER 3

THE WORK OF SIR WILLIAM HUGGINS

By GEORGE E. HALE

In these days of great observatories, completely equipped with powerful instruments and free from the handicaps of city smoke and glare, it is easy to underestimate the opportunities of the amateur, and the possibilities of research with limited instrumental means. An anonymous author in the *Edinburgh Review*, after expressing his doubts as to the value of some of the instruments of a new observatory, remarks: "Is there not in truth a double danger, that extravagance of scientific enterprise may defeat its own purpose by overproduction in one place, and killing moderate ambitions elsewhere?"[1] This is a fair question, which would seem to find an answer in the life and work of such a man as Sir William Huggins.

If we define an amateur as one who works because he cannot help it, Huggins may be classed in that great English group of amateurs which includes such men as Faraday, Darwin, and Rayleigh—men who have worked for no other motive than intense love for research, undeterred by obstacles or by dearth of instrumental means. Half a century ago large and expensive telescopes, though few in number, were not altogether lacking. In 1856, when Huggins acquired his first telescope, of 5 inches aperture,

[1] "The Tercentenary of the Telescope," *Edinburgh Review*, January 1910.

15-inch refractors were in use at Pulkowa and Harvard, and Lord Rosse's 6-foot reflector had yielded important discoveries at Parsonstown. Two years later Huggins purchased the 8-inch refractor with which he made his visual observations of stellar and nebular spectra. Lassell carried a 4-foot reflector to the favorable climate of Malta in 1863, but if this disturbed Huggins, who discovered the gaseous nature of the nebulae in the following year, we find no indication of the fact in his publications. On the contrary, we see him busy with a score of new projects, one of which, startling to the conservative mind of his day, was the establishment of a small physical and chemical laboratory as an adjunct to his observatory. In 1870 a 4-foot reflector, with modern equatorial mounting, was set up under the almost unexplored southern heavens at Melbourne. Huggins, in the same year, mounted an 18-inch mirror, borrowed from the Royal Society, on the smoke-obscured London hilltop where he lived and worked, with no increase of aperture, to the day of his death. The development of great refractors, and the return of the reflector to favor found him keenly alive to every advance and warmly sympathetic with those of his colleagues who were more favored than he. But there was no relaxation of effort or failure of interest; no discouragement because of limitations of instruments or of atmosphere. Discovery after discovery was communicated to the Royal Society, and the possibilities of further work in astrophysics suggested by every new development in physics or chemistry were discerned and eagerly debated. Indeed, he seemed to find in his modest equipment a strong incentive to greater effort, for in one of his presidential addresses to the Royal Society he remarks: "We must not forget that, in the case of institutions as well as of individuals, a powerful and healthy stimulus to the exertion needful for success arises from the necessity of coping with and overcoming difficulties, whether of a monetary or other kind."[1]

It is evident that Huggins would not have sympathized with the attitude of Sir James South. When Professor O. M. Mitchel visited him in 1842, South was involved in difficulties with a firm

[1] *The Royal Society*, p. 67.

of instrument makers, which had failed to complete the mounting of his large object-glass. Mitchel describes his visit as follows:[1]

One apartment was examined after another, until finally we reached a large room surmounted by a dome of great size and of an expensive construction, while fragments of the framework for mounting a great equatorial were scattered around.

"Here, sir," exclaimed Sir James, "you behold the wreck of all my hopes. Here I have expended thousands, and flattered myself that I was soon to possess the finest instrument in Europe; but it is all over, and there's an end."

I remarked that the object-glass was still in his possession, and might yet be mounted so as to realize his hopes and expectations.

"No," said Sir James, "Struve has reaped the golden harvest among the double stars, and there is little now for me to hope or expect."

It would be difficult to appreciate the feelings which at that moment were sweeping through the mind of the astronomer. Long-cherished visions of fame and high distinction, nay, perhaps of grand discoveries in the heavens, which for years had played round his hopes of the future, had fled forever. Another had reaped the golden harvest, and like Clairaut, who wept that there was not for him, as for Newton, the problem of the universe to solve, Sir James South could almost weep to think that another's eye had been permitted to sweep over the far-distant realms of space which he had long hoped might remain his own peculiar province.

Thirty years later, an American amateur, possessing only a 6-inch telescope, began a search for double stars in the city of Chicago. His days were spent in the law courts, his nights in sweeping the heavens, in the wake of many a telescope far larger than his own. But his labors were rewarded by the discovery with the 6-inch telescope of 451 new double stars, before he gave up his court duties to accept a position in a great observatory. Here, again, the man rose above his environment, and proved that devoted interest and true enthusiasm are more powerful than costly instruments.[2]

However, the fears of the Edinburgh reviewer must not be dismissed too lightly, for the point he raises is one of high impor-

[1] *Ormsby MacKnight Mitchel: Astronomer and General.* A biographical narrative. By his son, F. A. Mitchel, 1887.

[2] See the Introduction to *General Catalogue of 1290 Double Stars Discovered by S. W. Burnham,* where the South incident is quoted, and a list of Burnham's discoveries with various telescopes may be found.

tance. If it be true that modern observatories, with their expensive equipment, tend to discourage the serious amateur, then it may be doubted whether the best use is being made of the funds they represent. For the history of science teaches that original ideas and new methods, as well as great discoveries resulting from the patient accumulation of observations, frequently come from the amateur. To hinder his work in any serious way might conceivably do a greater injury than a large observatory could make good.

We may ask, however, whether conditions are materially different today from what they were in former times. There have always been observatories far transcending in equipment the resources of the isolated amateur or those of the average professional observer. But contributions from the minor instruments have continued to enrich astronomy, and if there are signs now of reduced activity they are certainly not conspicuous.

The reasons for the amateur's success should not be far to seek. Indeed, we see them most admirably illustrated in the career of Sir William Huggins. The purchase of his 8-inch refractor antedated by only a year the interpretation of the solar spectrum by Kirchhoff and Bunsen. Almost simultaneously Darwin published the *Origin of Species*, and the great controversy regarding evolution was begun. Thus a new method of analysis, which took no account of the distance of the luminous source, suddenly became available. A man of Sir William's imagination could not fail, while reflecting on this novel ally, to divine some of the less obvious applications of the spectroscope. Nor would he be without curiosity as to the bearing of the new method on the problem of inorganic evolution. Quick to appreciate his opportunity, he lost no time in applying the spectroscope to the analysis of starlight. Next he discovered in the gaseous clouds of the nebulae the raw material from which the stars may be evolved. And later, after a successful application of photography, he arranged his spectra in the probable order of stellar development.

This faculty to perceive new possibilities remained with Sir William to the end. No physicist of the new school showed greater enthusiasm over the rapid unfolding of the electron theory or the disclosures connected with radium. None watched with more

eagerness the discovery of new gases, or considered more carefully their bearing on astrophysical problems. When Runge found the D_3 line of helium double in his vacuum tube, Huggins instantly turned his spectroscope to the sun, though he had given up systematic solar work years before. Persisting in spite of bad weather and barely adequate instrumental means, he soon succeeded in detecting the duplicity of the solar line. So it was with every new advance, in whatever field. Sir William read of it at once, discussed it with Lady Huggins, called it to the attention of his correspondents, and carefully examined into its bearing on astrophysics. I have scores of letters and postcards illustrating this characteristic of his remarkably alert mind.

Thus we may explain the success of the founder of modern astrophysics. Discoveries like Kirchhoff's are almost unique, and few can ever have the opportunity of applying such an instrument as the spectroscope for the first time in a virgin field. But the chance of profiting by new advances always exists, and Huggins would have been a great investigator if his work had begun a quarter of a century later. Consider some of the numerous activities of his mind. In his earliest comparisons of terrestrial and stellar spectra (1863) he had in view the possibility of detecting displacements of lines indicating stellar motions, and four years later he made the first serious attempt in this direction.[1] The extraordinary developments of this department of stellar spectroscopy in the hands of Vogel, Keeler, Campbell, and others are too familiar to call for mention here. The laboratory work, begun by Sir William in 1863 for the purpose of interpreting stellar spectra,[2] was continued until 1905. In 1903 Sir William and Lady Huggins announced their remarkable discovery of the spectrum produced by the spontaneous luminous radiation of radium.[3] In 1870 Huggins had a large magnet constructed for the express purpose of detecting such modifications of spectral lines as he anticipated might be produced by a magnetic field.[4] Insufficient dispersion prevented the effect from being seen, and it was not until 1896 that it was first found by Zeeman. To Huggins we owe the first obser-

[1] *Scientific Papers*, p. 229.
[2] *Ibid*, p. 397.
[3] *Ibid.*, p. 444.
[4] *Ibid.*, p. 458.

vations of sun-spot spectra which distinguished between the different degrees of widening of different lines. He was the first to observe the form of a solar prominence through a widened slit, and his earliest attempts to use the spectroscope for this purpose were made in 1867, before the publication of the classic results of Janssen and Lockyer.[1]

One ordinarily thinks of Sir William in his home on Upper Tulse Hill, where, with the sole aid of his learned and devoted wife, he carried on his study and research. None who has entered that quiet library, its shelves overflowing with books, can ever forget the sight of the aged philosopher in his natural environment. But in spite of the fascination of his work, Huggins did not forget his obligations to the larger world of science. His broad interests and his years of activity in the Royal Society were recognized by his election to the presidency of that illustrious body in 1900. A quotation from one of his annual addresses on St. Andrew's Day will not be out of place here:

> On one central eminence, dominating alike the past, the present, and the future, science has for some years firmly entrenched herself—the position that through all the ages the cosmos has advanced, and is still advancing, by a process of orderly evolution. In the domain of the living the fact of progress by means of evolution was finally established by our illustrious countryman Darwin, and his prophet Huxley. In heredity and variation, the great discovery of Mendel, in the hands of one of our medallists of today, promises to bring the biologist nearer to his main quest, the fundamental nature of living things. In physics, only a few years ago Professor J. J. Thomson took by storm the outworks of the central citadel of Nature—the chemical atom. His later brilliant attacks, aided by the new artillery of radioactive radiations, may be said to have carried the keep itself. Material mass gives place to the electric mass of moving electric charges. On this view the chemical elements, each with its individual properties, but all falling into family groups according to a periodic law, have their origin in differences in the number of electrons and in the figures of their giddy dances, whirling within the atom. Material nature becomes simplified into electricity and ether—or, is it only ether? Passing from the atom to the heavens, within the memory of those living, science has taught us so to read the sunbeams and starbeams as to enable us to apply the methods of the laboratory to the heavenly bodies. By means of their radiations alone we discuss their chemical constitution and their orbital and other motions, which were before unknowable. By each discovery the

[1] *Scientific Papers*, p. 305.

vision of the world has become more glorious, the wonder of it more amazing, while chambers and palaces of Nature still unexplored remain the exhaustless heritage of all coming generations.[1]

In reading such addresses as this we appreciate why Huggins was so staunch a supporter of the Royal Society. He believed strongly in the specialized societies which, a century ago, began to develop from the older body as the need for them appeared. He argued for their continuance as separate organizations, and advocated their independence and complete autonomy. But he saw that the Royal Society was needed in his time quite as much as in the early days, when it included the whole of British science. Indeed, the very fact of specialization, with the consequent danger of narrow vision and limited range of interest, is a powerful argument for a society which insists on the unity of Nature. Moreover, the suggestive value in other branches of science of methods of procedure developed in a special field is perhaps greater now than ever before. Sometimes in its original form, but more often modified to suit different conditions, a new method of research may offer unlimited opportunities to the investigator. The wise student is always on the watch for such possibilities, and the weekly meetings of the Royal Society, where papers dealing with every department of science are presented, afford the best means of securing useful suggestions.

In *The Royal Society, or Science in the State and in the Schools*, Sir William has brought together his four addresses as president. The first of these, delivered in 1902, is entitled "Supreme Importance of Science to the Industries of the Country, Which Can Be Secured Only through Making Science an Essential Part of All Education." He perceived what the average British manufacturer has so strangely ignored: the great commercial importance of education in science. With the fruits of English discovery passing into the hands of the Germans, whose universities have so long fostered and spread abroad the spirit of research, the apathy of the British public is hard to understand. Huggins, speaking in plain language, pointed to the chief source of weakness—"the too

[1] "The Advisory Relation of the Royal Society to the State" (address delivered at the anniversary meeting, November 30, 1904), *The Royal Society*, p. 89.

close adherence of our older universities, and through them of our public schools, and all other schools in the country downward, to the traditional methods of teaching of mediaeval times." He pleaded for a decrease of examinational restrictions, and for the encouragement of research and independent reasoning on the part of the student. He contrasted the listlessness and lack of interest in their work which characterize many graduates under the existing system, with the enthusiastic devotion to knowledge for its own sake aroused in the student by observation and research. As a result of the public interest excited by this address, the Council of the Royal Society adopted a resolution urging the universities to make such modifications in their regulations as will "insure that a knowledge of science is recognized in schools and elsewhere as an essential part of general education."

The address of 1903 traced the rise of the specialized societies and discussed their relation to the Royal Society, while that of the following year dealt with "The Advisory Relation of the Royal Society to the State, and the Responsible Public Duties Which Rest Permanently upon the Society." Both of these furnish much food for thought to those who are concerned with the development of our own National Academy. They demonstrate beyond a doubt that the Royal Society is by no means a mere survival from a former state of science, but a great and ever-active force, occupying a place which no other body could fill.

The last address, showing the influence of science on the life and thought of the world, is on an equally high plane with the others. In it Sir William gives an eloquent account of the awakening of scientific thought during the reign of Queen Victoria. As he recounts the work of Darwin, we can easily understand how his mind was directed from the first toward the problem of stellar evolution, in the study of which he made his richest contributions to science.

Without attempting to outline the events of a long and active life, filled with successful research, we have glanced at two phases of Sir William's career. The one, dealing with his public services, forms an important chapter in the recent history of the Royal Society, which counted him as one of the ablest in its long line of

eminent leaders. The other includes his personal investigations, which lie at the base of astrophysics. The place he has left cannot be filled. With him passes the pioneer period of celestial spectroscopy. Qualitative tests, common to this and to all other branches of science in their early stages, gave way in his own and his colleagues' hands to the precise methods which now characterize spectroscopy. The term "astronomy of precision" no longer serves to differentiate the old astronomy from the new, for rigorous and exact measurement is equally common in both. Huggins contributed largely to this end, as from the outset he measured the lines of terrestrial, stellar, and nebular spectra as accurately as the means at his disposal permitted. Giving no evidence of advancing age, he moved forward into the second period of astrophysics, never losing the alert mind of the pioneer.

Every investigator may find useful and inspiring suggestions in the life and example of Sir William Huggins. Their surest message and strongest appeal will be to the amateur with limited instrumental means, and to the man, however situated, who would break new ground.

THE VISUAL AND PHOTOGRAPHIC RANGES AND THE PROVISIONAL ORBITS OF *Y PISCIUM* AND *RR DRACONIS*

By HARLOW SHAPLEY

A large majority of the eclipsing binaries whose light decreases more than one magnitude at principal minimum are composed of bright primary stars with larger faint companions. When the loss of light is greater than two magnitudes the faint companion is very often more than four times the volume of the bright component; and in such a case the luminosity per unit area of the former may be less than one-fiftieth that of the primary star, and is very rarely greater than one-twentieth. It is probable that the division of mass between the components of the average eclipsing system is not very unequal, and, therefore, that the greater volumes of the fainter components indicate relatively low densities. Assuming an equal division of the mass for each system and uniformly illuminated stellar disks, I have found from thirty-five systems of spectral classes B and A that the average density of the bright component (whose light determines the spectrum) is one-seventh that of the sun; while the average density of the twenty-nine faint components whose spectra are not known is less than one-thirtieth the solar density. If we consider the stellar disks darkened, according to the cosine law, to zero at the limb, rather than uniformly bright, the average density of the brighter component is reduced to about one-eleventh, but the average for the faint companions is only slightly increased. For the eclipsing systems of large range of variation the "equal mass" density of the faint components is often below this average; and, further, if we take into account the spectroscopic evidence which indicates, wherever the second spectrum can be measured, that the bright component is universally the more massive, we are led to believe in the extremely great rarity of these large, relatively dark companions and to assign them to a period earlier in the natural stellar evolution than is occupied by their denser primaries.

The determination of the secondary spectra in the systems which present deep eclipses, and hence possess great inequalities of surface intensities, is of importance, therefore, in the study of the development of the close binary systems. When the component stars differ in brightness more than one magnitude, the secondary spectrum is not discernible at maximum light. During the constant minimum phase of a total primary eclipse the light is wholly that of the faint companion, but, because of the faintness of the star at that time, the duration of the total obscuration of the bright star is too short, except perhaps in a few cases,[1] to permit the making of a spectrogram. It is possible, however, to estimate indirectly the color of the faint companion by determining photographically and visually the range of variation at the principal minimum. To be an effective test the eclipse must be total, or so nearly total that the light of the bright star which still remains at the deepest phase of the eclipse does not predominate over the light of the faint companion. There are many eclipsing stars suitable for an investigation of the difference between photographic and visual ranges. At present results have been obtained for *Y Piscium* and *RR Draconis*, two eclipsing binaries of exceptionally great range. The photographic curve of *Y Piscium* was determined at the Harvard College Observatory by Miss Cannon, who discovered the variable in 1911. One-half the observations during changing light are published in *Harvard Circular No. 165*. The remainder were made recently from multiple-exposure plates and have been generously furnished in manuscript by Professor E. C. Pickering. The photographic curve of *RR Draconis* was determined by Professor Seares with the 60-inch reflector at Mt. Wilson. His light-curve is discussed in detail, and compared with the partial visual curve obtained at the Laws Observatory, in *Contributions from the Mt. Wilson Solar Observatory*, No. 64. The paper has been kindly sent to me in manuscript in advance of publication. The visual curves I have determined with the sliding-prism polarizing photometer of the Princeton University Observatory.

Y Piscium.—The period determined by Miss Cannon in *Harvard Circular No. 165* is verified by the later photographic and visual

[1] Blažko, *Annales de l'Observatoire astronomique de Moscou*, II Série, **5**, 10, 1911.

observations, and but a small adjustment of the initial epoch is necessary. The elements of light-variation are, therefore,

$$\text{Min.} = \text{J.D. } 2410002.844 + 3^{d}.76582 \text{ E, G.M.T.}$$

The published photographic observations (not including those for which only an upper limit of the brightness was estimated) have been combined with the manuscript estimates into the normal points of the following table. The published observation of August 1, 1904, is rejected because of the abnormal residual, which probably

NORMAL PHOTOGRAPHIC MAGNITUDES OF *Y PISCIUM* DURING CHANGING LIGHT

Phase	Mag.	No. Obs.	O.−C.	Phase	Mag.	No. Obs.	O.−C.
$-0^{d}.170$.....	9.30	2	+0.04	$+0^{d}.064$.....	10.72	4	−0.03
.084.....	10.40	3	+ .07	.073.....	10.50	3	− .05
.069.....	10.63	3	− .01	.080.....	10.40	3	− .01
.042.....	11.53	3	+ .14	.100.....	10.07	3	+ .02
.025.....	11.83	3	− .10	.117.....	9.73	3	− .05
−0.005.....	12.30	3	− .07	.139.....	9.53	3	.00
+0.014.....	12.40	3	+ .14	+0.173.....	9.17	3	−0.06
+0.036.....	11.60	3	+0.01				

results from a defect in the photographic plate. The manuscript observations were made from plates secured August 21, September 1, 20, and October 24, 1911. The photographic brightness at maximum is $9^{mg}.00$, and since the star is of spectral class A the same value was adopted as the normal visual magnitude. The minimum is $12^{mg}.40$. The range of $3^{mg}.40$ means a loss of more than 95 per cent of the photographically effective light of the system, and though the minimum shows no constant phase, the eclipse must necessarily be nearly, if not exactly, total.

The light-curve is not sufficiently well defined by these observations to permit a definitive solution for the orbital elements, but it is satisfactory for the preliminary derivation of an orbit that will represent closely the actual conditions in the system. Following the method of Russell for solving a partial eclipse of uniformly illuminated disks,[1] I have first obtained from the intensity-curve which best represents the whole curve at principal eclipse one

[1] *Astrophysical Journal*, **35**, 326 ff., 1912.

relation between the ratio of the radii, k, and the percentage of the small star eclipsed at the deepest phase of minimum, α_0; namely,

$$\chi(k, \alpha_0, \tfrac{1}{4}) = \frac{\sin^2\theta(\frac{1}{4})}{\sin^2\theta(\frac{1}{2})} = 1.77 ,$$

where $\theta(\frac{1}{4})$ and $\theta(\frac{1}{2})$ are the true longitudes in the orbit of the eclipsing body at the times when the loss of light is respectively one-fourth and one-half the total loss. A second numerical relation between these quantities could be secured from the equation

$$\alpha_0 = 1 - \lambda_1 + \frac{1 - \lambda_2}{k^2}$$

(where λ_1, λ_2 are the intensities at the two minima) if the secondary minimum had been observed, and without information concerning the range at both eclipses the problem is indeterminate. The maximum possible depth of the secondary (corresponding to central transit of equal stars) is less than $0^{mg}.05$, so its detection is beyond the accuracy of the observations. Because of the great range at primary, however, and because of the shape of the light-curve, the indeterminateness in this case lies between very narrow limits. Assuming different values for the depth of secondary minimum, $1-\lambda_2$, we can readily compute the possible values of k and α_0, using a table of the χ-function, and obtain the following results, which extend from the limiting and improbable case of a completely black companion to the other extreme of a grazing total eclipse.

$1-\lambda_2=$	0.000,	0.004,	0.008,	0.013,	0.017
$\alpha_0=$	0.956,	0.97,	0.98,	0.99,	1.00
$k=$	0.59,	0.595,	0.60,	0.61,	0.615

In all cases the large star is in front at principal eclipse. For the derivation of the remainder of the elements I have adopted as an example the mean of the above values, $k=0.60$, $\alpha_0=0.98$; the secondary minimum is accordingly less than one-hundredth of a magnitude. In this instance, at principal minimum 2 per cent of the light of the small bright star remains; and since the maximum loss at that time is 0.956 of the normal light, the total light of the bright star, L_2, is 0.975, and L_1 is 0.025. Of the light remaining at minimum phase, four-sevenths is that of the faint component.

The surface intensity of the bright star (in photographic light) is approximately 100 times that of its companion; the inclination of the orbit to the tangent plane is $i=83^\circ.9$; the radius of the bright star in terms of the distance of centers is $r_2=0.144$, and of the faint one, $r_1=0.240$. The least apparent distance of centers is $\cos i=0.107$. Assuming the components equally massive, the density of the bright star is 0.16 of the solar density, and that of the faint one is 0.03.

It is useless to seek for more precise values of the elements from this curve, or even from one determined with the highest accuracy possible, without some knowledge of the distribution of brightness over the stellar disks. The above values are derived on the limiting hypothesis of uniform disks. We can go to the other limit and determine the elements on the assumption of disks completely darkened at the limb. The true values will lie between these two sets and will probably be closer to the "completely darkened" elements than to the "uniform" set just derived.[1] Using tables and equations especially suitable for this problem,[2] I have readily derived from the "uniform" elements the following values on the assumption that the disk of the brighter star is darkened to zero at the edge: $k=0.77$, $r_1=0.231$, $r_2=0.178$, $i=85^\circ.7$, $\cos i=0.075$, ratio of surface intensities 67:1, and the secondary minimum would be just less than $0^{mg}.02$. The values of a_0, L_1, L_2 are the same as before. Only 2 per cent of the light of the bright star remains at deepest phase, though a much larger percentage of its disk remains unobscured. The "equal mass" densities are now 0.08 and 0.04.

The theoretical light-curves computed from these two sets of elements are practically identical. The representation of the observations is shown by the column of residuals in the table of normal magnitudes. The original estimates of brightness were made to the nearest tenth of a magnitude, and consequently the average deviation of a normal point from the computed curve of ± 0.05 is entirely satisfactory. The geometrical aspect of the system is shown in Fig. 1. The upper diagram represents the appearance on the assumption of uniform disks, and the lower

[1] Cf. discussion of the orbits of *W Delphini*, *S Cancri*, *SW Cygni*, and *U Cephei*, *Astrophysical Journal*, **36**, 269, 1912.

[2] *Astrophysical Journal*, **36**, 249–250, 1912.

represents stars completely darkened at the limb. In each case the system is represented at greatest elongation and at principal minimum. The components are relatively distant from each other and no perceptible gravitational elongation in the line of sight is to be expected. Assuming each component to have a mass equal to that of the sun, we may draw the sun to scale in the diagram and thus easily represent the relative diameters and densities. The geometrical relations in the system are practically unchanged if we

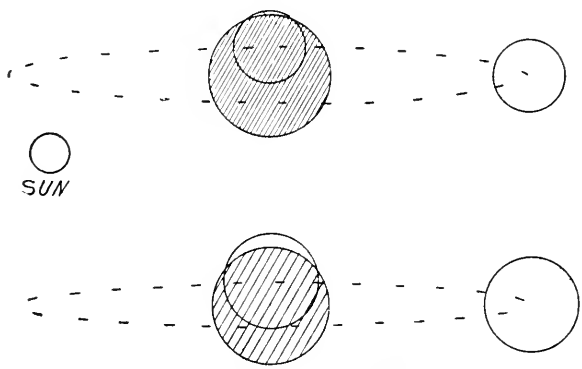


FIG. 1.—The system of *Y Piscium*

adopt other possible values of $1-\lambda_2$, but the effect on relative surface intensities and the color index will be shown later to be very considerable.

My photometric observations of *Y Piscium* are given in the table below. The comparison star is $a = B.D. + 7°5058$, ($9^{mg}.5$). During

PHOTOMETRIC OBSERVATIONS OF *Y PISCIUM*

Date	G.M.T.	Mag.	Date	G.M.T.	Mag.
1912 Sept. 8........	14^h55^m	11.79	1912 Sept. 8.......	17^h53^m	10.01
	15 12	11.90	9........	13 46	8.99
	15 28	12.04		13 58	8.92
	15 54	11.88		14 11	8.91
	16 12	11.62	Oct. 6........	18 50	9.06
	16 37	11.21		19 6	9.00
	16 51	10.99		19 23	8.94
	17 6	10.69	7........	16 30	9.02
	17 36	10.23		16 43	9.17

normal brightness of the variable I found the mean comparison star difference to be $a-v=+1^{mg}.60$. Each point in the table is the mean of 16 measures combined in the customary way so that the systematic errors of the instrument were eliminated and the variable atmospheric extinction minimized. The times are not corrected for the equation of light. The range of the variable is $3^{mg}.05$ with an uncertainty probably not exceeding $0^{mg}.05$. The difference in range, photographic minus visual, is, therefore, $+0^{mg}.35$, and this positive color index is at once an indication of the more advanced spectral type of the faint companion. If, as adopted above, the eclipse is partial, and various assumptions are made regarding the percentage of maximum eclipse, we find:

	Percentage of Light of Bright Star Remaining at Mid-Eclipse, $1-\alpha_0$				
	4.0	3.0	2.0	1.0	0.0
Visual magnitude of fainter star	13.20	12.77	12.47	12.25	12.05
Photographic magnitude of fainter star	15.00	13.64	13.08	12.67	12.40
Color index (relative to primary of spectrum A), $ph-v$	1.8	0.87	0.61	0.42	0.35
$\frac{J_2}{J_1}$(visual) uniform disks	132	85	65	50	41
$\frac{J_2}{J_1}$(visual) darkened disks	80	53	39	31	25

From this table we see that nothing more definite can be said concerning the color index of the faint companion than that it exceeds $0^{mg}.35$; that is, that the star is of a spectral type more advanced than F0.[1] It is not improbable that the color index and the relative surface brightness are much the same as those found below for *RR Draconis*, for which (by the "darkened" solution) $\frac{J_2}{J_1}=42$ and $ph-v=0^{mg}.84$. Moreover, we may find, from a further investigation of this question, that for stars like *Y Piscium* the problem can be worked backward to advantage; for if the color index should ultimately be possible of prediction for whole types of eclipsing stars of great range, the observed difference in range for the deep partial eclipses would enable us to derive highly accurate values of the percentage of the bright star's light not eclipsed, and hence also to obtain precise values of the other constants of the system.

[1] See King's $ph-v$ determinations, *Harvard Annals*, **59**, 180.

RR Draconis.—The range of variation of this eclipsing star remained unknown for many years. At minimum light it was invisible in the seven-inch equatorial of the Laws Observatory.[1] On June 20 of the present year, however, I succeeded in following it through minimum light with the 23-inch telescope, and on August 7 Professor Seares followed it during principal eclipse photographically at Mt. Wilson. My observations are given in the table below, each point representing as before 16 comparisons. The

OBSERVATIONS OF *RR DRACONIS*

Date and Epoch	G. Hel. M.T.	Phase	$v-*$	Mag.	Wt.	O.−C.	Remarks
1912 March 16 (866)	$16^h 15^m$	$+6^h 50^m$	1.04	9.98	3	0.00	Sky good
	16 26	+7 1	1.02	9.96	4	−0.02	
	16 59	+7 34	1.04	9.98	4	0.00	
	17 10	+7 45	1.04	9.98	4	0.00	Getting thick
	17 22	+7 57	1.04	9.98	4	0.00	
	17 36	+8 11	0.92	9.86	2	−0.12	Very thick
April 10 (875)	16 7	−4 48	1.12	10.06	2	+0.08	Reading uncertain
	16 19	−4 36	1.06	10.00	4	+0.02	Sky fine
	16 37	−4 18	1.04	9.98	4	0.00	
May 3 (883)	14 29	+2 0	0.91	10.76	3	−0.09	Sky poor
	14 45	+2 16	0.70	10.55	4	−0.05	
	14 58	+2 29	0.61	10.46	4	+0.01	
	15 10	+2 41	0.48	10.33	4	+0.03	
	15 36	+3 7	0.20	10.05	4	−0.04	
	15 48	+3 19	0.12	9.97	4	−0.06	
	16 0	+3 31	0.14	9.99	4	0.00	
	16 11	+3 42	0.14	9.99	4	+0.01	
June 20 (900)	13 59	−1 34	2.52	11.46	4	+0.04	Sky good
	14 13	−1 20	2.91	11.85	4	+0.05	
	14 25	−1 8	3.16	12.10	4	−0.06	Field high
	14 46	−0 47	3.78	12.72	4	−0.09	
	15 7	−0 26	4.00	12.94	4	0.00	Measures slow but good
	15 36	+0 3	3.97	12.91	4	−0.03	
	16 5	+0 32	4.02	12.96	4	+0.02	
	16 38	+1 5	3.32	12.26	4	0.00	
	16 59	+1 26	2.78	11.72	4	+0.09	

comparison stars were $c = B.D. + 62^\circ 1644$, ($8^{mg}.5$), $d = B.D. + 62^\circ 1642$, ($9^{mg}.3$), and the maximum light measures give $v-c = +1^{mg}.04$, $v-d = +0^{mg}.13$. The mean times of the observational groups are corrected for the equation of light. The heliocentric date of the minimum observed by me on June 20 is J.D. 2419574.648, G.M.T. It is exactly 900 periods after the initial epoch derived by Seares.[2]

[1] *Laws Observatory Bulletins*, Nos. 6 and 9, 1905, 1907.

[2] *Ibid.*, No. 9, 1907.

The correction, O.—C., to Hartwig is +17 minutes; to the ephemeris from the final elements of *Laws Observatory Bulletin*, No. 9, is −7.3 minutes; and to the ephemeris computed from the elements derived by Seares at Mt. Wilson is +4 minutes. Accordingly, the elements of light-variation I have adopted are:

$$\text{Min.} = \text{J.D. } 2417026.682 + 2^{d}.831073 \text{ E, G.M.T.}$$

and this formula is used to compute the phases in the table of observations. The maximum light of the variable is $9^{mg}.98$, adopted from Seares's determination at the Laws Observatory, and the visual range is $2^{mg}.96$. I have used the visual observations as the basis of a derivation of provisional elements on the limiting hypotheses of uniform and completely darkened disks. The results are as follows:

	Uniform	Darkened
Ratio of stellar radii	0.40	0.58
Radius of bright star	0.099	0.131
Radius of faint star	0.249	0.226
Inclination of orbit	82°.2	85°.7
Least apparent distance of centers	0.136	0.076
Ratio of surface intensities	90:1	42:1
"Equal mass" density of bright star	0.86	0.37
"Equal mass" density of faint star	0.05	0.07
Depth of secondary minimum	$0^{m}.01$	$0^{m}.02$

The principal eclipse is total and lasts for $7^{h}10^{m}$. The light is constant at minimum for $1^{h}20^{m}$, and its value at that time, $12^{mg}.94$, is the visual magnitude of the faint star. The light of the small bright star is 0.934, and of the faint star is 0.066, in terms of the total light of the system. The spectrum of the bright component is classified at Harvard as A5?.[1] The photographic range of variation is $3^{mg}.80$. The color index of $0^{mg}.84$ is that of a G4-type spectrum, provided the bright star is of type A; and corresponds to a G8-type if the primary spectrum is A5.[2] In any case we may safely say that the faint companion is a considerable distance below

[1] Note added February 12, 1913. Professor Pickering writes in a recent letter that Miss Cannon has examined the spectrum of *RR Draconis* again and finds it to be of type A.

[2] King, *op. cit.*

the brighter star in the spectral series. It is quite possible that the visual range would be measured greater or less by another observer, depending on the sensitiveness of the eye to the light the reddish star. The various values of the ranges of some eclipsing stars given by different observers is perhaps to be partly accounted for in this way. Some observers have noted a reddish color of the light at minimum phases, and Blažko[1] has found spectroscopic evidence of a yellow companion to *U Cephei*. The system of *RR Draconis* is represented diagrammatically in Fig. 2 with the "equal

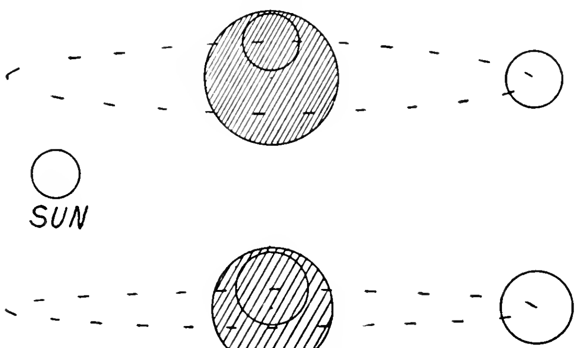


FIG. 2.—The system of *RR Draconis*

mass" sun drawn in to scale. The relatively high density of this system is to be noted.

Although the uncertainty of some of the numerical results obtained for the above stars is somewhat large, the investigation has given a definite indication of the probable outcome of the future study of the spectra of the faintly illuminated, low-density components of certain eclipsing binaries. Preliminary results for other stars, whose ranges of variation are being investigated, confirm the conclusions reached concerning *Y Piscium* and *RR Draconis*.

PRINCETON UNIVERSITY OBSERVATORY
October 28, 1912

[1] *Op. cit.*

DISTRIBUTION OF ENERGY IN THE SPECTRA OF PLATINUM, PALLADIUM, AND TANTALUM

By GEORGE VEST McCAULEY

In an early investigation of the emissive properties of certain solids Paschen[1] concluded that the spectral distribution of energy could be fairly well expressed by the empirical relation

$$E_{\lambda T}=C_1\lambda^{-a}e^{-\frac{C_2}{\lambda T}} \qquad (1)$$

for which the wave-length of maximum emission is given by the equation

$$\lambda_m T=\frac{C_2}{a}=\text{constant} \qquad (2)$$

where C_1, a, and C_2 are constants depending upon the substance, while T and λ denote absolute temperature of the source and wave-length respectively.

Subsequent attempts to verify this law for metallic radiators have led to widely different results. Lummer and Pringsheim,[2] working with platinum, but only for a very narrow range of temperatures, obtained results that seemed to confirm Paschen's conclusions except that they found the product, $\lambda_m T$, to be only 2620, which is somewhat less than the value obtained by Paschen. Lummer and Kurlbaum,[3] on the other hand, and more recently, W. W. Coblentz,[4] have shown that the exponent "a" is not constant for any one metal, but must be regarded as a function of the temperature if Paschen's law is to express metallic emission. A further result of Coblentz's[5] investigation was that the wave-length of maximum emission, when computed from a single isothermal energy-curve by means of the easily obtained relation,

$$\lambda_m=\frac{\lambda_1\lambda_2\log\dfrac{\lambda_2}{\lambda_1}}{(\lambda_2-\lambda_1)\log e}, \qquad (3)$$

[1] *Weid. Annalen*, **58**, 455, 1896; **60**, 662, 1897.

[2] *Ber. d. Deutsch. phys. Ges.*, **1**, 215, 1899.

[3] *Ibid.*, **17**, 106, 1898.

[4] *Bul. Bur. Standards*, **5**, 339, 1909.

[5] *Phys. Rev.*, **29**, 553, 1909.

previously used by Paschen,[1] was not constant, but increased with increased differences of λ_1 and λ_2. Since equation (3) demands only the general assumption that C_1, a, and C_2 be functions only of the temperature, this result seemed to indicate that even in this more general form Paschen's law was inadequate.

By assuming the empirical law,

$$E_{\lambda T}=C_1\lambda^{-a}\frac{1}{e^{\frac{C_2}{\lambda T}}-1} \tag{4}$$

which is similar to the energy distribution law deduced by Planck[2] for a "black body," and computing the wave-length of maximum emission for a given isothermal curve, Coblentz[3] obtained results that were more nearly constant than with Paschen's form of the law.

In order further to determine the relative agreement of the two assumed laws (1) and (4) with experimental data, the present investigation was undertaken. It was proposed also to compare the emission of the metals studied with that of a "black body," a comparison which had hitherto been impossible because of the lack of an accurate method for measuring the true temperatures of incandescent metal filaments.

In selecting a criterion with which to test the two laws (1) and (4), the writer wished to avoid any that demanded an accurate knowledge of the wave-length of maximum emission. This seemed desirable in view of the difficulty of locating the exact position of this maximum because of atmospheric absorption bands.

The desired end was attained by the following simple solution of equations (1) and (4):

Let E_1, E_2, and E_3 be the emissivities at a given temperature T for the wave-lengths λ_1, λ_2, and λ_3 such that

$$\lambda_3=2\lambda_1=4\lambda_2 .$$

Then since C_1, a, and C_2 are assumed to be functions only of the temperature, equation (1) admits of the solution

$$a=\frac{2\log\frac{E_1}{E_3}+\log\frac{E_1}{E_2}}{\log 2} \tag{5}$$

[1] *Wied. Annalen*, **60**, 665, 1897.

[2] *Vorlesungen über die Theorie der Wärmestrahlung*, Leipzig, 1906.

[3] *Phys. Rev.*, **31**, 317, 1910.

and similarly equation (4) admits of the solution

$$a=\frac{2\log\frac{E_1}{E_3}+\log\left\{\frac{E_1}{E_2}+\frac{2E_3}{E_1}+\sqrt{\left(\frac{E_1}{E_2}\right)^2-\left(\frac{2E_3}{E_1}\right)^2+\frac{4E_3}{E_2}}\right\}}{\log 2}-1 \qquad (6)$$

which should be constant for a given isothermal curve, if the assumed laws are correct.

For comparing the emission of the metals with that of a "black body," the true temperatures of the metal filaments were first measured and the "black-body" emission for all wave-lengths and this same temperature computed by means of Planck's "black-body" distribution law,

$$E_{\lambda T}=C_1\lambda^{-5}\frac{1}{e^{\frac{C_2}{\lambda T}}-1}\,. \qquad (7)$$

Then by assuming Kirchhoff's law

$$\frac{E_{\lambda T}}{\epsilon_{\lambda T}}=A_{\lambda T}=(1-R_{\lambda T}) \qquad (8)$$

where $E_{\lambda T}$, $A_{\lambda T}$, and $R_{\lambda T}$ represent the emissive, absorptive, and reflecting powers respectively of the metal, while $\epsilon_{\lambda T}$ denotes the corresponding emissivity of a "black body," the reflecting powers of the metals were determined as a function of wave-length and absolute temperature.

METHOD AND APPARATUS

The energy-curves were obtained with the usual spectro-bolometric arrangement shown in Fig. 1. The radiating source at O was inclosed in a suitable vacuum chamber V_1 separated by a rock-salt window W_4 from the large vacuum chamber V_2 which inclosed the entire spectrometer and bolometer. Thus the whole path of the radiant energy was *in vacuo* to eliminate atmospheric absorption as far as possible. The pressure in the spectrometer inclosure was maintained at about 15 mm of mercury by means of a water aspirator.

The spectrometer was of the Wadsworth[1] prism-mirror fixed arm type, furnished for part of the work with a rock-salt prism of

[1] *Phil. Mag.*, **38**, 337, 1894.

$59°57'44''$ refracting angle, 7-cm refracting edge, and 11-cm base, for the rest with a rock-salt prism of $60°5'35''$ refracting angle, 8-cm refracting edge, and 6-cm base.

The slit-width was such as to give an image at the bolometer strip of the same width as the strip itself and was maintained constant for all parts of the spectrum to insure the best possible agreement of measured and actual energy as pointed out by Runge.[1]

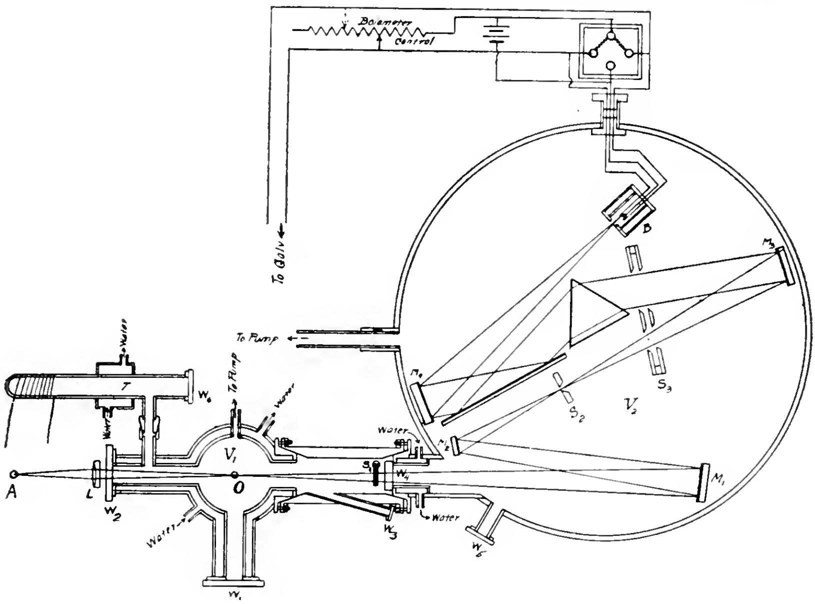


FIG. 1

The galvanometer used was described in a previous paper by Weniger.[2] It had a total resistance of about 7 ohms, a period of approximately 6 sec. with which it was practically aperiodic, and a current sensibility of 4×10^{-10} amperes for a deflection of 1 mm at a distance of 1.5 m. Suitable resistances were placed in series with the galvanometer to reduce its sensibility for work in the various parts of the spectrum.

Two bolometers were employed with strips 12 mm long, blackened with camphor smoke. With the large prism the bolom-

[1] *Zeit. f. Math. und Phys.*, **42**, 205–213, 1897.

[2] *Phys. Rev.*, **31**, 393, 1910.

eter width was 0.2 mm and with the small prism 0.5 mm. The remaining arms of the bolometer bridge were made of No. 22 manganin wire to minimize thermal effects. These coils were wound non-inductively, in the usual way, on a water-filled brass cylinder of about 300 cc capacity and were inclosed in a small wooden box outside the spectrometer vacuum for convenience of adjustment. The bridge balance was controlled by shunting the proper coil with a high resistance, a method recently employed by Abbot and now in general use in this laboratory.

The vacuum chamber V_1 (Fig. 1), in which the filaments were operated, consisted of a water-cooled brass tank of 12.5 cm internal diameter and 20 cm high supported from the large tank V_2 by means of a brass sleeve. The base, to which was attached all the electrical connections and mountings for the filaments, was also water-cooled and detachable. The tank and connecting sleeve were provided with windows W_1, W_2, and W_3 for purposes of calibration and temperature measurements. A steel tube, T, with a water-cooled jacket was fitted to the tank and used for heating calcium[1] to absorb residual oxygen and nitrogen. A triple metal screen S_1 was placed just in front of the rock-salt window W_4 and operated as a shutter through a packed joint in the lower side of the sleeve. All joints were conical and ground to fit, so that a thin film of tallow and beeswax mixture rendered them absolutely tight.

The metal filaments were mounted on heavy copper posts and were capable of being displaced laterally to a slight extent by rotating the base of the tank. This provided for focusing the filaments on the slit and for rotating them to one side during the galvanometer-bolometer calibration to be described later. The copper post clamping the lower end of the filament passed through a packed joint and was provided with an adjusting screw by means of which the filaments could be kept straight at all temperatures.

THE ENERGY-CURVES

The galvanometer deflections were read for successive angles of incidence differing by one minute of arc for a considerable distance on each side of and through the region of maximum emission.

[1] Soddy, *Proc. Roy. Soc.*, **78**, 439, 1906.

For the extreme short and long wave-length portions of the spectrum, the angle of incidence was varied by five minutes of arc. The sensibility of the galvanometer was adjusted, whenever possible, so that the deflections, which were observed to the nearest millimeter, were 10 cm or more. The zero reading was taken before and after each deflection to correct for drift which was small and uniformly steady in one direction during the time necessary to obtain several energy-curves.

The galvanometer deflections were found to be not proportional to the energy falling upon the bolometer strip, which necessitated a proportionality correction determined as follows:

A wide filament carbon lamp, furnished by the General Electric Co., was placed at A (Fig. 1) behind the source O. An achromatic combination of lenses L formed an image of the carbon filament at O, the metal filament being rotated now to one side. The prism was then turned $44'$ from the minimum deviation position of the D-line, corresponding to a wave-length of $1.6\,\mu$ or $1.7\,\mu$ depending upon the prism used, and the temperature of the carbon filament raised to 1457° C. "black-body" temperature as measured with an optical pyrometer using wave-length $0.658\,\mu$. By means of rotating sectors the radiation from the carbon filament was reduced in intensity in known ratios, and the corresponding deflections of the galvanometer observed. The unit of intensity was arbitrarily chosen as one-twentieth of the intensity of the carbon lamp radiation with no sector interposed. In this way a calibration-curve was obtained connecting galvanometer deflections with units of intensity by means of which it was possible to transform at once from deflection-displacement to intensity-displacement curves.

This calibration was made each time that a series of observations was taken and was made for each sensibility of the galvanometer. Energy-curves thus obtained at widely different times could be compared readily and were not affected by sensibility changes of the bolometer or galvanometer for any given arrangement of the spectrometer.

When using the narrower bolometer strips and corresponding slit-width it was found unnecessary to apply the slit-width correction of Runge (*loc. cit*), the first term of which made less than 1 per

cent change in the intensity in the region of maximum emission where the correction is greatest. With the wider bolometer strips, however, it was necessary to use the first term of the correction.

The dispersion correction for converting prismatic to normal energy-curves was obtained in the usual manner from the dispersion-curve of rock-salt and the computed spectrometer readings for the minimum deviation corresponding to various refractive indices.

The dispersion-curve used in the calibration was obtained from the data[1] of Martin's, Rubens, Rubens and Snow, Langley and Abbot,[2] and was corrected for *vacuo* by means of the dispersion-curve of air determined by Kayser and Runge,[3] extrapolating to long wave-lengths by means of the dispersion formula

$$n=1.00028817+1.316\,\lambda^{-2}+31600\,\lambda^{-4}$$

which these investigators found would express their results as far as the dispersion had been determined.

The above extrapolation was deemed justifiable in view of the fact that it gave correct wave-lengths for the experimentally located positions of the atmospheric absorption bands at $1.1\,\mu$, $1.4\,\mu$, $1.8\,\mu$, and $2.6\,\mu$ in the solar spectrum and for the CO_2 emission band at $4.4\,\mu$ of the bunsen burner.

The mean wave-length of the D-lines was chosen as the zero of reference; and the emission band at $4.4\,\mu$ from the bunsen burner, together with the sylvite absorption bands at $3.2\,\mu$ and $7.2\,\mu$, were used as checks on the accuracy of the spectrometer adjustments. The zero setting and check were always made first in air, using for this purpose the dispersion-curve of rock-salt with respect to air. Then after evacuating the spectrometer tank, the bunsen burner emission was observed again to check the vacuum correction applied to the dispersion-curve. In each case the band at $4.4\,\mu$ was located to within less than 0.5 per cent of the computed position.

THE "BLACK-BODY" ENERGY-CURVES

The "black-body" emission with which the metallic emission was compared was not measured directly, but was computed by means of Planck's distribution law in the following manner.

[1] Kayser, *Handbuch der Spectroscopie*, 4, 493.

[2] *Annals of Astrophys. Obs.*, I.

[3] *Astron. and Astrophys.*, 428, 1893.

With an arbitrary value of the constant C_1 in the above law, the "black-body" emission was computed for the absolute "black-body" temperature T_b at which the filament was operated and for the particular wave-length, 0.658 μ, at which the "black-body" temperature was measured. Equating this computed "black-body" emission to the observed metallic emission for this same wave-length, by virtue of the definition of "black-body" temperature, a value of C_1 was obtained which would refer the computed "black-body" emission to the same unit as the observed metallic emission. With this new value of the constant C_1 a "black-body" energy-curve was then computed for the absolute true temperature T_t of the metal filament. The inverse ratio of the ordinates of this curve to those of the observed energy-curve determined the reflecting powers according to equation (8).

TEMPERATURE MEASUREMENTS

The metals were obtained in sheet form ranging from 0.1 to 0.2 mm thick. After rolling to about 0.05 mm they were cut into strips 3.5 cm long by 7 mm wide and folded into wedge-shaped filaments with 12° openings. The true temperatures were measured through the window W_1 (Fig. 1) in the manner described by Professor Mendenhall[1] with an optical pyrometer of the Holborn-Kurlbaum type[2] calibrated from the palladium point as measured by Day and Sosman.[3]

In the case of tantalum the "black-body" temperatures were measured simultaneously with the "true" temperatures through the window W_3 by Dr. Forsythe and the writer as explained in the paper[4] referred to above. The relation of "true" to "black-body" temperature for this metal is shown in curve *a* (Fig. 2). In the case of platinum and palladium this relation was obtained from data by Professor Mendenhall,[5] and Waidner and Burgess[6] respectively and is shown in curve *b* (Fig. 2).

[1] *Astrophysical Journal*, **33**, 91, 1911.

[2] C. E. Mendenhall, *Phys. Rev.*, **33**, 74, 1911.

[3] *Amer. Jour. Sci.* (4), **29**, 93, 1910.

[4] *Astrophysical Journal*, **33**, 91, 1911.

[5] *Loc. cit.*

[6] *Bul. Bur. Standards*, **3**, 202, 1907.

EXPERIMENTAL RESULTS

Tantalum.—This metal was obtained from Siemens & Halske, and after rolling was polished with No. 0000 emery paper and washed free from grease and moisture with alcohol. The vacuum in which it was operated was maintained for two days with a Pfeifer rotating mercury pump running continuously, and the calcium was kept at a cherry-red heat for five or six hours prior to

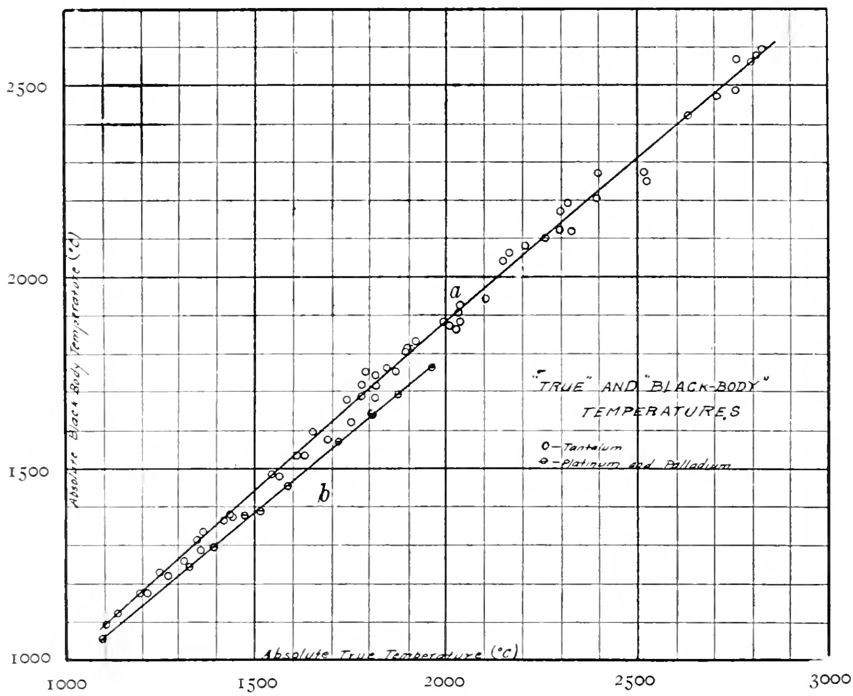


FIG. 2

making observations. These precautions were found necessary because of the great influence of residual gases and grease on this metal at temperatures above 700° C. In the preliminary work it was found that the slightest trace of residual gas would cause the resistance to increase rapidly, and the metal would become brittle, losing its smooth metallic surface. This was also observed by Coblentz[1] and Pirani.[2] With continuous pumping from a per-

[1] *Bul. Bur. Standards*, 5. 374, 1909. [2] *Ber. d. Deutsch. phys. Ges.*, 5, 308, 1910.

fectly tight vacuum tank and with heated calcium, it was possible to operate the filaments throughout the entire range of temperatures to within a few degrees of the melting-point of the metal. Even at the very highest temperatures the current and voltage would remain constant, which was considered good working-conditions. After making the observations for five or six isothermal curves, the filaments were found to be ductile with a bright metallic surface as initially, except that they showed the effects of flaking produced by the high temperatures.

The values of "a" computed from equations (5) and (6) were not constant but diminished rapidly at first, reaching an approximately constant value at long wave-lengths. The departure from constancy was more marked in the case of the first of the assumed laws. It must be concluded, therefore, that previous measurements of the temperature variation of this exponent, based upon the assumption that a law of form (1) completely expressed the observed facts, are open to adverse criticism. No consistent variation of "a" with temperature was observed in this instance for a given region of the spectrum. The variation observed with increasing wave-length means nothing more than an erroneous initial assumption regarding the law of energy distribution. Had the assumption demanded the "constants" of (1) and (4) to be functions of wave-length, quite other conditions than (5) and (6) would have resulted for testing the respective laws.

The reflecting powers of tantalum as computed in the manner previously described from a comparison with "black-body" emission are shown in Fig. 3. Curve *a*, shows the values obtained by Coblentz[1] from direct reflection experiments made at room temperature. It is to be observed that the reflecting power diminishes with increasing temperature for wave-lengths greater than $0.7\,\mu$, the decrease being most rapid in the region from $0.8\,\mu$ to $2.0\,\mu$, wherein occurs a minimum reflecting power for high temperatures. For wave-lengths less than $0.7\,\mu$ there seems to be an increase of reflecting power with temperature. This increase was also observed from measurements of the "true" and "black-

[1] *Bul. Bur. Standards*, **7**, 207, 1911.

body" temperature at 0.658 μ with optical pyrometers.[1] The reflecting powers obtained by this method are shown in Fig. 4, curve *a*.

The variations of the product $\lambda_m T$ with temperature are indicated by Fig. 5. The wave-lengths λ_m were not computed by means

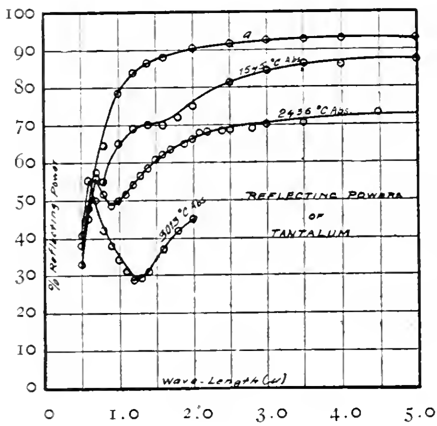


FIG. 3

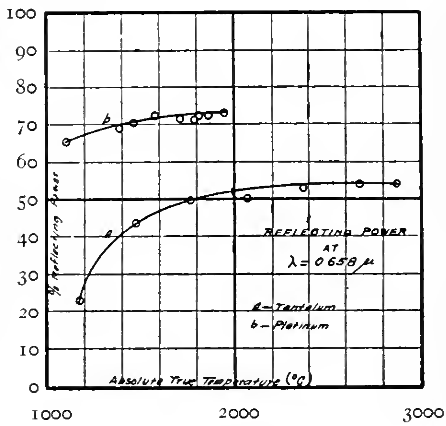


FIG. 4

of any formula resulting from an assumed energy distribution, but were taken directly from the experimental curves. The relation

$$\lambda_m T = \text{const.}$$

is seen at once not to obtain for tantalum. The wave-length of maximum emission shifts much more slowly toward shorter wave-

[1] Let E and E' be the emissivities of the metal from within the wedge and from the outside surface respectively as measured with the pyrometer. Let T_t and T_b be the corresponding "true" and "black-body" temperatures. Then by definition of these temperatures we have by Wien's "black-body" distribution law

$$\frac{E'}{E} = \frac{C_1 \lambda^{-5} e^{-\frac{C_2}{\lambda T_b}}}{C_1 \lambda^{-5} e^{-\frac{C_2}{\lambda T_t}}} = e^{\frac{C_2}{\lambda}\left(\frac{1}{T_t} - \frac{1}{T_b}\right)}$$

or

$$\log \frac{E'}{E} = \frac{C_2}{\lambda}\left(\frac{1}{T_t} - \frac{1}{T_b}\right) \log e$$

Then by assuming Kirchhoff's law

$$\log \frac{E'}{E} = \log a = \log (1 - R) = \frac{C_2}{\lambda}\left(\frac{1}{T_t} - \frac{1}{T_b}\right) \log e$$

from which the reflecting power (R) may be computed as soon as T_t and T_b are known.

lengths with increasing temperature than for a "black body," especially at the higher temperatures. Furthermore, tantalum has its maximum emissivity for low temperatures at a shorter wave-length, and for high temperatures at a longer wave-length than does a "black body."

A few of the spectral energy-curves for tantalum are shown in Fig. 6. They are seen to be perfectly continuous with no bands of

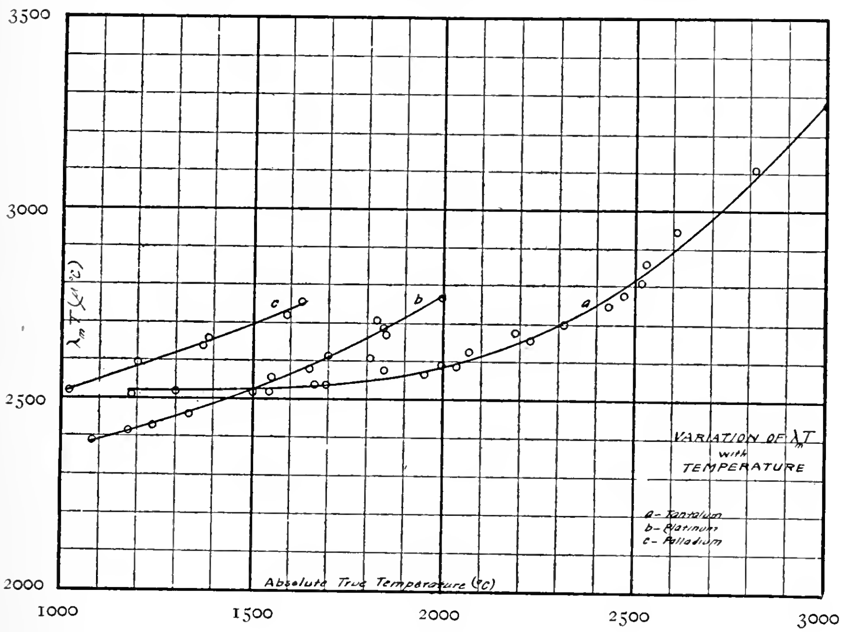


FIG. 5

selective emission. In general form the curves do not differ strikingly from "black-body" energy-curves, of which one is shown in curve *a* (Fig. 6) to the same scale. The emissivity of tantalum, however, diminishes more rapidly than that of the "black body" at the same temperature in the infra-red.

Platinum.—The filaments for this part of the work were cut from platinum obtained from the firm of Baker & Co. They were polished, first with a rounded steel tool on a smooth plate glass, then with No. 0000 emery paper, and cleaned with a dry cloth. They were operated under a pressure of less than 0.1 mm of mercury, merely to eliminate atmospheric absorption as far as possible.

The values of "a" computed as for tantalum showed similar variations indicating a better agreement in the case of the second of the assumed laws. Here, also, no consistent variation of this exponent with temperature was observed.

A comparison of the spectral energy-curves with those of a "black body" gave the reflecting powers shown in Fig. 7. Curve a was obtained from values observed by Coblentz[1] at room temperature. The same general variations occur here as for tantalum except that the reflecting power in the visible spectrum is more nearly constant for platinum. This constant value in the visible is better illustrated in curve b (Fig. 4) which gives the reflecting powers computed from the relation between "true" and "black-body" temperatures at 0.658 μ as determined by Professor Mendenhall.[2]

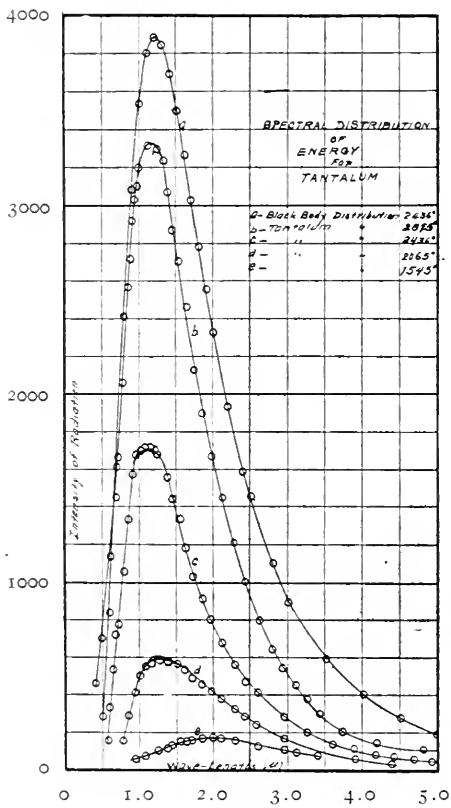


FIG. 6

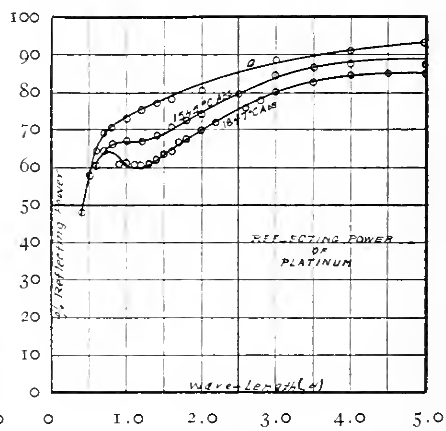


FIG. 7

The variations of $\lambda_m T$ with temperature are shown in Fig. 5. It will be seen that the same general tendency for this product to

[1] *Bul. Bur. Standards*, **7**, 207, 1911.

[2] *Astrophysical Journal*, **33**, 91, 1911.

increase with temperature obtains here as for tantalum, the increase being more rapid at the higher temperatures.

The spectral distribution-curves are shown in Fig. 8. They are seen to be very similar to those of tantalum, showing a rapid decrease of emission in the infra-red as compared with "black-body" emission shown in curve *a*.

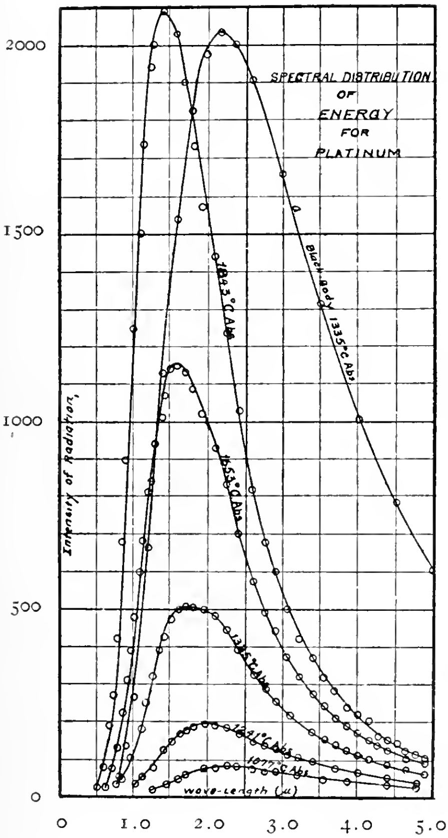


FIG. 8

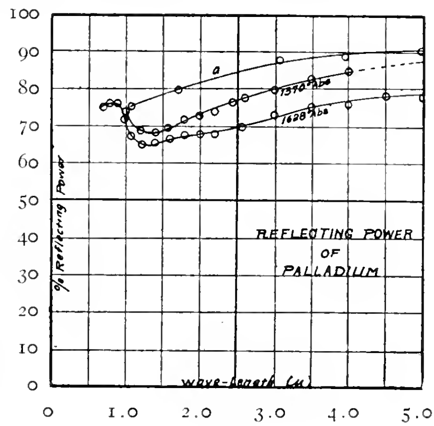


FIG. 9

Palladium.—This metal was obtained from Eimer & Amend, and after being rolled to the required thickness was polished with No. 0000 emery paper. The filaments were operated under the same conditions as the platinum. Some difficulty was experienced in keeping the wedge from opening at the higher temperatures, but by repeated trials it was possible to obtain energy-curves for temperatures ranging from 1000° to 1600° abs.

Owing to the small amount of energy available in the long and short wave-lengths for temperatures at which this metal was operated, it was possible to obtain values of "*a*" through only a narrow region of the spectrum. Even in these narrow limits, however,

the evidence seemed to be in favor of the second of the assumed laws.

The reflecting powers computed as for tantalum and platinum are shown in Fig. 9. Curve *a* gives the values observed by Coblentz.[1]

The dependence of $\lambda_m T$ upon temperature is seen from Fig. 5 and is similar to that of platinum. The values are higher for a given temperature than for either of the other metals, indicating that there is no direct relation between atomic weight and wave-lengths of maximum emission as suggested by Kayser,[2] from the early work of Jacques.

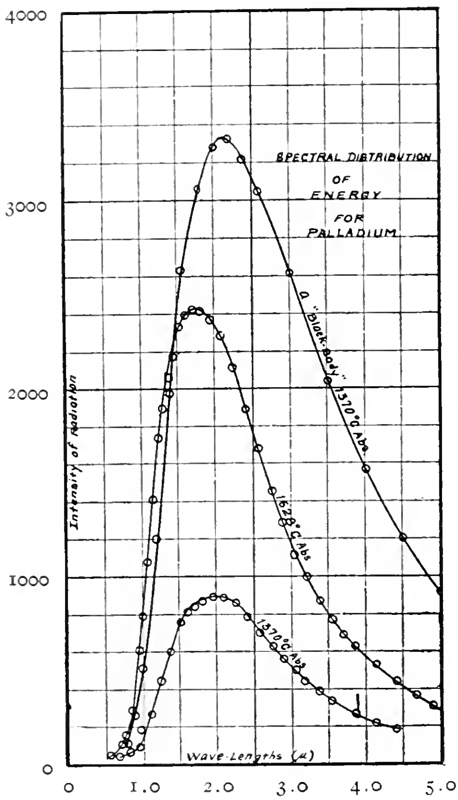


FIG. 10

In Fig. 10 is shown the energy distribution for two temperatures together with that of a "black body."

Owing to the fact that these curves as well as those for platinum were obtained at a time when the humidity of the atmosphere was high, a slight fogging of the rock-salt window separating the two vacuum chambers was practically unavoidable when the smaller chamber was opened to replace the filaments. Consequently the water vapor absorption band at $1.8\,\mu$ shows much stronger in these curves than in the tantalum curves, making the wave-length of maximum emission for some of the curves less certain.

DISCUSSION OF RESULTS

The constant decrease in the values of "*a*" as the longer wave-lengths are considered proves without a doubt the inadequacy

[1] *Bul. Bur. Standards*, **2**, 470, 1906.

[2] *Handbuch der Spectroscopie*, **2**, 92.

of the assumed laws (1) and (4). Even for the second of these, which shows the better agreement, the departure from observed values is considerable. Assuming a constant mean value of "*a*" as determined for a given isothermal curve, the computed emission is in general from four to seven times smaller than the observed values at 6.0 μ. The agreement is, of course, better for shorter wave-lengths, but correspondingly worse for longer ones.

Reflecting powers determined in the manner here described are apt to be in error by several per cent, as the "black-body" curves were obtained from a knowledge of the emissivity of the metals at a part of the spectrum where bolometric measurements are somewhat uncertain. The error in the present instance arising from this source was estimated to be from 2 to 10 per cent, being less, of course, at high temperatures. Absolute values only are affected in this way, hence the reflecting powers obtained in this manner show in a qualitative way the relative emissivities of a metal for various wave-lengths and temperatures.

The diminution of reflecting power with increasing temperature for long wave-lengths is in qualitative agreement with the measurements of Hagen and Rubens[1] on the reflecting power of platinum for the residual rays of fluorite as a function of temperature.

The shift of wave-length of maximum emission (Fig. 5) is less rapid for tantalum than for platinum or palladium for low temperatures. Hence temperature estimates[2] of tantalum made from the relation

$$\lambda_m T = 2620,$$

which Lummer and Pringsheim[3] found for platinum, is correct only for a temperature in the neighborhood of 2000° abs.

Fig. 11 shows the agreement of the values of $\lambda_m T$ for platinum with those observed by Paschen.[4] The values obtained by the previous investigator are lower in general than those of the present writer, which, however, are more consistent among themselves. The slight disagreement may be due to the different temperature scales employed or to conduction losses from the thermocouples

[1] *Ann. d. Phys.*, **11**, 888, 1903.

[2] W. W. Coblentz, *Bul. Bur. Standards*, **5**, 375, 1909.

[3] *Loc. cit.*

[4] *Ann. d. Phys.*, **60**, 70, 1897.

used in the earlier work for measuring temperatures. The constant value found by Lummer and Pringsheim seems to have been in error, due possibly to the small temperature range employed, or to lack of polish of the radiating surface. This latter would tend to give a constant value of $\lambda_m T$ as indicated by Pachen's[1] results for such poorly reflecting substances as iron oxide, copper oxide, and lampblack.

SOURCES OF ERROR

The usual errors attendant upon spectro-bolometric measurements from scale readings and subsequent reduction to energy were

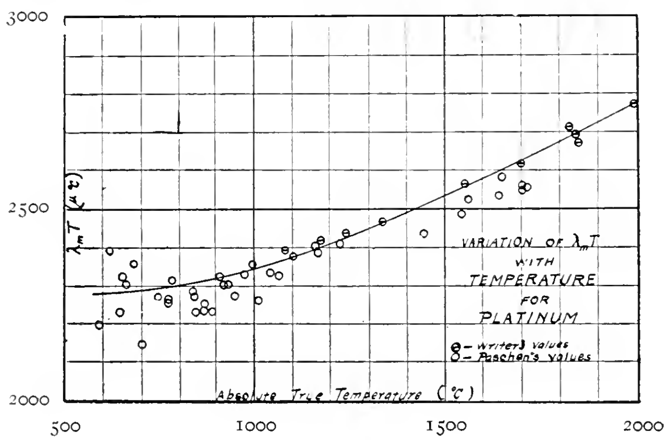


FIG. 11

manifest here. These were minimized as far as possible by employing large deflections, and were estimated to be less than 1 per cent except for the long and short wave-length extremities of the curves. The use of a rock-salt prism further reduced the possibility of error in making the dispersion correction in the region of maximum emission. The point of inflection of the dispersion-curve, at which the dispersion correction is difficult to determine, occurs at about $3.0\,\mu$ for rock-salt and hence is well beyond the wave-length of maximum emission except for very low temperatures.

Of the constant errors, the reduction of the rock-salt dispersion to *vacuo* is perhaps the most questionable. The CO_2 emission

[1] *Ann d. Phys.*, **58**, 455, 1896; **60**, 663, 1897.

check previously described, however, seemed to justify such a correction. That there are no regions of anomalous dispersion for air as far as $20.0\,\mu$ is known from determinations of the dispersion of rock-salt, fluorite, sylvite, quartz, and other substances with respect to air for this region of the spectrum. It is possible that the dispersion-curve of air may have a point of inflection in the infra-red, similar to that of rock-salt and fluorite. This would tend to diminish the corrected values of the refractive indices in the dispersion-curve of rock-salt and would thereby increase the corrected wave-lengths in the infra-red. If such a point of inflection does occur in the dispersion-curve for air, it must be beyond $4.4\,\mu$ or else the departure from the assumed law is small, as the precise location of the CO_2 emission band indicates.

To make sure that no constant error was being introduced by using the bunsen burner emission as a check for the vacuum correction to dispersion and for the zero displacement, it was necessary to show that the position of this band was not altered by the absence of CO_2 in the path of the ray. This was accomplished by observing its position in air containing CO_2 and then in air freed from this gas. Its position was found to be unaltered by the absence of CO_2.

Zero changes of the spectrometer, which are usually very troublesome and almost inevitable, were reduced to a minimum in this instance by the use of iron fittings and mirror supports for the spectrometer. Every part of the spectro-bolometer was carried by heavy iron rods and castings from the base of the spectrometer.

A final source of error is that due to the possible non-black character of the bolometer for certain regions of the spectrum. Any error from this source, however, as shown by Royds,[1] would be of the order of 1 per cent or less beyond $0.8\,\mu$ and is therefore negligible in this instance.

SUMMARY

The results of the present investigation may be summarized as follows:

1. The distribution of energy in the spectra of tantalum, platinum, and palladium was determined for temperatures ranging from 1000° abs. to the respective melting-points of the metals.

[1] *Phil. Mag.*, **21**, 172, 1911.

2. The assumed metallic radiation laws

$$E_{\lambda T}=C_1\lambda^{-\alpha}e^{-\frac{C_2}{\lambda T}}$$

and

$$E_{\lambda T}=C_1\lambda^{-\alpha}\frac{1}{e^{\frac{C_2}{\lambda T}}-1}$$

were tested by a simple method and were shown to be inadequate to express the emission of the metals.

3. The metals studied were shown to acquire a minimum reflecting power in the early infra-red, becoming more marked at high temperatures.

4. The dependence of reflecting power upon absolute temperature was shown to be similar for all three metals in the infra-red. In the visible spectrum, however, this dependence was found to be different for the different metals.

5. The product $\lambda_m T$ was found to be not constant, but increased rapidly with temperature. For tantalum the absolute value of this product was found to be greater than for a "black body" above 2600° abs.

6. No direct relation was found to exist between atomic weight and wave-length of maximum emission.

In conclusion the writer wishes to express his thanks to Professor Mendenhall for the many helpful suggestions offered and for the special apparatus necessary to carry on the investigation; also to Dr. W. E. Forsythe for his assistance in parts of the work.

University of Wisconsin
August 1911

VACUUM TUBE DISCHARGE IN A MAGNETIC FIELD

By NORTON A. KENT AND ROYAL M. FRYE

While experimenting with a hydrogen vacuum tube placed in a magnetic field, it was found that when the field was created, the visual appearance of the discharge was entirely altered. While it was to be expected that the field would tend to alter the course of the ions, the sweeping nature of the alteration in spectrum, revealed by the direct-vision spectroscope, was a surprise. A search was made through the literature of the subject.[1] The most important results recorded in previous investigations may be briefly summarized as follows:

When a vacuum tube is set vertically in a horizontal magnetic field, there generally results a change in the visual appearance of the tube and also a change in spectrum analogous to that attending the insertion of a Leyden jar in the circuit. The discharge in the field is concentrated in certain parts of the capillary, causing increased resistance; the effect varies with the strength of the field, is most prominent directly between the poles, and is sometimes annulled altogether by increase of temperature. Pressure of the gas, impressed voltage, and kind of glass used modify the result; while reversal of the direction of the discharge (or of the field) produces merely a temporary change. Various gases have been used. With nitrogen Chautard observed that the spectrum developed in the field was the ordinary band spectrum except that the red and orange regions were almost entirely absent. This result he attributed to the change in the effective diameter of the tube. In the case of sulphur, Van Aubel noted a change from a band to a line spectrum. In addition to these spectroscopic investigations, much work has been done upon the influence of the magnetic field upon the resistance of the tube.

In short, in papers previously published, certain writers have hinted at the true cause of the observed changes in spectrum, but, strange as it may seem, not one of them has stated clearly and

[1] See bibliography on p. 189.

fully the substances to which the resulting spectra are in general due. Some observers have noted that the effect is similar to that obtained by the introduction of capacity, resulting in a change from one spectrum of the gas to another of the same gas—which, indeed, is true in some cases. Several have presented the decrease in the effective cross-section of the capillary as sufficient cause for the changes in the spectra. Moreover, attention has been called to the presence of the sodium lines.

As, however, no previous investigator had fully identified the lines of the resultant spectra, a further study of the subject appeared desirable.

APPARATUS

In the experiments performed in this laboratory there were used the following pieces of apparatus:

A Weiss electromagnet; diameter of core 10 cm and conical pole tips of 1 cm diameter. The gaps were of 7 or 9 mm. The current throughout the greater part of the work was 16.3 amperes, which corresponded to a field of about 25,000 gausses with the 7 mm gap, and 21,000 with the 9 mm. Field intensities were measured by a bismuth spiral.

A Cox induction coil operated by means of a mercury break. During a large part of the work, two one-quart Leyden jars were employed in parallel with the vacuum tube. Sometimes, however, they were removed and a spark gap placed in series with the tube.

A Hilger single-prism spectrograph, of dispersion about 10 Ångströms per millimeter in the violet and nearly 100 in the red—much too small to obtain the Zeeman effect. By means of a shutter a comparison spectrum could be thrown on both sides of a central region. The plates used were Cramer instantaneous isochromatic.

Tubes of sodium glass, containing oxygen, hydrogen, nitrogen, and carbon monoxide. Further, Dr. Theodore Lyman of Harvard very kindly loaned us some "end-on" tubes containing hydrogen and argon, with impurities. He also constructed for us and filled with hydrogen several tubes of various kinds.

RESULTS

In this investigation, the record of which was made on forty plates averaging nine exposures each, a few typical gases were employed to illustrate the several effects noted: (*a*) hydrogen as an elementary substance, (*b*) carbon monoxide as a typical compound gas, and (*c*) nitrogen and argon, as good examples of elementary gases having plural spectra. Several combinations of these were also used. Unless otherwise stated, the tubes were set vertically in a horizontal field and the spectrum was photographed from the side. Several photographs, however, showed that it made no difference whether the tubes were viewed "end on" or "side on."

a) The hydrogen tube when not in the field gave the series spectrum alone, and showed no trace of an impurity. In a field of about 25,000 gausses, however, only the F line remained, much weakened and shattered, while there appeared strong lines—chiefly sodium—and weaker ones belonging to the oxygen spark spectrum (see Plate VII, 3). The effect was verified by other tubes. Many tubes became partly shattered between the pole tips and several completely so. These shattered regions were located within the tube at the front and back, as viewed by an eye placed at the slit of the spectroscope. The more completely the discharge was rectified the more unequally shattered were these two regions—an effect to be expected from the heat which must have been generated locally by the sand-blast action of the deflected ions. Viewed vertically through the window of an end-on tube, the discharge appeared to be concentrated in two filaments which hugged the front and rear of the tube and were more unequal in size the more complete the rectification.

The appearance of the sodium lines in the spectrum is thus not at all surprising. However, one would hardly expect the glass to liberate free sodium without, at the same time, liberating an equivalent amount of oxygen in the nascent state. By reason of the fact that oxygen tubes generally soon cease to give the oxygen spectrum, we are confirmed in our belief that this gas is generated continuously while the tube is in the magnetic field. Further, since the entire mass of hydrogen present is not greater than 0.000005 gm, the mass of material from the glass may easily

exceed this, and thus explain the removal of most of the hydrogen lines. The relative strengths of the sodium and oxygen lines may be partially due to conditions which favor the sodium as a carrier, but work on a later tube (carbon monoxide) indicated that the relative number of ions determined the relative strength of the lines. There were in this case twice as many sodium as oxygen ions ($Na_2O = 2\ Na + O$).

The first effect of gradually increasing the field, using a hydrogen tube of about 1 mm internal diameter, was to broaden the hydrogen lines, then gradually to suppress the weakest, and finally to cause all but the F line to disappear. Meanwhile the oxygen and sodium spectra became visible (see Plate VIII, 28, 29).

By comparing photographs of the spectra of four different sized capillaries approximately 0.2, 0.5, 1, and 3.4 mm internal diameter, in equal fields, the gas being in each case hydrogen, a similar effect was noticed. With the largest capillary the hydrogen lines were simply broadened, but showed no tendency to weaken. Indeed, the result was an actual enhancement (see Plate VII, 8). In the size next smaller the lines were weaker, until, in the two smallest, even the F line had disappeared (see Plate IX, 24). With each decrease in size more sodium lines appeared, and with the smallest capillary there was in addition a considerable region of continuous spectrum. Moreover, in general, whenever the sodium lines are weak, no oxygen lines appear, thus strengthening the hypothesis that the oxygen comes from the glass.

b) In the case of the compound gas, carbon monoxide, with the field off, the regular band spectrum was given, but with the field on, the bands disappeared and were replaced by oxygen and sodium lines, the former here being the stronger (see Plate VII, 6*a*, 6*b*). The preponderance of oxygen may be explained by the decomposition of the carbon monoxide. This recombines, however, when the field is removed, as is shown by the reappearance of the original band spectrum.

Tubes containing hydrogen with a little carbon monoxide as impurity gave combination spectra, as expected (see Plate IX, 22). The oxygen lines were nearly as strong as the sodium. The tubes which became shattered between the poles subsequently gave sodium lines with the field off as well as on.

c) The effect of the field on argon and nitrogen tubes resembled that due to the introduction of capacity. With the creation of the field, the red line spectrum of the argon tube changed to the blue; and the band spectrum of nitrogen was replaced by its line spectrum. Nitrogen, however, behaves in a rather peculiar manner. The observation of Chautard, mentioned above, was confirmed only in the case of a very strong field suddenly applied. Under these circumstances the red and orange parts disappeared from the band system and a few of the spark lines developed (see Plate VIII, 33, 1, 2). But, when a very weak field was progressively increased, the positive band spectrum began to weaken and there appeared a spectrum, most of the lines of which belonged to the blue argon spectrum[1] (see Plate VIII, 3–6). In most cases, the removal of the field was accompanied by an immediate return to the original spectrum; but in this case, three successive three-minute exposures with the field off were marked by only a *gradual* reappearance of the nitrogen band spectrum (see Plate VIII, 7–9).

These changes in the nitrogen spectrum were accompanied by noteworthy visual changes. With no field, the capillary was reddish white, the electrode regions, violet. A small increase of field resulted in no change. At a certain point, however, a violent commotion took place in the tube and the discharge was temporarily nearly choked off. It subsequently became perfectly regular but the color in the capillary changed to an intense blue, while the electrode regions became red, and a green fluorescence appeared at one end of the tube. (It may be mentioned that the creation of the field in nearly every case increased the resistance of the tube enough to render the discharge almost unidirectional.) After the removal of the field, the tube retained its changed appearance for two exposures; then the discharge regained its original color—in agreement with the photographic results. The tube tended to heat more when the field was off than when it was on.[2]

[1] This was an old tube and the manufacturer informs us that it was filled by the "air" method.

[2] The influence of temperature was investigated directly by running a tube in the field first through intervals to keep it as cool as possible and then continuously to produce high temperature. The spectrum showed no change but the discharge was intermittent in the latter case, due probably to increased pressure.

With reference to argon, the following may be noted: A tube evidently containing pure argon (at least no hydrogen lines were visible under ordinary conditions), when subjected to a gradually increasing field, showed at a certain point three hydrogen lines; introducing two one-quart Leyden jars suppressed these lines; and, finally, removal of the jars and a further increase of field resulted in a progressive disappearance—the weakest line first. The above-mentioned conditions appeared to be the only ones under which the remote trace of hydrogen present could be detected.

SUMMARY

The vacuum tube discharge is unquestionably a complex phenomenon, owing to the large number of variables to be considered—potential, current, resistance, temperature, pressure, frequency, and composition of the gaseous mixture contained in the tube. The magnetic field constitutes a new variable and has a decided effect upon those already mentioned. Ordinarily, in a given tube, the size of the capillary is constant, but the magnetic field, by forcing the ions to use only a small part of the capillary, virtually changes the effective cross-section. This increases the resistance of the tube, and diminishes the value of the current flowing. Secondly, the mechanical bombardment of the walls of the tube by the ions liberates material from the glass and adds to the complexity of the gaseous mass under investigation; and, thirdly, these same collisions are probably one of the factors which result in the decomposition or dissociation of the bombarding ions into simpler forms. In general, then, these three factors result in (*a*) the production of the spectra either of substances already present in the tube as impurities, or of dissociation products either (*b*) of the original gas or (*c*) of the glass. As specific examples of each case, we may cite (*a*) the enhancement of the hydrogen lines in an argon tube (which contained hydrogen as an impurity) when this tube was subjected to the proper conditions; (*b*) the change from the band to the line spectrum of nitrogen or the production of the oxygen spectrum in a carbon monoxide tube; and (*c*) the production of not only the sodium but the oxygen spectrum from the

glass. To these new facts may also be added the peculiar phenomena observed with the nitrogen tube.

The writers wish to acknowledge their indebtedness to Dr. Theodore Lyman of Harvard, for loaning old vacuum tubes and filling new ones. We are also indebted to Mr. Charles H. Smith, who assisted us for some days at the beginning of the investigation, and to Mr. A. Herman Wigren, who aided in determining the field strengths employed.

The following are the most important papers dealing with the spectroscopic side of vacuum tube discharge in a magnetic field:

A. de la Rive, *Annales de chimie et de physique* (3), **54**, 238, 1858.
J. Plücker, *Poggendorf's Annalen*, **179**, 88, 151; also **180**, 113, 1858.
A. Trève, *Comptes rendus*, **1**, 36, 1870.
L. Daniel, *ibid.*, **1**, 183, 1870.
P. Secchi, *ibid.*, **1**, 431, 1870.
A. J. Ångström, *ibid.*, **2**, 3369, 1871.
J. Chautard, *ibid.*, **1**, 1123, 1874.
———, *ibid.*, **1**, 1161, 1875.
———, *ibid.*, **2**, 75, 1875.
———, *ibid.*, **1**, 272, 1875.
P. G. Melani, *Nuovo Cimento*, IV, **5**, 329, 1897.
E. Van Aubel, *Journal de physique*, **7**, 408, 1898.
J. Berndt, *Annalen der Physik*, IV, **8**, 625, 1902.
P. Zonta, *Nuovo Cimento*, V, **7**, 321, 1904.
N. Vaccaro, *ibid.*, V, **11**, 222, 1906.
J. E. Purvis, *Proc. Camb. Phil. Soc.*, XIII, **6**, 354, 1906.

Physical Laboratory
Boston University
June 1912

EFFECT OF REFLECTION FROM A MOVING MIRROR ON THE VELOCITY OF LIGHT

By A. A. MICHELSON

According to the undulatory theory of light the velocity of light is independent of the velocity of the source, and of the velocity of a mirror at which it is reflected.

According to the emission theory the resultant velocity from a moving source is increased by the component of the velocity of the source. But it appears that different forms of emission theory require different results on reflection from a moving mirror. If the light corpuscles are reflected as projectiles from an elastic wall, then the velocity of light should be increased by twice the component of the velocity of the mirror.[1]

The following arrangement was devised for the purpose of deciding the question experimentally.

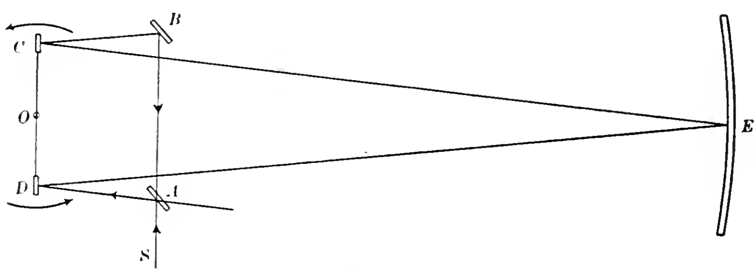


Fig. 1.—Diagram of apparatus

Light from a source at S falls on a lightly silvered mirror A. The reflected pencil goes to a revolving mirror D, thence to the concave mirror E, to the second mirror C revolving about the same axis O, whence it proceeds to the plane mirror B and is reflected back to A. The transmitted pencil pursues the same path in the

[1] An alternative hypothesis, that the velocity of light should be increased by the component of the velocity of the mirror, is suggested by R. C. Tolman but shown to be inadmissible (R. C. Tolman, "Some Emission Theories of Light," *Physical Review*, August 1912).

opposite direction, returning via DA to the starting-point, where it meets the first pencil, producing interference fringes which are observed by means of a telescope with micrometer eyepiece.

According to the undulatory theory the velocity of light is unaffected by the velocity of the mirror while the emission theory[1] requires that

$$\overline{V}=V+rv$$

where $\overline{V}$ is the velocity of light after reflection, V the velocity before reflection and v the component of the velocity of the mirror in the direction of the reflected pencil, and $r=2$ according to the elastic impact theory; while $r=1$ if the mirror surface acts as a new source.

The time occupied by the pencil DEC is

$$T_1=\frac{2(D+d)}{V_1}$$

while that taken by the pencil CED is

$$T_2=\frac{2(D-d)}{V_2}$$

where D is the distance OE, d=distance the revolving mirror moves while light passes over DEC, and V_1 the resultant velocity of the first pencil, V_2 that of the second.

The difference in time is therefore

$$T_1-T_2=2\left[\frac{D+d}{V+rv}-\frac{D-d}{V-rv}\right].$$

But

$$\frac{d}{2D}=\frac{v}{V}$$

whence[2]

$$T_1-T_2=4\frac{D}{V}(2-r)\frac{v}{V}.$$

[1] According to the theory of Ritz (*Annales de chimie et de physique*, Ser. VIII, 13, 1908) the velocity of light is not affected by reflection from a moving mirror, but is affected by the motion of the source.

[2] Omitting quantities of the second order; v should be replaced by $v \cos \alpha$, but since α is only 3°, the factor $\cos \alpha$ may be taken equal to unity.

The corresponding displacement of the interference fringes is

$$\Delta=\frac{V(T_1-T_2)}{\lambda}=4\frac{D}{\lambda}(2-r)\frac{v}{V}$$

For $r=0$ $\qquad \Delta=8\frac{D}{\lambda}\frac{v}{V}$

" $r=1$ $\qquad \Delta=4\frac{D}{\lambda}\frac{v}{V}$

" $r=2$ $\qquad \Delta=0$

The experiment was tried under the following conditions.

The revolving mirrors were mounted on the shaft of an electric motor the speed of which (measured by a speed counter) could be varied from zero to 1800 revolutions per minute. The distance between centers of the mirrors was $l=26.5$ cm;[1] the distance OE was 608 cm. The light of the carbon arc (in one experiment, sunlight) was filtered through a gelatine film transmitting light of mean wave-length $\lambda=0^{\mu}.60$.

The formula for the displacement,

$$\Delta=8\pi n\frac{l\,D}{\lambda\,V}\quad\text{if } n \text{ is turns per sec.}$$

or

$$\Delta=\frac{8}{60}\pi N\frac{l\,D}{\lambda\,V}\quad\text{if } N \text{ is turns per min.}$$

gives with these data and $r=0$ a displacement of 3.76 fringes for 1000 revolutions per minute.

Following is a table of results of observations reduced to this speed.

It appears therefore that within the limit of error of experiment (say 2 per cent) the velocity of a moving mirror is without influence on the velocity of light reflected from its surface.

Assuming that the effect is actually nil, this interference method may be used to measure the velocity of light with an order of accuracy equal to that of the improved Foucault method or of the "combination" method proposed in an article in the *Phil. Mag.*,

[1] This agrees within less than 1 per cent with the distance calculated from the measured distance AB of the interferometer mirrors.

March 1902. Any one of these three methods is capable of furnishing results of the order of accuracy of one part in one hundred thousand; and differential measurements (e.g., with the light of the

	Δ	Weight
1	3.8	1
2	3.1	1
3	3.2	1
4	4.3	2
5	3.8	2
6	3.93	3
7	3.83	4
	3.81 = weighted mean 3.76 = calculated displacement	

two limbs of the sun) can be obtained with a still higher degree of precision; and thus the effect of a moving source on the velocity of light could be determined.

Ryerson Physical Laboratory
University of Chicago

THE TEMPERATURE OF A WEDGE-SHAPED CAVITY AND ITS USE AS A BLACK BODY

By B. J. SPENCE

Some time ago, in this *Journal*,[1] Professor C. E. Mendenhall described the properties of the radiation emitted by the interior of a wedge-shaped cavity at various temperatures. He found by means of a previously calibrated optical pyrometer that the radiation emitted by such a cavity when the angle of the wedge is small is quite approximately black-body radiation. The simple theory underlying the problem shows that this must be true. By means of the wedge, Professor Mendenhall was also able to compare the temperature of the interior of the wedge with the corresponding black-body temperature of the exterior surface of the wedge, thus gaining a knowledge of the radiation properties of the particular metal of which the wedge was composed. The above statements hold true only so long as the true temperature of the wedge is the same without as within. The results of the experiment warrant the assumption.

If the temperature of the wedge could be determined by means of a thermo-couple inserted somewhere within the wedge, its usefulness would be materially increased. For example, it could be used as a black body for the calibration of optical pyrometers, thus obviating the necessity of the cumbersome and costly black-body furnace. Again, one may determine the relation between the true temperature of a substance and its corresponding black-body temperature, using a pyrometer to determine the black-body temperature. The thermo-couple would indicate the true temperature of the substance.

The following describes, in brief, a method which has satisfactorily measured the temperature of the wedge. A strip of platinum foil about 3.5 cm long and 2 cm wide was wrapped about a piece of very fine bored and thin-walled silica tubing of the same length as the platinum foil, in such a manner that the tube was

[1] **33**, 91, 1911.

completely enveloped and still allowed the platinum foil to form a wedge. The manner of wrapping may be best understood by a reference to the diagram, Fig. 1. The silica tube running the length of the apex of the wedge formed an insulation for the thermo-couple which was threaded through it and retained the junction at the center of the tube.

The wedge was mounted in a brass frame in such a manner that a current could be passed along the length of the wedge. The frame was so constructed that the expansion of the platinum due to heating could be taken up. The thermo-couple was made of two strands of No. 40, B.S. gauge platinum, and platin 10 per cent rhodium and was carefully calibrated, using the melting-points of zinc, antimony, and copper. With couples of wire of larger diameter it was found that the temperature indicated was not the true temperature but a temperature somewhat lower owing to the conduction of the heat away from the junction. In fact this was the reason a wedge 3.5 cm long was used, the attempt being to obtain a place within the tube of uniform temperature distribution and also to minimize the effect of conduction of the thermo-element. Consequently with such an arrangement, it was assumed that the temperature indicated by the couple was the true temperature of the platinum when the heating current was passed through it.

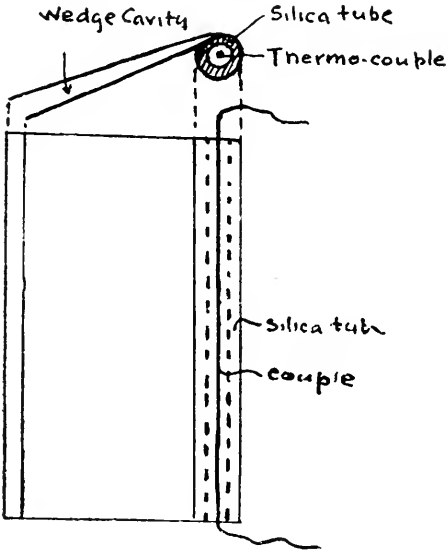


FIG. 1.—Diagram of wedge

Three methods were available for testing the truth of the above assumption. The first method employed the melting of tiny pieces of gold (Kahlbaum) on the outer surface of the wedge. The couple indicated the true temperature of melting gold as accurately as

could be desired. For example, the melting-point of gold taken as 1063° C. gave rise to a thermal electromotive force of 12,140 micro-volts as determined by the standard melting-point calibration. The electromotive force corresponding to the melting of gold on the outside of the wedge was 12,134 micro-volts. This value is an average of 30 determinations of which the maximum deviation of any determination from the mean was not over 60 micro-volts.

The thermal electromotive force corresponding to the melting-point of palladium (Kahlbaum) was also determined. An average value of a large number of determinations indicated a temperature corresponding to 20,700 micro-volts. The value demanded by the extrapolation of the calibration points of the couple was not computed because such an extrapolation is of no value quantitatively.

To further determine the type of radiation from the wedge interior it was viewed with a Holborn-Kurlbaum optical pyrometer which had been previously calibrated by means of a black-body furnace. The current through the pea lamp of the pyrometer corresponding to the melting-point of gold as determined by the furnace calibration was 499 arbitrary units. The current corresponding to the melting-point of gold on the outer surface of the wedge when the interior of the wedge was viewed with the pyrometer was 502 arbitrary units, thus indicating that the radiation from the interior of the wedge approximated more nearly that of the black body at the melting-point of gold than did the black-body furnace.

Finally the difference between the true temperature of the wedge as determined by the thermo-couple and the corresponding black-body temperature indicated by the radiation from the exterior of the wedge was sought for the range from the melting-point of gold to the melting-point of palladium at 1549°.2 C. The black-body temperature was determined with the pyrometer. The results are included in the accompanying table. Column 1 indicates the true temperature determined by the thermo-couple, column 2 the black-body temperature determined by the pyrometer, and column 3 the difference between the true and black-body temperatures.

The values given in the third column agree very closely with the

values obtained by Waidner and Burgess[1] and also those of Professor Mendenhall.[2] Unfortunately no means were available so that the electromotive force of the thermo-couple used in the experiment could have been determined for a temperature corresponding to the palladium melting-point. Had the couple been a standard platinum-platin 10 per cent rhodium couple, a direct

True Temperature	Black-Body Temperature	Difference
985° C.	910° C.	75° C.
1050	960	90
1108	1010	98
1168	1060	108
1227	1110	117
1280	1155	125
1335	1205	130
1390	1250	140
1440	1288	148
1490	1335	155
1530	1365	165

comparison could have been made with the standard couple used by Day and Sosman (*Pub. Carnegie Institution*, No. 157, p. 118). However, the close agreement between the values of the electromotive force at the melting-point of gold and the value given from the standard melting-point determination led one to the conclusion that the electromotive force indicated for the palladium point is very close to the true value of the electromotive force at 1549° C. The difference between the true and apparent temperatures of the platinum for various heating currents also led to the conclusion that the method of measuring the temperature of the wedge with the thermo-couple is a valid one.

UNIVERSITY OF NORTH DAKOTA
November 1912

[1] *Bul. Bur. Stand.*, 3, 202, 1907. [2] *Loc. cit.*

THE SPECTRA OF SPIRAL NEBULAE AND GLOBULAR STAR CLUSTERS[1]

THIRD PAPER

By E. A. FATH

Since the writing of the second paper[2] a number of additional spectrograms have been obtained with the Mount Wilson 60-inch reflector.

The spectrograph used was the same as before except for the last plate, taken in May 1912. For this a new spectrograph, mounted directly in the axis of the telescope, was available. The collimator lens of the new instrument has an effective aperture of 3.6 inches and a focal length of 18 inches. The prism is of UV flint, has an angle of 30 degrees, and will include the full beam. The camera lens has an aperture of 4 inches and a focal length of 8 inches. It is a special lens of portrait type made for the Yerkes Observatory and loaned for this investigation. For this courtesy the writer desires to express his great appreciation. There was no opportunity, at the time, to test the relative speed of the two spectrographs but it seems very probable that the new one has about twice the speed of the other. Lumière "Sigma" plates were used throughout. The method of measurement of the spectrograms was the same as that employed before.

SPIRAL NEBULAE

N.G.C.	Dates of Exposure	Total Exposure
1023.......	1910 Nov. 29, 30, Dec. 1, 2	20^h40^m
1023.......	1911 Oct. 19, 20, 21, 22	38 14
3031.......	Jan. 4, 5, 6	22 39
4594.......	Mar. 29, 30, 31, Apr. 1	17 8
4736.......	May 2	7 40
4826.......	1912 Feb. 10, 11, 12	16 15
5194.......	1911 Apr. 28, 29, 30, May 1	29 20
7331.......	1910 Aug. 27, 28	12 18

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 67.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 49; *Astrophysical Journal*, **33**, 58, 1911.

Owing to a discontinuation of the observations, a description is given of a number of spectrograms which are underexposed, although the information to be derived from such plates is very meager.

As before we shall consider the two classes of objects separately.

N.G.C. 1023.—Absorption lines corresponding to F, 436 $\mu\mu$, G, H, and K are present. The line at 436 $\mu\mu$ may be H_γ, but as it is about 2 $\mu\mu$ to the red of the H_γ position this identification does not appear probable. The grain of the plate having the longer exposure is exceedingly troublesome. The spectrum is very narrow in each case. The general appearance of the spectrum is similar to that of the sun.

N.G.C. 3031.—This is a very good plate. In appearance it exactly resembles the spectrum of a K-type star. There are absorption lines at F, 454, 448, 440 $\mu\mu$, G, groups at 419 and 406 $\mu\mu$, H, and K.

N.G.C. 4594.—This negative is very weak. It is not safe to say more than that the color-curve indicates a spectrum of approximately solar type.

N.G.C. 4736.—As stated in the second paper, there is a difference between the Mount Hamilton[1] and the Mount Wilson plates obtained by the writer, the former being apparently somewhat out of focus while the latter is in good focus and shows absorption lines like the sun. Another plate was taken to check the two preceding. The definition of the third plate is not quite as good as in the first Mount Wilson plate and shows an appearance somewhat like the Mount Hamilton plate, namely, a broadening and slight intensification immediately to the red of H, at 406 $\mu\mu$. Otherwise it agrees fully with the first Mount Wilson plate in showing the prominent solar lines. The latter plate, upon careful examination, also shows the effect at 406 $\mu\mu$ but by no means so prominently as the last. In order to bring out this brighter band it therefore seems necessary to suppress the absorption lines.

N.G.C. 4826.—This plate is very weak. The color-curve is similar to that of a solar-type star.

N.G.C. 5194.—The spectrum is weak in spite of an exposure of nearly 30 hours. The large mirror was in very poor condition at

[1] *Lick Observatory Bulletin*, No. 149.

the time this exposure was made. This doubtless was the cause of the faintness of the spectrum. The color-curve is similar to that of a K-type star. Lines at G and H are certainly present.

N.G.C. 7331.—The plate is underexposed. Lines corresponding to F, G, and H are found. Two other absorption lines are present, one on either side of G, but their position cannot be determined with any accuracy because of the faintness and narrowness of the spectrum. This plate agrees with the Mount Hamilton plate obtained in 1908 which shows lines at G and H.

In the last four years the writer has investigated the spectra of eleven nebulae, *N.G.C.* 224, 650–1, 1023, 1068, 3031, 4594, 4725, 4736, 4826, 5194, and 7331, of which all but 650–1 are certainly spiral. For the most part the spectra are either G- or K-types. In the case of some of the nebulae the spectrum obtained came largely from the nucleus, but this is certainly not the case with the great *Andromeda* nebulae in which a spectrum of solar type was obtained over a strip five minutes of arc in length.

N.G.C. 1068 and 4736.—These are peculiar. The first shows bright nebular lines together with dark lines, although the type of absorption spectrum is not evident. In the case of the second there is increased radiation in the vicinity of 406 $\mu\mu$ near the position of one of the bright lines in some of the Wolf-Rayet stars. These two cases are the only ones in the series of plates secured by the writer at Mount Hamilton and at Mount Wilson which give definite evidence of what may be termed "gaseous" radiation. In a paper[1] published when the present one was practically completed, Professor Max Wolf reports finding emission lines in a number of spiral nebulae, many of which correspond to emission lines in Wolf-Rayet stars. Though the writer's plates show definite emission lines in but two cases, the two series cannot be considered contradictory. Wolf's plates were on about double the scale (spectra 6 mm in length) of those here described, and apparently a finer-grained plate was used.

N.G.C. 5904.—The following absorption lines are present: F, H_γ, G, H, K, and a band at 419 $\mu\mu$ which may possibly be double.

[1] *Sitzungsber. Heidelberger Akad. d. Wiss.*, Abt. A, 1912, No. 15.

The appearance is practically that of a solar spectrum except that H_γ is more prominent here than in the sun.

GLOBULAR CLUSTERS

N.G.C.	Dates of Exposure	Total Exposure
5904.......	1911 May 3, 4, 5	16^h17^m
6093.......	June 1, 2, 3	15 40
6205.......	1912 May 21, 22, 23	15 35
6254.......	1911 May 30, 31	13 05

N.G.C. 6093.—This spectrum is of good density. Absorption lines corresponding to solar F, G, H_δ, H, and K are present. The line corresponding to H_δ is stronger than in the sun. Aside from this the spectrum resembles the solar type. H_γ may be present, for G looks broader than normal.

N.G.C. 6205.—This exposure was made with the second spectrograph as noted above. The exposure of 15^h35^m therefore corresponds to about 31^h when compared with the others. It is the first spectrogram of satisfactory density of this cluster which I have obtained and seems to explain the discrepancy between the first plate obtained at the Lick Observatory[1] and the first Mount Wilson plate.[2] The Lick plate, although faint, indicated stars of various spectral types. These could be picked out because the guiding was such that the cluster was kept quite closely in one position with respect to the slit. The cluster being comparatively coarse, the spectra of but few stars were obtained. On the other hand, for the first Mount Wilson plate, the guiding was such as to allow the cluster to drift slightly in both α and δ. A more nearly average type of spectrum was thus obtained and the F-type was indicated. The plate, however, was not of satisfactory density, so another was taken. In this case a special effort was made to obtain an average spectrum by making the image of the cluster move about over the slit. The spectrum obtained shows absorption lines at H_β, H_γ, G, H_δ, H, and K and a band at $420\,\mu\mu$. H_γ and G appear as a

[1] *Lick Observatory Bulletin*, No. 149.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 49; *Astrophysical Journal*, **33**, 58, 1911.

close double line while H and K are not fully separated. The plate thus indicates either a type of spectrum between F and G, or stars ranging from the one type to the other. If we take the latter, we have approximate agreement among the three plates. A re-examination of the first Mount Wilson plate brought out the fact that the line called H_γ is broader than the others and therefore favors the possibility of its being double as shown in the last plate.

N.G.C. 6254.—This is a comparatively coarse cluster and the spectrum of but one star can certainly be identified. It is of solar type and shows the F, G, H, and K lines. The spectrum of another star is on the plate but it is much fainter than the former, which it appears to resemble. The lines F, H, and K are probably present. G may also be present but the very slight width of the spectrum and the troublesome grain of the plate make this very uncertain.

The four clusters described above agree in showing the solar line G as well as lines of hydrogen and calcium. We thus have three possibilities—the clusters consist of: (1) stars of F and G types, (2) stars of a type intermediate between F and G, possibly F8 in the Harvard classification; (3) a combination of 1 and 2. The latter hypothesis seems most probable.

After the presence of the G line had been recognized in these four cases the spectrograms of the clusters discussed in the second paper were carefully examined again. With the exception of the plate of *N.G.C.* 6205, mentioned above, they gave no evidence of being composed of other than F-type stars.

The number of globular clusters investigated thus numbers twelve, approximately 11 per cent of those of appreciable size and brightness. As stated in the second paper, only the brighter stars of the clusters had any effect on the photographic plate. The result obtained thus far may be stated as follows: *As a whole the brighter stars of the globular clusters investigated have spectra ranging only from the F- to the G-type.* The clusters observed are nearly all that can readily be reached in latitude 34° north and in which the brighter group of stars are fairly condensed and of sufficient brightness to be observed in a reasonable time. Unfortunately these do not include the large group of clusters near 18 hours and

south of $-20°$. It would be of great interest to know if these, too, show the same small range of spectral type as the northern clusters, and it is hoped that some southern observatory may undertake to answer this question.

SMITH OBSERVATORY
BELOIT, WIS.
November 1912

THE *ALGOL* VARIABLE *RR DRACONIS*[1]

SECOND PAPER

By FREDERICK H. SEARES

The comparison of the photographic results for *RR Draconis* presented in the first paper[2] with the incomplete visual light-curve indicated that the photographic variation is greater than the visual. An examination of the relative dimensions and light-intensities of the two objects composing the system made it practically certain that the difference must be real. This result has since been confirmed by photographs during the minimum of September 10, 1912, made alternately on ordinary and isochromatic plates, the latter being used with a yellow filter. The present paper contains an account of these later observations.

The plates used were Seed "Gilt Edge 27" and Cramer "Instantaneous Isochromatic." The filter is one prepared for the determination of visual magnitudes, and when used in connection with the Cramer plate, the curve of color sensitiveness for spectral types not too far advanced appears to agree satisfactorily with that of the eye. The results derived with this combination will, in what follows, be referred to as photovisual[3] magnitudes.

During the period of variation on September 10, 18 plates, divided equally between the Seed "27" and the Cramer "Iso," were made with the full aperture of the 60-inch reflector. Five one-minute exposures were made on each of the Seed plates, while four exposures of three minutes each were given to all the Cramer plates excepting the last, for which the exposures were two minutes. The total number of exposures is therefore 81. They cover the

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 68.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 64; *Astrophysical Journal*, **36**, 368, 1912.

[3] The increasing importance of visual magnitudes determined by photographic methods suggests the desirability of a special designation for results so derived. Though etymologically unsatisfactory, the term *photovisual* is suggested as a convenient notation, the term *visual* being reserved for the indication of results to which hitherto it has generally been applied.

minimum and parts of both branches of the curve, and extend over an interval of $5^h 27^m$. The atmospheric conditions were satisfactory throughout. The transparency was high, and the steadiness ranged from 3 to 4 on a scale of 10. Further details are given in Tables IX–XI, but before proceeding to their discussion it is necessary to consider the magnitude scale of the comparison stars.

The relative photographic magnitudes of the comparison stars have already been well determined. Referred to an arbitrary zero point, they are shown in the second column of Table IV of the first paper and of Table III below. The results of a provisional determination of the zero point from a plate of duplicate exposure on the field of the variable and on the Pole are given in the last column of Table IV of the first paper. As a control upon this comparison two further photographs of duplicate exposure have since been made on Seed "27" plates. In addition, a series of photographs on Cramer "Iso" plates has been made for the derivation of the photovisual scale used for the reduction of the isochromatic plates exposed during the minimum on September 10. A list of the additional photographs made for scale determinations is given in Table I. Those on "Iso" plates include exposures with the various apertures used for the determination of the relative photovisual magnitudes, and also duplicate exposures on the field of the variable and on the Pole, the latter serving to fix the zero point of the photovisual scale.

Turning first to the revision of the zero point of the photographic scale, we have available plates 902 and 903. The images of the comparison stars for the variable and the Polar Sequence stars shown on these plates were measured with the photometric scale in the usual manner. These exposures were made with a 32-inch diaphragm, and as the distances of the stars from the axis were moderate, we may regard the field as free from aberration. The mean scale readings, uncorrected for distance error, are therefore used for the reduction. These are given in Table II. The numbers and magnitudes of the polar stars are from *Harvard Circular*, No. 170, with the exception of (350) and (362), for which the numbers refer to *Harvard Annals*, **48**, as these objects are not included in the Polar Sequence. Their magnitudes are from an investigation of the Polar Sequence not yet completed, but as the

results for this part of the scale are in close agreement with those of Pickering, they may be used for the present purpose.

TABLE I

PLATES FOR MAGNITUDE SCALE OF COMPARISON STARS

Plate	Date	G.M.T.	No. Exp.	Exp. Time	Aperture	Region	Seeing
	1912						
P 845, "Iso"..	Sept. 15	19^h22^m–19^h33^m	4	3^m	60, 32	Variable	3
846, "Iso"..	15	19 40 –20 0	6	3	60, 32, 14	Variable	3
847, "Iso"..	15	20 5 –20 23	3	3	60	Var., Pole	3–4
856, "Iso"..	16	18 17 –18 32	4	4	60, 32	Variable	5
857, "Iso"..	16	18 41 –19 0	4	4	60, 32	Variable	5
858, "Iso"..	16	19 7 –19 23	3	3	60	Var., Pole	5
900, "Iso"..	Oct. 7	15 40 –16 24	3	10	32	Var., Pole	4
901, "Iso"..	7	16 37 –17 20	3	10	32	Var., Pole	3
902, "27"...	7	17 31 –17 57	3	4	32	Var., Pole	3
903, "27"...	7	18 0 –18 22	3	4	32	Var., Pole	2–3

TABLE II

COMPARISON WITH THE POLE—PHOTOGRAPHIC RESULTS

POLAR SEQUENCE				VARIABLE AND COMPARISON STARS		
H.C. 170		Scale Reading		Star	Scale Reading	
Star	Mag.	902	903		902	903
14	10.52	9.0	8.3	*v*	6.6	6.5
7*r*	10.57	9.4	8.6	1	6.9	6.8
(350)	10.63	9.5	8.8	2	13.1	13.8
5*s*	10.68	9.5	8.9	3	13.7	14.0
16	11.26	11.6	11.5	4	13.7	14.4
19	12.28	14.6	15.6	5	14.5	14.4
7*s*	12.31	14.5	14.7	6	15.1	15.2
20	12.59	14.8	16.4	7	14.8	15.0
(362)	12.78	16.2	17.8	8	15.5	16.5
11*r*	12.83	17.2	18.0	9	16.2	...
23	13.20	18.0	....	10	17.4	16.8
12*r*	13.34	18.5	....	11	17.6	....
24	13.50	18.5	....	..	....	....

The scale readings for the polar stars were plotted against the magnitudes in the second column of Table II, and from the resulting curves were interpolated the magnitudes of the comparison stars and the variable. These results require a correction for extinction.

The zenith distances for the variable for the mean times of exposure of plates 902 and 903 were respectively 48°7 and 52°5.

The corrections are therefore +0.10 and +0.05 mag. The corrected values of the interpolated magnitudes are given in fourth and fifth columns of Table III. The corresponding quantities for plate 792, derived by adding the correction for extinction to the results in the eighth column of Table IV of the former paper, are also given in Table III. The magnitudes of the variable and of star No. 1 from plates 902 and 903 are unreliable, as they represent

TABLE III

PHOTOGRAPHIC MAGNITUDES OF COMPARISON STARS

Star	Prov. Mag.	Interpolated Mag.			Reduction to P.S.			Adopted Mag.	Mag. from P.S.	Diff.
		792	902	903	792	902	903			
v...	0.76	.....	(9.78)	(9.99)	....	(9.02)	(9.23)	9.64	(9.88)	(−24)
1...	0.90	.....	(9.89)	(10.08)	....	(8.99)	(9.18)	9.78	(9.98)	(−20)
2...	3.03	12.07	11.96	12.01	9.04	8.93	8.98	11.91	12.01	−10
3...	3.12	12.27	12.13	12.05	9.15	9.01	8.93	12.00	12.15	−15
4...	3.23	12.18	12.13	12.15	8.95	8.90	8.92	12.11	12.15	− 4
5...	3.40	12.40	12.40	12.15	9.00	9.00	8.75	12.28	12.32	− 4
6...	3.68	12.73	12.58	12.35	9.05	8.90	8.67	12.56	12.55	+ 1
7...	3.70	12.58	12.49	12.32	8.88	8.79	8.62	12.58	12.46	+12
8...	4.02	12.90	12.69	12.65	8.88	8.67	8.63	12.90	12.75	+15
9...	4.03	12.96	12.87	.....	8.93	8.84	....	12.91	12.92	− 1
10...	4.06	12.96	13.19	12.72	8.90	9.13	8.66	12.94	12.96	− 2
11...	4.28	13.00	13.23	.....	8.72	8.95	....	13.16	13.12	+ 4
12...	4.58	.....	.....	.....	....	....	....	13.46	.....	....
13...	4.80	.....	.....	.....	....	....	....	13.68	.....	....
14...	4.89	.....	.....	.....	....	....	....	13.77	.....	....
Means..........................					8.95	8.91	8.77			
Adopted mean reduction to Polar Sequence........................						8.88				

an extrapolation of the magnitude-curve defined by the Polar Sequence stars. On this account they are not used in the derivation of the reduction constant, though they may be accepted for a further determination of the relative brightness of the variable and star No. 1. The value previously found was $v-\text{No. }1=-0.15$ mag. with weight 6. From plates 902 and 903 we derive −0.10 mag. with weight 2. The mean is −0.14 mag. On this account the provisional magnitude of the variable in Table III is 0.76 instead of 0.75, as previously given.

The comparison of the interpolated magnitudes with the pro-

visional results in the second column of Table III gives the values of the reduction constants shown in columns six, seven, and eight. The respective means of 8.95, 8.91, and 8.77 mags. are in satisfactory agreement, and yield as a mean constant for the reduction of the provisional magnitudes to the zero point of the Polar Sequence, 8.88 mags. The final values for the normal brightness of the variable and for the magnitudes of the comparison stars are in column nine of Table III. As a result of the revision, the magnitude of the variable has been decreased by 0.06, the comparison stars by 0.07.

It will be noted that the scale of the adopted magnitudes has been independently derived, and, excepting the zero point, is in nowise based on that of the Polar Sequence. It is of interest, however, to compare the two scales, which is easily done by forming the mean of the interpolated magnitudes for each star. The results are in the column following the adopted magnitudes in Table III. The differences in the last column, with the exception of the first two which are without significance, are all small. There is a suggestion of a small systematic difference, but the interval covered is too short to afford a reliable indication.

Proceeding now to a consideration of the photovisual scale, we have available for the determination the diaphragm plates 845, 846, 856, and 857, all of which were made during normal brightness of the variable. The scale readings for these appear in Table IV. The values in parentheses for star No. 1 are those actually observed, corrected for distance error by means of the average curve ordinarily used for the full aperture of the 60-inch reflector. But, as pointed out in the former paper, these corrections are uncertain in the case of bright stars at any considerable distance from the axis, on account of the large influence of even comparatively small deformations of the mirror. Star No. 1 is of this character, and to avoid the difficulty, the following procedure was followed: The variable during normal light is very nearly of the same brightness as star No. 1 but is situated at the center of the plate. If, therefore, the value of the difference in brightness of the two objects can be independently derived, it will be possible to apply this difference to the scale readings for the variable and derive for star No. 1 values

which will be more reliable than those observed. The required differences may be found from the exposures made with the 32- and 14-inch diaphragms, the results for which are also entered in Table IV. Two additional values are also given by the polar comparison plates, 900 and 901, listed in Table VI, both of which were made with the 32-inch diaphragm. Since the difference

TABLE IV

MEAN READINGS FOR PHOTOVISUAL SCALE PLATES

Star	845		846			856		857	
	S_{60}	S_{32}	S_{60}	S_{32}	S_{14}	S_{60}	S_{32}	S_{60}	S_{32}
v	6.4	13.6	6.2	13.6	16.4	8.5	13.5	7.5	12.9
1	(7.5)	13.7	(7.4)	13.6	17.3	(7.8)	13.6	(7.5)	13.2
	6.5	....	6.3	....	....	8.6	....	7.6	....
2	13.5	....	13.7	....	....	13.7	....	13.0	....
3	14.5	....	14.2	....	....	14.4	....	13.7	....
4	14.4	....	14.0	....	....	14.1	....	13.6	....
5	11.9	....	11.9	....	....	12.5	....	11.8	....
6	....	....	16.8	....	....	16.1	....	15.3	....
7	14.6	....	15.1	....	....	14.3	....	14.3	....
8	16.7	....	16.8	....	....	15.6	....	15.0	....
9	16.4	....	16.3	....	....	16.2	....	15.3	....
10	....	....	....	....	....	16.5	....	15.7	....

is very small, amounting to only 0.04 or 0.05 mag., we may disregard the small fluctuations in the value of the scale interval from plate to plate and for different parts of the scale. We thus find No. $1-v=+0.14$ scale interval, which, added to the 60-inch exposure values for the variable in the first line of Table IV, gives the required values for star No. 1. These were used in the determination of the magnitude scale instead of those directly observed.

The data in Table IV permit of five separate determinations of the scale of photovisual magnitudes. The exposures were too short for those with the diaphragms to show images for any of the stars excepting the variable and No. 1, and, as these are of the same brightness, we have, in effect, to assume that the photometric scale with which the images were measured is a uniform scale of magnitudes whose constant is to be determined for each plate. Fortunately this is sensibly the case. The resulting mean magnitudes referred to an arbitrary zero point, and the deviations in hundredths

of a magnitude, for each determination of the scale are in Table V. The average deviation, including the effect of relative scale error over a range of three magnitudes, is ±0.045 mag.

TABLE V

PROVISIONAL PHOTOVISUAL MAGNITUDES

Star	Provisional Mag.	Residuals				
		845	846		856	857
		60–32	60–32	60–14	60–32	60–32
v	1.73	−5	−4	+8	+2	0
1	1.80	−2	0	0	+5	−2
2	3.76	+3	−3	−4	0	+2
3	3.98	−2	+6	+4	−6	−1
4	3.92	−5	+5	+4	0	−4
5	3.29	+1	−1	0	0	−1
6	4.65	...	+2	−2	−7	+8
7	4.12	+9	−5	−7	+12	−9
8	4.58	−3	−5	−9	+6	+11
9	4.58	+5	+9	+5	−18	+1
10	4.80	...	...	...	−8	+8
Average deviation	....	±0.039	±0.040	±0.043	±0.058	±0.043

The zero point of the photovisual scale was derived from the polar comparison plates 847, 858, 900, and 901. The arrangement of Table VI, which contains the scale readings, is similar to that of Table II. It is to be noted, however, that the visual magnitudes of the Polar Sequence stars from *Harvard Circular*, No. 170, are here used instead of the photographic. Precisely the same method has been followed in deriving the reductions to the Pole as was used for the photographic magnitudes. The zenith distances of the variable and the corresponding corrections for extinction were:

	Plate			
	847	858	900	901
Zenith Distance	54°7	48°2	37°5	43°8
Extinction	+0.02	+0.10	+0.19	+0.15

The magnitudes interpolated from the curves defined by the data for the Polar Sequence stars and corrected for extinction are in

columns three to six of Table VII. The provisional magnitudes in the second column are as given in Table V, except that the value for the variable has been revised to include the results from plates

TABLE VI

COMPARISON WITH THE POLE—PHOTOVISUAL RESULTS

POLAR SEQUENCE						VARIABLE AND COMPARISON STARS				
H.C. 170		Scale Reading				No.	Scale Reading			
No.	Mag.	847	858	900	901		847	858	900	901
10.......	8.94	....	....	8.7	8.8	*v*	6.3	8.3	12.0	11.8
7*r*......	9.84	....	....	11.6	11.8	1	7.6	8.0	11.7	11.7
5*s*......	9.86	7.5	7.3	11.7	11.8	2	14.2	14.0	16.0	16.1
14.......	10.44	....	....	13.2	13.3	3	14.8	14.5	17.4	17.3
15.......	10.78	....	....	13.7	13.8	4	14.9	14.4	16.7	17.1
16.......	11.07	....	....	14.5	15.0	5	12.6	12.7	14.6	14.7
19.......	12.22	....	15.3	18.3	18.2	7	15.4	15.6	17.8	....
11*r*......	12.24	14.1	14.7	18.2	17.7	8	17.2	17.1	....	....
20.......	12.65	16.4	16.8	....	.....	9	17.4	16.7	....	....

TABLE VII

PHOTOVISUAL MAGNITUDES OF COMPARISON STARS

Star	Provisional Mag.	Interpolated Magnitude				Reduction to Polar Sequence				Adopted Mag.	Mag. from P.S.	Difference
		847	858	900	901	847	858	900	901			
v........	1.76	9.47	10.27	10.26	10.07	7.71	8.51	8.50	8.31	9.98	10.02	− 4
1........	1.80	9.92	10.17	10.15	10.03	8.12	8.37	8.35	8.23	10.02	10.07	− 5
2........	3.76	12.16	12.04	11.68	11.72	8.40	8.28	7.92	7.96	11.98	11.90	+ 8
3........	3.98	12.36	12.19	12.16	12.22	8.38	8.21	8.18	8.24	12.20	12.23	− 3
4........	3.92	12.39	12.16	11.91	12.15	8.47	8.24	7.99	8.23	12.14	12.15	− 1
5........	3.29	11.62	11.74	11.18	11.15	8.33	8.45	7.89	7.86	11.51	11.42	+ 9
6........	4.65	.....	.....	.....	.....	....	....	....	....	12.87	.....	....
7........	4.12	12.53	12.51	12.28	.....	8.41	8.39	8.16	....	12.34	12.44	−10
8........	4.58	12.96	12.88	.....	.....	8.38	8.30	....	....	12.80	12.92	−12
9........	4.58	13.00	12.80	.....	.....	8.42	8.22	....	....	12.80	12.90	−10
10........	4.80	.....	.....	.....	.....	....	....	....	....	13.02	.....	....
Means...........................						8.29	8.33	8.14	8.14			
Adopted mean reduction to Polar Sequence........................						8.22						

900 and 901, which, having been made with the 32-inch diaphragm, are available for the determination of the relative brightness of the variable and star No. 1. The arrangement of Table VII and the

method of reduction to the zero point of the Polar Sequence is the same as that for Table III. As before, the last column gives the difference between the adopted magnitudes resulting from the independent determination of the scale and the means of the interpolated magnitudes which are based upon the Harvard values for the Polar Sequence stars. With the exception of a small constant difference due to the particular distribution of the data used, the agreement between the Harvard visual scale and the photovisual results obtained with the "Iso" plate and yellow filter is satisfactory.

TABLE VIII

ADOPTED MAGNITUDES

Star	Photographic Mag.	Photovisual Mag.	Color Index	Spectrum	
				H. Mags.	Mt. W. Mags.
v...(Normal Brightness)	9.64	9.98	−0.34	B4	A2
1	9.78	10.02	−0.24	B6	A4
2	11.91	11.98	−0.07	B9	A8
3	12.00	12.20	−0.20	B6	A5
4	12.11	12.14	−0.03	A0	A9
5	12.28	11.51	+0.77	F8	G6
6	12.56	12.87	−0.31	B4	A2
7	12.58	12.34	+0.24	A6	F5
8	12.90	12.80	+0.10	A3	F1
9	12.91	12.80	+0.11	A3	F1
10	12.94	13.02	−0.08	B9	A8
11	13.16	.....	......	...	...
12	13.46	.....	......	...	...
13	13.68	.....	......	...	...
14	13.77	.....	......	...	...

In Table VIII are collected the final results for the normal brightness of the variable and for the various comparison stars. The fourth column contains the color index, and the fifth, the corresponding spectrum interpolated from the curve of Parkhurst.[1] The apparent preponderance of very early type spectra raises a question as to whether the two scales, if extended to stars of the sixth magnitude, would coincide for stars of the type A0. There is, of course, some uncertainty in the zero points of the adopted scales owing to the small number of comparisons with the Pole. The

[1] "Yerkes Actinometry," *Astrophysical Journal*, **36**, 217, 1912.

average deviation of the separate values of the reduction constants from their means are respectively 0.07 and 0.08 mag. One would not expect, therefore, a relative error greater than a tenth of a magnitude, assuming both the photographic and visual magnitudes used for the Polar Sequence stars to be correct. But an error even of this amount would not be sufficient to account for the deviation of the spectra from the normal distribution.

The results become more satisfactory, if, instead of the Harvard photographic magnitudes for the Polar Sequence stars, we use the values from an investigation with the 60-inch reflector for which preliminary results have recently been obtained. The agreement of the Mount Wilson scale with that of Harvard is excellent for the stars between the tenth and fifteenth magnitudes, but for the brighter stars there is a well-marked divergence amounting to about 7 per cent. If the two scales be brought into coincidence at the sixth magnitude in accordance with the international convention, it appears that the Mount Wilson magnitudes for the fainter stars are fainter than those of Harvard. The difference increases uniformly from the sixth to the tenth magnitude, but from there on is constant and equal to 0.37 mag. Since all of the Polar Sequence stars used above for the determination of the zero point of the photographic scale are fainter than the tenth magnitude (see Table II), the results in the second and fourth columns of Table VIII may be reduced to the Mount Wilson system by the simple addition of 0.37 mag. The spectra corresponding to this modification are given in the last column of Table VIII. It will be observed that the distribution is now normal, and, further, that the spectrum for the variable is now in sensible agreement with the Harvard value A5?.[1] The result is of interest, but not conclusive, for we have practically no reliable information as to the distribution of the spectra of the fainter stars; and, moreover, we are here dealing with only a small group of objects. But as the main purpose of the present discussion is the determination of the relative values of the photographic and the visual variation of *RR Draconis*, the uncertainty is of no immediate consequence.

[1] *Annals Harvard College Observatory*, **56**, 189.

The data relating to the observations during the period of variation are in Tables IX–XI. The scale readings have been corrected for distance error, and those for the variable have been reduced to the mean exposure by the method used for the first series. As before, there is no evidence of systematic difference

TABLE IX

MEAN SCALE READINGS OF COMPARISON STARS—PHOTOGRAPHIC RESULTS

Star	813	815	817	819	821	823	825	827	829
1	5.0	5.8	5.2	4.6	6.3	6.3	5.6	5.6	5.4
2	12.5	12.0	....	10.7	12.0	11.5	11.6	11.9	11.6
3	12.4	12.2	....	11.4	12.4	12.1	11.9	11.8	11.8
4	12.5	12.4	12.1	11.6	12.8	12.2	12.2	12.0	12.0
5	12.8	12.7	12.1	11.5	12.4	12.1	12.2	12.0	12.1
6	13.5	13.0	12.3	12.3	13.5	13.3	12.7	12.6	12.8
7	13.3	12.9	12.4	12.3	13.3	13.0	12.6	12.5	12.5
8	13.6	13.4	13.1	12.9	14.0	13.5	13.3	13.2	13.4
9	13.7	13.4	13.2	13.2	14.3	13.9	13.3	13.4	13.5
10	13.7	13.7	13.2	13.8	14.1	13.9	13.5	13.5	13.7
11	14.1	13.5	13.1	13.8	14.0	14.0	13.8	13.6	14.1
12	14.9	14.3	14.1	14.6	14.7	14.6	14.5	14.3	14.8
13	15.5	15.3	15.1	15.4	15.6	15.2	14.9	15.0	15.7
14	15.6	15.5	15.3	15.8	15.8	15.9	15.5	15.6	16.2

TABLE X

MEAN SCALE READINGS OF COMPARISON STARS—PHOTOVISUAL RESULTS

Star	814	816	818	820	822	824	826	828	830
1	7.8	7.8	7.8	8.3	9.7	7.8	8.0	7.8	9.1
2	14.0	13.6	13.7	14.0	14.3	13.0	13.6	13.4	15.1
3	14.6	14.2	14.3	14.6	15.3	13.7	14.0	13.8	15.6
4	14.6	14.1	14.2	14.5	15.0	13.6	13.8	13.8	15.4
5	12.9	12.2	12.3	13.1	13.6	11.8	12.3	11.9	13.4
6	16.4	16.2	16.2	15.9	16.6	15.0	15.4	15.5	....
7	15.1	14.7	14.6	15.0	16.0	14.2	14.5	14.4	17.1
8	16.4	16.0	15.9	16.1	17.0	15.0	15.8	15.5	17.8
9	16.4	16.0	15.8	15.8	16.8	14.8	15.2	15.4	17.8
10	17.1	16.6	16.7	16.8	....	15.6	15.7	16.1	....

between the first and last exposures on the same plate. The values of the brightness of the variable in the seventh column of Table XI were interpolated from the magnitude-curves formed with the adopted magnitudes of the comparison stars in Table VIII and the scale readings in Tables IX and X, the argument used being the scale reading in the sixth column of Table XI.

TABLE XI

MEAN SCALE READINGS AND MAGNITUDES OF *RR Draconis*

1912, September 10

Plate	Exposure	G.M.T.		Phase	s	Mag.	O.−C.
		PHOTOGRAPHIC RESULTS					
P 813......	1	$15^h20^m.5$	$0^d.639$	$-0^d.107$	7.4	10.39	+ 4
	2	22.5	0.641	−0.105	7.3	10.36	0
	3	24.5	0.642	−0.104	7.5	10.42	+ 4
	4	26.5	0.644	−0.102	7.1	10.31	−12
	5	28.5	0.645	−0.101	7.8	10.50	+ 6
815......	1	16 5.5	0.671	−0.075	9.2	10.93	− 7
	2	7.5	0.672	−0.074	9.6	11.07	+ 5
	3	9.5	0.674	−0.072	9.7	11.11	+ 2
	4	11.5	0.675	−0.071	9.8	11.15	+ 3
	5	13.5	0.676	−0.070	10.2	11.30	+15
817......	1	16 47.5	0.699	−0.047	11.9	12.34	+ 7
	2	49.5	0.701	−0.045	11.8	12.29	−11
	3	51.5	0.702	−0.044	12.1	12.44	− 1
	4	53.5	0.703	−0.043	12.1	12.44	− 9
	5	55.5	0.705	−0.041	12.6	12.66	+ 2
819......	1	17 22.5	0.724	−0.022	14.8	13.52	+ 6
	2	24.5	0.725	−0.021	14.7	13.50	+ 4
	3	26.5	0.727	−0.019	14.8	13.52	+ 6
	4	28.5	0.728	−0.018	14.7	13.50	+ 4
	5	30.5	0.729	−0.017	14.8	13.52	+ 6
821......	1	18 2.5	0.752	+0.006	15.2	13.52	+ 6
	2	4.5	0.753	+0.007	15.1	13.47	+ 1
	3	6.5	0.755	+0.009	15.0	13.43	− 3
	4	8.5	0.756	+0.010	15.0	13.43	− 3
	5	10.5	0.757	+0.011	15.1	13.47	+ 1
823......	1	18 36.5	0.776	+0.030	14.8	13.49	+ 6
	2	38.5	0.777	+0.031	14.5	13.35	− 7
	3	40.5	0.778	+0.032	14.5	13.35	− 5
	4	42.5	0.780	+0.034	13.8	13.00	−20
	5	44.5	0.781	+0.035	14.2	13.19	+ 5
825......	1	19 15.5	0.803	+0.057	11.2	11.78	+ 6
	2	17.5	0.804	+0.058	11.2	11.78	+14
	3	19.5	0.806	+0.060	11.0	11.69	+13
	4	21.5	0.807	+0.061	10.7	11.57	+ 7
	5	23.5	0.808	+0.062	10.1	11.35	−12
827......	1	19 49.5	0.826	+0.080	8.6	10.88	+ 1
	2	51.5	0.827	+0.081	8.6	10.88	+ 4
	3	53.5	0.828	+0.082	8.5	10.84	+ 2
	4	55.5	0.830	+0.084	8.3	10.76	− 1
	5	57.5	0.831	+0.085	8.2	10.72	− 4
829......	1	20 22.5	0.849	+0.103	7.0	10.46	+ 5
	2	24.5	0.850	+0.104	6.8	10.40	+ 2
	3	26.5	0.852	+0.106	6.8	10.40	+ 4
	4	28.5	0.853	+0.107	6.4	10.26	− 9
	5	30.5	0.854	+0.108	6.6	10.33	− 1

TABLE XI—*Continued*

Plate	Exposure	G.M.T.		Phase	s	Mag.	O.−C.
		PHOTOVISUAL RESULTS					
P 814......	1	$15^{h}41^{m}5$	$0^{d}654$	$-0^{d}092$	11.4	10.98	0
	2	45.5	0.657	−0.089	11.7	11.07	+ 4
	3	49.5	0.660	−0.086	11.8	11.11	+ 1
	4	53.5	0.662	−0.084	11.7	11.07	− 7
816......	1	16 28.5	0.686	−0.060	13.2	11.85	+ 9
	2	32.5	0.689	−0.057	13.0	11.77	−17
	3	36.5	0.692	−0.054	13.4	11.92	− 5
	4	40.5	0.694	−0.052	13.8	12.04	− 7
818......	1	17 2.5	0.710	−0.036	15.7	12.75	−12
	2	6.5	0.713	−0.033	15.6	12.71	−34
	3	10.5	0.715	−0.031	16.2	12.91	−25
	4	14.5	0.718	−0.028	16.5	13.00	−23
820......	1	17 43.5	0.738	−0.008	17.2	13.25	+ 2
	2	47.5	0.741	−0.005	16.9	13.14	− 9
	3	51.5	0.744	−0.002	17.4	13.32	+ 9
	4	55.5	0.747	+0.001	17.2	13.25	+ 2
822......	1	18 17.5	0.762	+0.016	17.8	13.18	− 5
	2	21.5	0.765	+0.019	17.8	13.18	− 5
	3	25.5	0.768	+0.022	18.1	13.30	+ 7
	4	29.5	0.771	+0.025	17.9	13.21	− 2
824......	1	18 50.5	0.785	+0.039	14.6	12.61	− 6
	2	54.5	0.788	+0.042	14.4	12.53	− 2
	3	58.5	0.791	+0.045	14.2	12.44	+ 4
	4	19 2.5	0.794	+0.048	13.8	12.26	− 2
826......	1	19 31.5	0.813	+0.067	12.7	11.66	− 1
	2	35.5	0.816	+0.070	12.2	11.48	− 4
	3	39.5	0.819	+0.073	12.5	11.59	+ 5
	4	43.5	0.821	+0.075	11.9	11.36	− 1
828......	1	20 4.5	0.836	+0.090	10.7	10.97	− 3
	2	8.5	0.839	+0.093	10.7	10.97	+ 1
	3	12.5	0.842	+0.096	10.3	10.84	− 6
	4	16.5	0.845	+0.099	10.6	10.94	+10
830......	1	20 37.0	0.859	+0.113	10.6	10.50	−13
	2	40.0	0.861	+0.115	11.1	10.66	+ 6
	3	43.0	0.863	+0.117	11.5	10.79	+22
	4	46.0	0.865	+0.119	11.0	10.62	+ 8

Both the photographic and the photovisual light-curves are defined for an interval of about a tenth of a day on either side of the minimum, so that the epoch is reasonably well established by both. The value, 1912, September 10, $17^{h}53^{m}5$ G.M.T., was given

by both curves. Reduced to the sun, the result is, September 10, $17^h 54^m.5$, or J.D. 2419656.7461 G.M.T. The representation of the minimum by the elements of the first paper is exact. The representation of the visual observations by Lehnert[1] is less satisfactory, however, as each of the three minima observed by him give the correction O.−C.=$+0^d.006$. A deviation in the same direction is also shown by the minimum observed visually by Mr. Harlow Shapley at Princeton in June 1912, and kindly communicated by letter. The correction in this instance is $+0^d.003$.

Both series of photographic observations, and the photovisual results of the latter series were grouped into the normal places shown in Table XII for the formation of mean light-curves. As all of the data thus far accumulated, including the Laws Observatory visual measures, indicate that the light-curve is symmetrical, the results for the two branches were combined in deriving the normal places. The resulting curves are shown in Fig. 1, and their ordinates are given in the second and third columns of Table XIII. The representation of the observations by the curves is indicated by the residuals in the last column of Table XI. The average deviation for the photographic series is ±0.057 mag.; for the photovisual, ±0.077 mag. Omitting plate 818, for which the results are discordant, the average deviation for the photovisual observations becomes the same as for the photographic.

The limits and the amounts of variation, and the extreme color indices are:

	Photographic	Photovisual	Color Index
Maximum	9.64	9.98	−0.34
Minimum	13.46	13.23	+0.23
Amplitude	3.82	3.25	0.57

The two series of photographic observations agree closely in the values given for the photographic range. The photovisual range differs considerably from the visual value of 2.96 mags. found by Shapley on June 21, 1912, with a photometer of the polarizing type. Part of this difference may be due to the lack of red-sensitiveness

[1] *Astronomische Nachrichten*, **191**, 201, 1912.

in the Cramer "Iso" plate, which for stars of more advanced spectral types can scarcely be expected to give exact agreement with normal visual magnitudes. Both series of observations are in agreement, howevei, in indicating a smaller value for the visual

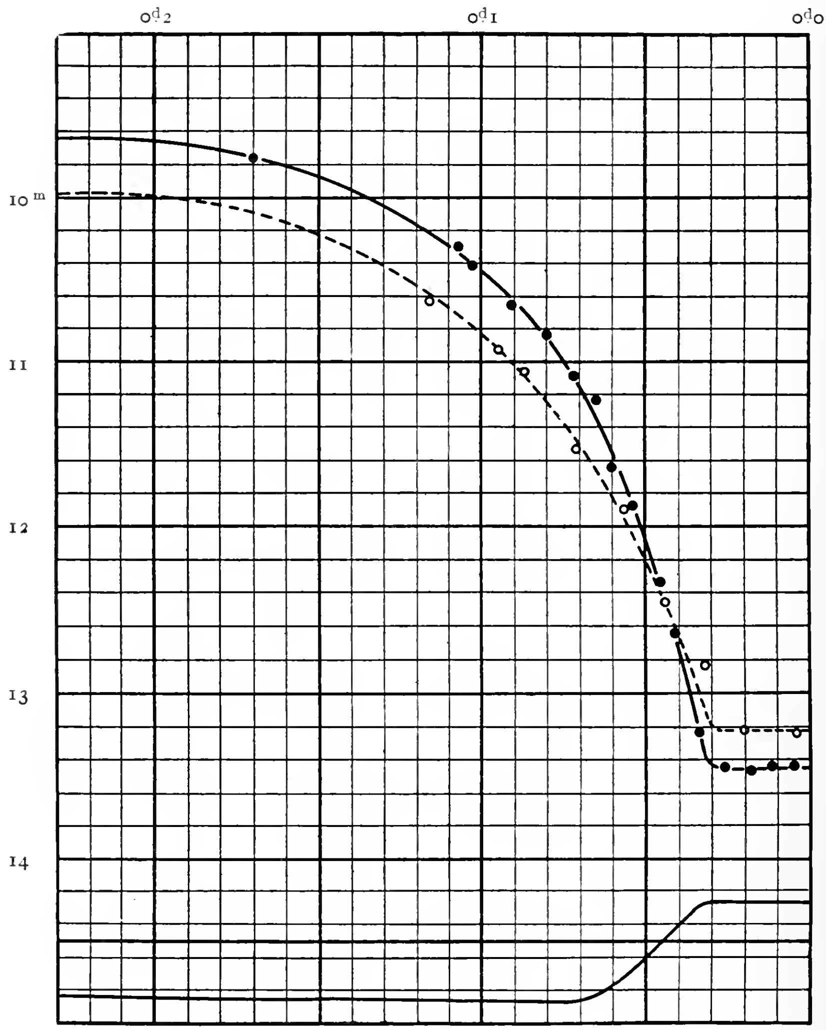


FIG. 1.—Photographic and photovisual light-curves for *RR Draconis*. The latter is the broken curve. The variation in the color index is shown in the lower part of the figure.

range, and there can be no doubt of the reality of the difference in the photographic and visual results.

TABLE XII

NORMAL PLACES FOR MEAN LIGHT-CURVE

Photographic				Photovisual			
Phase	Mag.	O.−C.	No. Obs.	Phase	Mag.	O.−C.	No. Obs.
0^{d}005....	13.45	− 1	6	0^{d}004	13.24	+ 1	4
12....	13.45	− 1	7	20	13.22	− 1	4
18....	13.48	+ 2	7	32	12.84	−25	4
26....	13.44	− 2	7	44	12.46	0	4
34....	13.24	0	7	56	11.90	− 6	4
41....	12.64	0	6	71	11.52	+ 2	4
46....	12.33	0	6	87	11.06	0	4
54....	11.88	0	6	95	10.93	+ 2	4
60....	11.65	+ 9	6	116	10.64	+ 5	4
65....	11.24	−10	6	0.220	9.98	0	4
72....	11.08	0	6	.....	.....	....	..
80....	10.84	− 3	7	.....	.....	....	..
91....	10.65	+ 3	7	.....	.....	....	..
103....	10.42	+ 1	7	.....	.....	....	..
107....	10.30	− 4	6	.....	.....	....	..
170....	9.76	0	4	.....	.....	....	..
0.220....	9.64	0	..	.....	.....	....	..

TABLE XIII

MEAN LIGHT-CURVES AND COLOR INDEX OF *RR Draconis*

Phase	Magnitude		Color Index		Phase	Magnitude		Color Index	
	Photo-graphic	Photo-visual	Mt. W.	Laws Obs.		Photo-graphic	Photo-visual	Mt. W.	Laws Obs.
≡0^{d}025..	13.46	13.23	+23	....	0^{d}090..	10.65	11.00	−35	−33
30..	13.43	13.20	+23	....	100..	10.45	10.81	−36	−34
35..	13.14	12.94	+20	−10	110..	10.30	10.66	−36	−33
40..	12.73	12.63	+10	−13	120..	10.17	10.53	−36	−34
45..	12.40	12.40	0	−17	130..	10.07	10.42	−35	−34
50..	12.08	12.18	−10	−22	140..	9.98	10.31	−33	−33
55..	11.80	12.00	−20	−27	150..	9.89	10.23	−34	−34
60..	11.56	11.82	−26	−21	160..	9.82	10.16	−34	−34
65..	11.34	11.67	−33	−34	170..	9.76	10.10	−34	−34
70..	11.16	11.52	−36	−35	180..	9.71	10.05	−34	−34
75..	11.00	11.37	−37	−36	190..	9.68	10.02	−34	−34
80..	10.87	11.24	−37	−37	200..	9.66	10.00	−34	−34
85..	10.76	11.11	−35	−33	210..	9.65	9.99	−34	−34
0.090..	10.65	11.00	−35	−33	≡0.220..	9.64	9.98	−34	−34

The variation in the color index is shown by the curve in the lower part of Fig. 1, and the numerical values are given in the fourth

column of Table XIII. The color index resulting from the comparison of the photographic results with the Laws Observatory visual light-curve in Table I of the first paper is also shown in Table XIII, in the last column. For this comparison the zero point of the visual curve was made to coincide with that of the photovisual results. An examination of the two series of values shows that for phase values greater than $0^{d}.060$ the three curves are practically identical. For values less than $0^{d}.060$ the Laws Observatory curve lies between the other two.

The photographic curve is probably well determined, and the same may be said of the photovisual curve between the phase values $0^{d}.050$ and $0^{d}.125$. The two points on the latter curve within the interval of constant minimum brightness are also probably reliable, but the intermediate portion of the curve is uncertain. It is impossible to improve the representation of the normal point at $0^{d}.032$ without leading to improbable values of the color index. On the other hand, there is no obvious reason why this point should be discordant, for plate 818, upon which it is based, is apparently equal in quality to the others. An examination of the residuals shows that they may be slightly reduced by applying a correction of $+0^{d}.002$ to the adopted epoch, but this raises difficulties with the photographic curve, unless we admit the possibility of a difference in the times of the photographic and photovisual minima. Although the modification of the photovisual epoch would reduce the average residual and lead to a correction for the adopted elements in the direction of that required by the visual observations of Lehnert and Shapley, it seems best to let the results stand as they are for the present.

If the photographic magnitudes be based upon the Mount Wilson system referred to above, it is to be noted that the maximum and minimum limits of variation become 10.01 and 13.83 mags., respectively; and that the extreme color indices are then +0.03 and +0.60. The corresponding spectrum at minimum is F5, and, as the minimum is a total eclipse of the bright body by the larger and darker companion, this value is to be assigned to the fainter object. The corresponding modifications in the photographic mean light-curve and in the color indices in Tables XII

and XIII are made by adding the value 0.37 mag. throughout. The effect upon the curves in Fig. 1 would be to displace the photographic curve downward, and that of the color index upward, by this amount.

But quite independently of any uncertainty as to the spectrum, the investigation shows that the darker component of the system is redder than the brighter by an amount corresponding to 1.3 times the interval separating successive spectral classes; if Shapley's value of the visual range be used, the excess of redness is measured by nearly two spectral classes. The result is one of very great interest, for Shapley, in his investigation of the orbits of eclipsing binaries, has brought forth strong evidence that the darker components of systems of the type of *RR Draconis* are much less dense than their brighter companions, and, hence, presumably mark an earlier stage in stellar evolution. In view of this fact, the exact nature of the spectrum of the darker components of such systems becomes immediately a matter of the greatest interest. Should it appear that we are justified in assigning spectral classes on the basis of the ordinary relations between color index and spectrum, we should arrive at the very significant result that at least some of the so-called advanced spectral types are very probably to be associated with the earlier stages of stellar development. Such a possibility has of course long been recognized, but thus far definite evidence has not been procurable. It is obviously impossible to observe spectroscopically an object as faint as *RR Draconis* at minimum, but there are other *Algol* variables whose examination should prove profitable.

I am greatly indebted to Miss High of the computing division of the observatory for the very careful measurement of the plates upon which the above results are based.

MOUNT WILSON SOLAR OBSERVATORY
December 16, 1912

Books Received

S. de Ball: *Lehrbuch der sphärischen Astronomie.* Pp. xv+387. Leipzig: W. Engelmann, 1912. M. 20.

James Weir: The Energy System of Matter. Pp. 200, Figs. 12. London: Longmans, Green & Co., 1912. $2.00 net.

T. Van Lohuizen: *Bijdrage tot de Kennis van Lijnen spectra.* Pp. 102, Pls. 2. Den Haag: 1912.

Svante Arrhenius: *Conference sur quelques thèmes choisi de la chimie physique pure et appliqué.* Pp. 112, Figs. 10. Paris: A. Hermann et Fils, 1912. Fr. 3.

Charles Davison: *The Origin of Earthquakes.* Pp. 144, Figs. 26. Cambridge: University Press, 1912. $0.40.

G. A. J. Cole: *Rocks and Their Origins.* Pp. 175, Figs. 20, Cambridge: University Press, 1912. $0.40.

Schott: *Electro-magnetic Radiation.* New York: McGraw-Hill Co., 1912.

Anthony Zeleny and Henry A. Erickson: *A Manual of Physical Measurements.* Pp. 248, Figs. 125. New York: McGraw-Hill Co., 1912.

C. J. P. Cave: *The Structure of the Atmosphere in Clear Weather.* Pp. 144, Figs. 47. Cambridge: University Press, 1912.

H. Kayser: Vol. VI, *Handbuch der Spectroscopie.* Pp. 1065, Pls. 6. Leipzig: S. Hirzel, 1912. M. 60.

Wilhelm Schweydar: *Untersuchungen über die Gegzeiten der festen Erde und die hypothetische Magmaschicht.* Leipzig: B. G. Teubner, 1913. M. 3.50.

Dr. Sigmund Riefler: *Tables of the Weights of Air, γ_t^b, of the Air-Pressure Equivalents β_t^b, and of Gravity, g.* Pp. 102. Berlin: Julius Springer, 1912. M. 6.

Frank Harris: *Gravitation.* Pp. 108, Figs. 17. New York: Longmans, Green & Co., 1913. $1.00.

E. Rutherford: *Radioactive Substances and Their Radiations.* Pp. 699, Pls. 8, Figs. 123. Cambridge: University Press. 15*s*. net.

Max Wolf: *Stereoskopbilder vom Sternhimmel.* Leipzig: Verlag vom Johann Ambrosius Barth, 1913. M. 5.

Resultados del Observatorio Nacional Argentino en Córdoba, Vol. 20, Catálogo de 5791 Estrellas. Cordoba: Publicado por el Observatorio, 1911.

Annuaire pour l'an 1913, publié par le Bureau des Longitudes. Paris, Gauthier-Villars. Contains statistics relating to astronomy, geography, weights and measures and meteorology (the physical and chemical tables appearing in the alternate volumes).

Annales de l'Université de Lyon, Nouvelle Série; I, Fascicule 32, 1912; contains, 1, Observations of the system of Jupiter by Guillaume; 2, Meridian observation of Struve double stars by Merlin and Flajolet; 3, Micrometric observations of Struve double stars by Cadet, Lagrula, and Guillaume.

Fascicule 33 by Luizet is in two parts, 1, "Les Céphéides considérées comme étoiles doubles"; 2, "De l'étoile variable δ *Céphée*."

H. G. ZEUTHEN: *Die Mathematik im Altertum und im Mittelalter.* ("Die Kultur der Gegenwart." Herausgegeben von Paul Hinneberg.) Die mathematischen Wissenschaften, unter Leitung von F. Klein. Des Gesamtwerkes, Teil III, Abteilung I. Erste Lieferung. Pp. 95, Figs. 20. Leipzig: B. G. Teubner, 1912. M. 3.

THOMAS EDWARD HEATH: *Tracks of the Sun and Stars, A.D. 1900 to A.D. 37900:* Shown in Space of Three Dimensions and a Description of the Method Used for Drawing Them. Pp. 17+vi, Figs. 3, Charts 6. Sold by W. Wesley & Son, London, and by Flatters & Garnett, Manchester, 5*s*. net.

NOTICE

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention is given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

Articles written in any language may be accepted for publication, but unless a wish to the contrary is expressed by the author, they usually will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right unless the author requests that the reverse procedure be followed.

Accuracy in the proof is gained by having manuscripts typewritten, provided the author carefully examines the sheets and eliminates any errors introduced by the stenographer. It is suggested that the author should retain a carbon or tissue copy of the manuscript, as it is generally necessary to keep the original manuscript at the editorial office until the article is printed.

All drawings should be carefully made with India ink on stiff paper, usually each on a separate sheet, on about double the scale of the engraving desired. Lettering of diagrams will be done in type around the margins of the cut where feasible. Otherwise printed letters should be put in lightly with pencil, to be later impressed with type at the editorial office, or should be pasted on the drawing where required.

Where unusual expense is involved in the publication of an article, either on account of length, tabular matter, or illustrations, arrangements are made whereby the expense is shared by the author or by the institution he represents.

Authors will please carefully follow the style of this JOURNAL in regard to footnotes and references to journals and society publications.

Authors are particularly requested to employ uniformly the metric units of length and mass; the English equivalents may be added if desired.

If a request is sent *with the manuscript,* one hundred reprint copies of each paper, bound in covers, will be furnished free of charge to the author. Additional copies may be obtained at cost price. No reprints can be sent unless a request for them is received before the JOURNAL goes to press.

The editors do not hold themselves responsible for opinions expressed by contributors.

The ASTROPHYSICAL JOURNAL is published during each month except February and August. The annual subscription price is $5.00; postage on foreign subscriptions, 62 cents additional. Business communications should be addressed to the *University of Chicago Press, Chicago, Ill.*

All papers for publication and correspondence relating to contributions should be addressed to *Editors of the* ASTROPHYSICAL JOURNAL, the *University of Chicago, Chicago, Illinois, U.S.A.*

THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XXXVII MAY 1913 NUMBER 4

THE TOTAL SOLAR RADIATION DURING THE ANNULAR ECLIPSE ON APRIL 17, 1912

By W. H. JULIUS

SCHEME OF THE INVESTIGATION

The annular eclipse of the sun on April 17, 1912, offered a rare opportunity for investigating the total amount of radiation due to the entire "solar atmosphere," i.e., to the complex of layers of the sun lying outside the level generally indicated as the surface of the photosphere.

Every part of the solar atmosphere emits some proper radiation and scatters some photospheric light, and it is only natural to suppose that the lowest layers bear the greatest share in that radiation and scattering. Now, at a total eclipse the base of the atmosphere is always wholly or partly screened by the moon; whereas during the annular phase of the eclipse of April 1912 even the lowest strata of the atmosphere all round the disk contributed to the remaining radiation. From the minimum value through which the remaining radiation passes at the instant of centrality one should be able to calculate an upper limit, which the radiation, emitted and scattered by the entire solar atmosphere, certainly does not exceed.

Since a reliable determination of such an upper limit would afford an important criterion for testing fundamental ideas regarding the nature of the photosphere, the principal aim kept in view in devising our actinometric apparatus was that the minimum of

the radiation-curve should come out as sharply and definitely as possible.

On former occasions (during the eclipses of 1901 in Karang Sago, Sumatra, and of 1905 near Burgos) we measured the march of the total radiation by means of a thermopile directly exposed to the sun's rays, without making use of any lenses or mirrors to concentrate the beam. If circumstances had then allowed us to find the true shape of the radiation-curve, it would have been possible to calculate from those data trustworthy values for the radiating power of successive concentric zones of the solar disk.[1] Unfortunately the weather did not favor the Sumatra and Burgos observations; so we desired to make similar observations again. The apparatus had proved satisfactory, and sensitive enough to give measurable indications of heat even during totality; for at Burgos a break in the clouds had permitted us to state that at mid-eclipse the unscreened part of the corona radiated less than 1/200,000 of the output of the uneclipsed sun or 2/5 of that of the full moon.[2]

For observing the radiation during the annular eclipse we therefore decided to follow substantially the same plan, though with some alterations in the apparatus. This time the minimum would not be so low. From a close discussion of the Burgos results we presumed it to lie somewhere between 1/10,000 and 1/1000. So the galvanometer could be taken less sensitive, but, on the other hand, the steadiness of the zero could be improved and the period of oscillation shortened.

Quickness of indication was, indeed, a very important condition, which not only the galvanometer but also the recipient of the radiation had to satisfy, if the minimum were to be observed exactly.

At the observing station near Maastricht,[3] selected by the

[1] W. H. Julius, "A New Method for Determining the Rate of Decrease of the Radiating Power from the Center toward the Limb of the Solar Disk." *Astrophysical Journal*, **23**, 312, 1906.

[2] *Proc. Roy. Acad. Amst.*, **8**, 503, 1905.

[3] A preliminary account of the observations made by the Netherlands Expedition on April 17, 1912, is to be found in *Proc. Roy. Acad. Amst.*, **14**, 1195, meeting of April 26, 1912. Cf. also: Nyland, "De eklips van 17 April 1912." *Hemel en Dampkring*, **10**, 1, May 1912.

Eclipse Commission of the Royal Academy of Amsterdam, the annular phase of the eclipse was expected to last less than one second.[1] Our thermopile, used in Sumatra and Burgos, required 10 seconds for reaching a stationary temperature after being suddenly exposed to a constant source of radiation, and therefore would be too slow to catch the minimum, although quick enough to give the greater part of the radiation-curve with sufficient accuracy.

DESCRIPTION OF APPARATUS

We determined on arranging two separate equipments: a rapidly working one, and a slower one, both suited for measuring the intensity of radiation from the first until the fourth contact, but in some respects complementing each other. The slower set of apparatus consisted of a thermopile (the same as used before), a Siemens & Halske moving-coil galvanometer with accessories and suitable resistances. The thermopile was very carefully protected against all disturbing influences; it reacted only upon the radiation that passed through a long tube fitted with diaphragms and mounted parallactically, so as to be easily kept pointing toward the sun by means of a finding arrangement.[2] We had ascertained by a special inquiry, that for temperature differences between the solderings not greater than those produced by full sunshine, the electromotive force of the thermopile could be considered strictly proportional to the intensity of irradiation. The deflections of the Siemens & Halske galvanometer were observed visually by examining the positions of a bright index on a transparent scale. With a permanent shunt of 16 ohms the instrument was just deadbeat; one millimeter deflection then corresponded to 10^{-8} amp. The deflections were proportional to the current. The observer had the resistance box close at hand, in order to keep the image on the scale, and marked the epoch of each reading by means of a double-handed chronometer, one hand of which could be stopped and made to catch up again (a *chronographe rattrapante*). Many readings were also made, in the course of the eclipse,

[1] According to J. Weeder, *Proc. Roy. Acad. Amst.*, **14**, 947, 1912.

[2] A description of the instrument is given in *Total Eclipse of the Sun, May 18, 1901; Reports on the Dutch Expedition to Karang Sago, Sumatra*, No. 4: "Heat Radiation of the Sun during the Eclipse," by W. H. Julius (1905).

with the thermopile screened; the zero proved very satisfactorily constant.[1]

Our second actinometric set was especially intended to answer rapidly and to give a photographic record of the middle part of the radiation-curve. It included a bolometer and a galvanometer with a moving coil of extremely small moment of inertia. Both instruments have been designed and constructed by Dr. W. J. H. Moll, who also was in charge of this equipment on eclipse day. The bolometer consisted of many strips of very thin platinum (Wollaston sheet) coated with lampblack, and mounted so as to form two equal gratings, one of which received the radiation. A thick copper frame warranted quick equalization of temperature of all screened parts, while an envelope of nonconducting material protected it against rapid external changes. The whole was fastened to the end of a tube with diaphragms, which was directed toward the sun by an assistant.

As will appear from the photographic records, the galvanometer answered the purpose admirably (time of deadbeat swing less than one second; deflection 4 mm for 1 microvolt; zero steady within 0.1 mm); but the instrument being only a temporary one, adapted to the requirements of this eclipse and not yet to general use, Dr. Moll, who has since been improving the pattern, desires to publish full particulars at a later date.

In order to obtain reasonable bridge-currents within the very wide range of sensitivity imposed by the phenomenon, the observer varied the resistance of the principal bolometer circuit by steps, as the eclipse proceeded, and each time read the strength of the main current on a milliammeter, the resistance in the bridge being left unaltered. That the zero-reading of the sensitive galvanometer was very little influenced thereby was a proof of the symmetry of the arrangement.

OBSERVATIONS MADE WITH THE BOLOMETER

During the greater part of the eclipse the galvanometer deflections were only visually observed, by noticing the motion of the reflected image of a slit on a transparent scale; but from 5 minutes

[1] I am very much indebted to Professor J. J. A. Muller for his valuable assistance in manipulating the thermopile on eclipse day.

before until 5 minutes after centrality the image was received on a photographic recording drum.

On Plate X, Fig. 1, the photogram is reproduced 9/20 true size. Fig. 2 is the central part of it, 5/4 true size,[1] and Fig. 3 gives on the same scale a control of the volt-sensitivity of the galvanometer, effected immediately after the eclipse was over. It shows well the qualities of the instrument.

The vertical lines are time-signals, produced by a small electric lamp flashing up at intervals of ten seconds in front of the slit of the recording apparatus; the first line following the minimum of the curve corresponds to $0^h34^m57^s$ Leiden M.T.

Two of the zero-readings, obtained by screening the bolometer, are visible on the curve (Fig. 1), one at 0^h30^m, another at 0^h37^m. A straight line joining them may quite safely be taken to represent the zero during the interval. The ordinate of the minimum thus comes out to be a quarter of a millimeter. At 11^h30^m (6 minutes after first contact) a deflection of 6.1 mm was observed visually,[2] the intensity of the main current at that time being 1/195 of its value at the time of recording. Reduced to the latter value of the main current, the deflection corresponding to full sunshine would have been more than $195 \times 6.1 = 1190$ mm, or nearly 5000 times the deflection at minimum.

A few irregularities in the curve, especially at $0^h31^m20^s$ and at $0^h36^m40^s$, require explanation. They are not genuine, but simply due to an excusable negligence of the assistant who had to point the bolometer at the sun. The emotions of the event making him forget to keep the tube continuously in the right direction, he had twice suddenly to make up for the loss. Fortunately the minimum is unaffected.

DISCUSSION OF THE BOLOMETER RESULTS

If the apparatus had followed the radiation instantaneously, the minimum would have been lower yet. We may therefore

[1] The striped aspect of the curve is connected with the click of the recording apparatus.

[2] As a basis for calculation we purposely select this *small* deflection, because the great deflections of the provisory galvanometer were not strictly proportional to the current.

certainly conclude from these observations that at the central phase of the annular eclipse the solar radiation fell below 1/5000 of its ordinary value.

This remainder must in part be due to the unscreened ring of the disk. Assuming the apparent surface of that photospheric ring to be 1/2500 of the surface of the disk (which certainly is a low estimate), and its apparent radiating power per unit of disk-surface to be 1/4 of the average intrinsic radiating power of the disk, we may say that at the epoch of centrality the photosphere was still able to furnish us with at least 1/10,000 of the ordinary amount of radiation.

Consequently, less—and probably much less—than 1/10,000 of the sun's total radiation toward the earth is left as proceeding from the annular part of the *solar atmosphere* visible round the moon's edge.

So far, the inference is pretty sure, because it depends on the outcome of direct observations only.

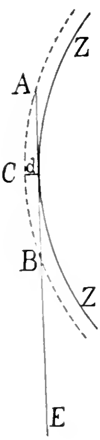


FIG. 1

What we want to deduce next, however, is an estimate of the radiation due to the entire solar atmosphere —or rather to the visible half of it. This we cannot do without making some simplifying assumptions concerning the absolutely unknown conditions prevailing in the sun.

Let ZZ (Fig. 1) be the photosphere (with radius r), ABE the direction toward the earth. If the radiating and scattering power of the solar atmosphere were distributed homogeneously through its whole depth d, the emission due to the hemispherical shell would bear approximately the same ratio to the atmospheric emission observed at mid-eclipse that the volume of the hemispherical shell ($2\pi r^2.d$) bears to the volume of the ring produced by the rotation of the segment ABC about the sun's diameter, that is parallel to AB, ($2\pi r \times$segment ABC).

That proportion is

$$p = \frac{rd}{\text{segment } ABC} \cdot$$

For small values of d the surface of the segment is nearly $2/3\ d \cdot AB$, and the ratio becomes

$$p=\frac{3}{2}\ \frac{r}{AB}\ .$$

Suppose we may replace the actual heterogeneous atmosphere by an ideal homogeneous one for which $d=2000$ km (1/350 of the sun's radius). The corresponding value of AB is about $0.15\ r$, giving for the ratio

$$p=\frac{3}{2}\ \frac{r}{0.15r}=10\ .$$

Our conclusion therefore is that less than 1/1000 of the sun's total radiation is emitted or scattered by parts of the celestial body lying outside the photospheric surface.

Even though we are free to admit an uncertainty of several hundred per cent in some of the estimates on which the above calculation is based, our result yet makes it impossible to maintain the current ideas on the nature of the photosphere.

Most solar theories, indeed, consider the photosphere to be a layer of incandescent clouds, whose decrease of luminosity from the center toward the limb of the solar disk would be caused by absorption and scattering of light in an enveloping atmosphere ("the dusky veil"). According to calculations made by Pickering, Wilson, Schuster, Vogel, v. Seeliger, and others, such an atmosphere should intercept an important fraction ($\frac{3}{4}$ to $\frac{1}{3}$) of the photospheric radiation. The atmosphere is of course in a stationary condition; receipts and expenses must balance each other. Now, what would become of that immense quantity of absorbed energy, of which only something of the order of magnitude 1/1000 is emitted and scattered? So long as we have no evidence of any other form of solar output, especially proceeding from the atmospheric layers, and comparable in magnitude with the sun's total radiation, we are forced to reject the cloud-theory of the photosphere.

The radial variation of the brightness of the disk depends on the nature of the photosphere itself, not of its envelope. A new interpretation of the photosphere, agreeing with this result, will be proposed in a subsequent paper.

OBSERVATIONS MADE WITH THE THERMOPILE

We now proceed to the discussion of the observations made for finding the shape of the entire radiation-curve. In this part of the work our thermopile arrangement had the advantage of the bolometric apparatus in point of proportionality, within wide limits, between radiation and galvanometer deflection.

The total resistance of the thermopile circuit had to be varied in a few steps from 1300 for full sunshine to 100 for the central quarter of an hour, and back again. Table I contains the deflections, all reduced to the lowest value of the resistance, and reckoned

TABLE I

Leiden Mean Time	Intensity of Radiation	Leiden Mean Time	Intensity of Radiation	Leiden Mean Time	Intensity of Radiation
$23^h 12^m 23^s$...	4960	$0^h 21^m 23^s$...	849	$0^h 44^m 54^s$...	612
13 52 ...	(1st contact)	23 42 ...	680	46 37 ...	733
15 35 ...	4950	25 3 ...	593	47 36 ...	805
23 5 ...	4725	28 1.4 ..	410	48 37 ...	872
25 16 ...	4625	28 34.6 ..	375	49 43 ...	945
28 2 ...	4460	29 10.4 ..	335	50 22 ...	993
29 40 ...	4360	30 50.6 ..	224	50 56 ...	1034
31 28 ...	4280	31 28.4 ..	183	54 2 ...	1222
37 15 ...	3950	31 57.0 ..	153	55 16 ...	1313
38 52 ...	3880	32 27.2 ..	122	55 58 ...	1386
40 27 ...	3765	33 5.8 ..	87	56 53 ...	1453
46 23 ...	3355	33 28.0 ..	66.5	57 50 ...	1550
48 31 ...	3170	33 52.6 ..	46	58 55 ...	1640
50 20 ...	3150	34 23.4 ..	21	59 57 ...	1738
51 43 ...	3075	(Minimum) ..	2.5	1 1 8 ...	1839
0 3 36 ...	2213	35 16.2 ..	20	3 16 ...	1962
5 8 ...	2075	35 52.0 ..	50	4 17 ...	2010
6 41 ...	1954	36 14.6 ..	70	5 38 ...	2095
8 7 ...	1828	36 36.6 ..	89	6 50 ...	2190
9 45 ...	1685	37 9.2 ..	119	7 49 ...	2325
11 38 ...	1551	37 40.8 ..	149	9 9 ...	2382
13 39 ...	1438	38 10.0 ..	178	38 40 ...	4340
14 53 ...	1342	38 29.7 ..	198	40 40 ...	4380
16 52 ...	1203	39 7.2 ..	238	42 10 ...	4425
17 49 ...	1107	39 43.8 ..	277	44 40 ...	4520
18 38 ...	1054	40 16.8 ..	317	46 10 ...	4600
19 28 ...	978	40 50.6 ..	356	52 10 ...	4590
20 8 ...	925	43 56 ...	545	54 10 ...	4660

from zero-positions that were found by interpolation between a series of zero-readings, made in the course of the eclipse with the thermopile shaded. The shift of the zero was small and regular.

Plate XI, Fig. 1 is a reduced copy of the original mapping of

Table I. The deflections observed between $0^h28^m10^s$ and $0^h41^m30^s$, plotted on a ten times larger scale, are shown on Plate XI, Fig 2. These latter observations give evidence of the exceptionally favorable condition of the sky especially during the middle part of the eclipse. When uniting the observational points by a curve, I was quite surprised to find it so perfectly smooth and symmetrical, for in our country a sky without even invisible haze is a rare occurrence.

The central part of this curve corroborates our conclusion drawn from the photographic curve, viz., that the minimum value of the radiation was 1/5000 of the maximum. Indeed, the real minimum value could not be reached by the slow apparatus; but if we prolong the lower parts of the falling and the rising branch of the curve downward as nearly straight lines (beginning at points corresponding to 10 seconds before and 10 seconds after centrality), they meet at *one* millimeter above zero; and according to Plate XI, Fig. 1 the maximum was represented by about 5000 mm.

The rest of the observations ran somewhat less regularly, both in the falling and in the rising phase of the radiation. From notes on sky-condition, made by other members of the party, we could afterward state that the depressions in the series of points exactly corresponded to hazy cloudlets passing before the sun. Yet some arbitrariness was left in the process of tracing the radiation-curve so as to answer to an ideally constant degree of transparency of the sky. We simply made the curve pass through the *highest* points (because the observed values could only be too small), and for the rest took care that the curvature should vary as regularly as possible.

Special attention may be drawn to the points *B* (Plate XI, Fig. 1) marked by small circlets. They are deduced from the Burgos observations of 1905[1] in the following way.

In the course of that eclipse the sun shone sometimes for a few minutes in a beautifully clear patch of sky between heavy clouds, and happened to do so during the phases in which the radiation passed through one-half of its maximum value. The exact epochs at which the intensity was $\frac{1,794,000}{2} = 897,000$ occurred

[1] *Astrophysical Journal*, **23**, 312, 1906.

$33^m 38^s$ before second contact and $33^m 43^s$ after third contact; so, on the average, $33\frac{2}{3}$ minutes were required for the moon to cover the second effective half of the solar disk.

Now, at Burgos the moon's edge took $77\frac{3}{4}$ minutes to cross the whole solar disk; at Maastricht, in 1912, it took $80\frac{3}{4}$ minutes. If, therefore, the ratio of the radius of the moon's disk to the radius of the sun's disk had been the same in both cases, then the time necessary for covering the second effective half of the solar disk would have been, at Maastricht, $33\frac{2}{3}\times\frac{80\frac{3}{4}}{77\frac{3}{4}}$ = very nearly 35 minutes.

But at Maastricht the moon's radius was practically equal to the sun's radius, whereas at Burgos the radii were in the proportion 132.8 : 126.8. This difference between the two cases implies that the interval of 35 minutes, calculated for Maastricht, is a little too great. Indeed, when drawing circles representing the sun and the moon in the right proportion and position, and taking the distribution of brightness on the disk into consideration, one easily concludes that the interval has to be taken about 25 seconds smaller, say $34\frac{1}{2}$ minutes.

Consequently, the results obtained in 1905 required that in 1912, at the epochs $0^h 0^m 20^s$ and $1^h 9^m 20^s$ (i.e., $34\frac{1}{2}$ minutes before and after centrality), the radiation should have shown half its maximum intensity, or $\frac{4960}{2}=2480$ scale divisions. This is indicated by the points *B*. The agreement with the actual observations of 1912 is indeed very satisfactory.

During the middle phase of the Burgos eclipse the conditions were, on the contrary, so unfavorable that the central part of the radiation-curve, there obtained, claims no confidence.

It was worth while, therefore, to found on our present eclipse-curve a renewed application of the method, formerly devised,[1] of determining the rate of decrease of the radiating power from the center toward the limb of the solar disk.

DISCUSSION OF THE THERMOPILE RESULTS

On a homogeneous piece of paper a circle of 40 cm in diameter, representing the sun, was drawn, and divided in the manner shown

[1] *Ibid.*, **23**, 312, 1906.

by the adjoined figure.[1] There are concentric zones, indicated by the numbers 1 to 12, and arcs representing the moon's limb in a series of positions. The width of the sickle-shaped strips bounded by these arcs is 1/20 of the sun's radius, excepting the strips *a*, *b*, *c*, *d*, for which it is 1/40.

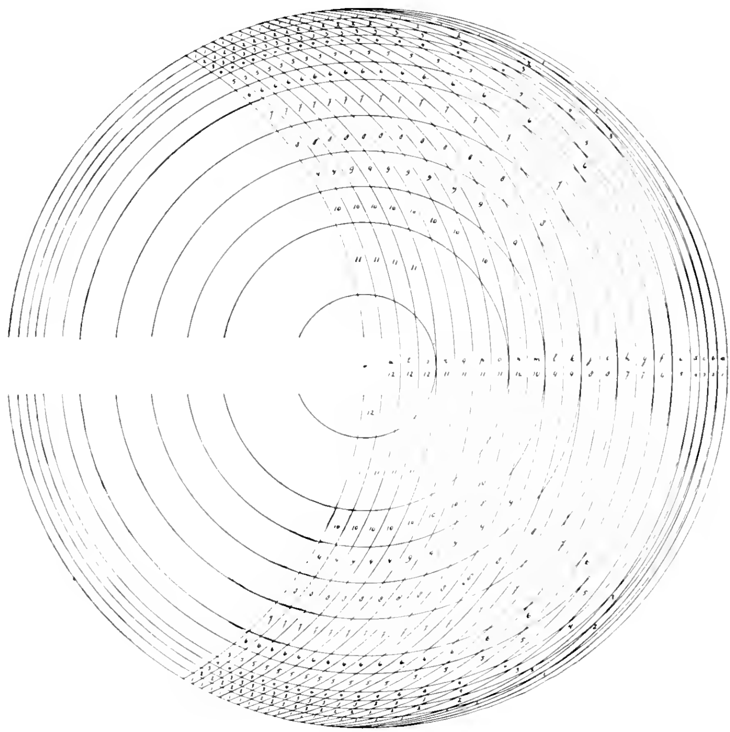


FIG. 2

In $40\frac{3}{8}$ minutes the moon's limb accomplished a distance equal to the sun's apparent radius; so the strips *a*, *b*, *c*, *d* required $1/40 \times 40\frac{3}{8}$ minutes each for reappearing from behind the moon, the strips *e* to *u* took $1/20 \times 40\frac{3}{8}$ minutes each. On our curve (Plate XI) we read the successive increments of the radiation, corresponding to the series of sickle-shaped strips. We shall denote these increments by the same letters as the strips.

[1] The figure is not a copy of the original drawing, as this could not be so much reduced on account of the delicacy of the lines.

The increment a is entirely due to radiation from zone 1; the increment b to radiation from the zones 1 and 2, etc.

Let us indicate by x_n the average intensity of the radiation with which a unit of disk-surface, belonging to zone n, supplies our thermopile. Then the increment h, for instance, will be composed as follows:

$$h=\theta_1 x_1+\theta_2 x_2+ \quad . \quad . \quad . \quad . \quad . \quad +\theta_7 x_7,$$

θ_1, θ_2, etc., being the surfaces of the parts which the corresponding zones contribute to the strip h. Though possible, it is extremely tedious to calculate these surfaces. We therefore determined them

TABLE II

Increments	Coefficients of:											
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
a= 47..	251.0	....	....	....	....	....	....	....	....	....	....	....
b= 53.5	83.0	168.4	....	....	....	....	....	....	....	....	....	....
c= 58.5	25.5	88.5	137.5	....	....	....	....	....	....	....	....	....
d= 62..	13.8	34.5	78.5	123.0	....	....	....	....	....	....	....	....
e=130..	15.7	37.5	59.6	113.0	264.0	....	....	....	....	....	....	....
f=135..	10.9	21.0	31.1	45.0	163.0	217.0	....	....	....	....	....	....
g=140..	8.5	15.0	19.5	27.0	80.0	146.0	192.0	....	....	....	....	....
h=144..	8.1	11.9	15.8	21.1	55.3	77.0	298.0	....	....	....	....	....
i=147..	7.7	10.3	12.3	15.9	42.0	55.5	198.0	146.5	....	....	....	....
j=150..	7.4	9.3	11.0	13.2	33.0	42.0	123.5	247.0	....	....	....	....
k=152..	7.1	8.4	9.2	11.9	28.7	34.8	93.5	168.5	120.0	....	....	....
l=153..	6.9	8.2	8.8	10.2	25.4	30.2	76.6	108.2	204.2	....	....	....
m=154..	6.9	8.1	8.5	9.8	22.4	27.5	66.0	86.5	142.0	98.2	....	....
n=154.5	6.8	8.0	8.3	9.5	20.8	24.9	58.0	73.4	96.4	165.3	....	....
o=154..	6.8	7.7	8.1	9.2	19.8	22.6	52.5	63.6	77.8	119.7	77.7	....
p=154..	6.8	7.6	8.0	9.0	19.1	21.1	49.1	57.3	66.2	82.2	134.0	....
q=154..	6.7	7.5	7.8	8.8	18.3	19.7	44.9	53.0	57.7	68.9	164.7	....
r=153.5	6.7	7.4	7.6	8.6	17.6	19.0	42.4	49.7	53.4	60.0	181.2	....
s=152.5	6.8	7.5	7.6	8.4	17.0	18.2	40.3	45.5	49.2	54.5	143.0	50.3
t=151.5	6.8	7.5	7.5	8.2	16.7	17.5	39.2	42.9	46.5	51.1	115.3	83.6
u=149.5	6.8	7.4	7.5	8.1	16.5	17.1	38.0	41.7	45.1	48.3	102.0	97.0

by cutting out and weighing the pieces of each strip. So the unit of area, adopted for measuring the surfaces, corresponds to a piece of our drawing-paper weighing 1 mg. Expressed in that unit, the coefficients θ_1, θ_2, ... θ_7 were found to be 8.1, 11.9 . . . . 298.0. Table II contains all the coefficients of x_1, x_2, x_3 . . . . x_{12} thus obtained. The first column gives the values of the increments of the radiation as read on the eclipse-curve. Every horizontal

row defines an equation. From the first equation we obtain x_1, from the second equation x_2, etc.

The results are collected in the second column of Table III. The third column shows the same values converted into percentages of the intensity prevailing in the center of the disk. After they had

TABLE III

Distance of Zone from Center of Disk	Average Radiating Power per Unit of Zone-Surface: Found Directly from the Equations	Average Radiating Power per Unit of Zone-Surface: Reduced to Value 100 at Center	Average Radiating Power per Unit of Zone-Surface: Found by Graphical Interpolation	Distance from Center of Disk
0.9875......	x_1 =0.18725	48.6	40.0	1.0
0.9625......	x_2 =0.2254	58.5	61.0	0.95
0.9375......	x_3 =0.2457	63.9	69.0	0.9
0.9125......	x_4 =0.2631	68.4	74.2	0.85
0.875.......	x_5 =0.2813	73.0	77.8	0.8
0.825.......	x_6 =0.2900	75.4	80.7	0.75
0.75........	$x_7 = \left\{\begin{matrix}0.3038\\0.3103\end{matrix}\right\} = 0.3071$	79.8	83.3	0.7
			87.4	0.6
0.65........	$x_8 = \left\{\begin{matrix}0.3221\\0.3305\end{matrix}\right\} = 0.3263$	84.8	91.0	0.5
0.55........	$x_9 = \left\{\begin{matrix}0.3463\\0.3432\end{matrix}\right\} = 0.3447$	89.5	93.8	0.4
			96.5	0.3
0.45........	$x_{10} = \left\{\begin{matrix}0.3519\\0.3562\end{matrix}\right\} = 0.3540$	92.0	98.3	0.2
0.3.........	$x_{11} = \left\{\begin{matrix}0.3656\\0.3694\\0.3681\\0.3734\end{matrix}\right\} = 0.3691$	95.9	99.5	0.1
			100.0	0.0
0.125.......	$x_{12} = \left\{\begin{matrix}0.3817\\0.3842\\0.3860\end{matrix}\right\} = 0.3840$	99.8		

been plotted on millimeter paper, a smooth curve was drawn, fitting the points as well as possible. On this "distribution-curve" the numbers of the fourth column were read as ordinates, belonging to the places defined in the fifth column. Our results are thus made more easily comparable with those obtained by other observers.

It is not surprising to find the shape of our distribution-curve sensibly different from the shape of any of the curves that represent Vogel's spectrophotometric measurements. Indeed, the latter show the distributions characteristic of special groups of rays, each covering a narrow part of the spectrum; they are germane, but yet vary considerably with the wave-length. The combined effect of all waves (invisible ones included), that are absorbed by our thermopile, must give a distribution-curve of another type, less simple than that to which Vogel's curves for nearly monochromatic light belong.

SUMMARY

During the annular eclipse of the sun on April 17, 1912, the variation of the total radiation has been observed near Maastricht under exceptionally favorable sky-conditions, with two mutually independent sets of apparatus.

One set, comprising a bolometer and a short-period recording galvanometer, served the purpose of finding as accurately as possible the proportion of the minimum to the maximum radiation. The ratio was found to be nearly 1/5000. On this result we based an estimate of the total amount of energy radiated and scattered by the entire solar atmosphere; we thus obtained a very small fraction of the solar output (about 1/1000).

It is impossible, therefore, to ascribe the fall of the sun's brightness from the center toward the limb of the disk to absorption or scattering of the light by an atmosphere, enveloping a body that otherwise would appear uniformly luminous. The cloud-theory of the photosphere is not borne out by the facts.

With the other set of apparatus, consisting of a thermopile and accessories, we obtained a sufficient number of reliable readings for constructing the whole radiation-curve, from the first until the fourth contact, with a fair degree of exactness. Besides confirming the value of the minimum as found with the bolometer, this curve procured the data necessary for once more determining the rate of decrease of the radiating power from the center to the limb of the solar disk.

Utrecht
January 1913

THE VARIATION WITH TEMPERATURE OF THE ELECTRIC FURNACE SPECTRUM OF IRON[1]

By ARTHUR S. KING

It is well known that the spectrum of a given element is usually subject to large variations depending on the physical conditions of the radiating source. The differences between flame, arc, and spark spectra have been studied, with a considerable divergence of opinion among investigators as to how largely these differences may be ascribed to temperature rather than to the electrical and chemical conditions which are manifestly present in different degrees in the several sources. Between the flame and the arc, overlapping in temperature some of the hotter flames, stands the tube resistance furnace, with which the investigation now to be described has been carried out. By the use of this apparatus, any desired steps of temperature may be obtained and measured, up to a point where the vaporization of the carbon tube becomes excessive. The resulting spectra show a progressive change in the relative intensities of lines with increase of temperature. It is the purpose of this paper to give the intensities of iron lines as produced by the furnace at low, medium, and high temperature, and a comparison in some detail with the arc spectrum, together with some mention of the spectra from other sources. The material available shows not only the approximate temperature at which a given line appears in the iron spectrum, but also the rate at which the line strengthens with increasing furnace temperature and with the change from furnace to arc. The classification based on these results will, it is hoped, aid in showing to what extent an observed variation in the spectrum of a given source is to be ascribed to temperature differences

APPARATUS AND METHODS

1. *The electric furnace.*—A general description of the tube resistance furnace used in the Pasadena laboratory was given in

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 66.

the first publication concerning it.[1] The furnace consists essentially of a tube of Acheson graphite, each end of which is clamped in a contact block which connects it with one of two water-cooled copper pipes leading in the current. The whole is inclosed in a heavy steel cylinder cooled by water-jackets, the electrode tubes passing through one head of the cylinder. Windows in each end allow light from the furnace tube to pass to the spectrograph and also permit of pyrometer measurements.

The contact blocks, instead of being entirely of graphite, as in the original design of the furnace, have been improved by making them of massive blocks of bronze 5 cm thick, sawed from top to bottom, and fitting around the upper or lower electrode tube as the case may be. In the center of each contact block is a cylindrical cavity into which fits a disk of graphite 5 cm in thickness and 7.6 cm in diameter, sawed across the vertical diameter and bored in the center to receive the end of the graphite tube, which is usually about 19 mm outside diameter, 12.5 mm inside diameter, and 30.5 cm long. These contact blocks give little variation in contact resistance, the surfaces being smooth and of large area. The two halves are fastened together by four horizontal bolts and when these bolts are moderately tight the smooth contact surfaces between bronze and graphite allow enough slipping to prevent breaking of the tube through relative expansion of the connections.

In the earlier experiments, the furnace tube was inclosed by a split graphite tube which was surrounded by loose carborundum as heat insulation. In the work done by the writer with the furnace filled with air or carbon dioxide at high pressure[2] such a jacket is almost indispensable on account of the rapid oxidation of the tube, if it is exposed directly to the compressed gas, and the strong convection of heat to the steel walls of the chamber. In comparing the spectra at various furnace temperatures, however, the chamber is regularly pumped out to a pressure of less than

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 28; *Astrophysical Journal*, **28**, 300, 1908.

[2] *Contributions from the Mount Wilson Solar Observatory*, Nos. 53 and 60; *Astrophysical Journal*, **34**, 37, 1911, and **35**, 183, 1912.

2 cm of mercury, and, when this is done, a much greater constancy of temperature is obtained by using no heat jacket. The tube then quickly reaches an equilibrium of temperature, which varies little during the time usually required for a photograph. With the carborundum jacket, on the other hand, the tube temperature increases steadily for some time, on account of the slow heating of the jacket, so that either a reduction of the impressed voltage or occasional breaking of the current for a few seconds is required to prevent too large a temperature variation during the run. Dispensing with the jacket naturally requires a larger current to produce the same temperature of the tube, but if sufficient power is available, the greater constancy of the temperature is a decided advantage.

The 50 K.W. transformer used for the furnace gives a range by 5-volt steps from 5 to 30 volts. For the temperatures employed in these experiments, potentials from 15 to 30 volts were generally used, the currents ranging from 800 to 1600 amperes.

2. *Measurement of temperature.*—Observations with a Wanner pyrometer were taken regularly during the operation of the furnace, the interior of the furnace tube being viewed through the window opposite to that directed toward the spectrograph. This method should give reliable values for the relative temperatures at which spectra were successively obtained, which was the chief object in the present work. The reading necessarily gives a mean value of the temperature from end to end of the tube, assuming that the departure from black-body conditions is not great, since the line of view can be inclined only slightly to the tube's axis, and thus the temperature of any particular part of the tube is uncertain. Attempts have been made to measure the temperature at the center of the tube by placing a graphite plug midway in its length. Readings of the pyrometer when directed at this plug have given temperatures 100° to 150° higher than the mean temperature when viewed without the plug. The temperature of the plug does not correctly give that of the adjacent wall of the tube, but is probably close to it by reason of the high heat conductivity of graphite. It is also an open question how nearly the temperature of metallic vapor inside the tube agrees

with the temperature of the tube itself. These points, however, do not invalidate the relative measures required in this investigation.

3. *Photography of the spectrum.*—Large-scale photographs of furnace spectra have been used as far as possible, having the obvious advantages of avoiding blends and showing more clearly the character and structure of the lines. The vertical Littrow spectrograph described in previous publications[1] was employed, the arrangement most used being with an objective of 30 feet (9.1 m) focus and a Michelson plane grating of 7.2×19 cm ruled surface, of which area only a part about 5×15 cm could be used for the best definition. The dispersion for the first order varied from 1.95 Å. per mm in the blue to 2.06 Å. per mm in the red. The range from λ 3880 to λ 6600 was covered on this scale. A large part of the spectrum, including the portions where lines are most numerous and variable, was later photographed in the second order with a dispersion close to 0.95 Å. per mm. The plates were 9.2×43.2 cm in size, the emulsions most used being the Seed "Gilt Edge 27," the Cramer "Inst. Isochromatic," and the Seed "27" bathed with the Wallace three-dye solution of pinacyanol, pinaverdol, and homocol.[2] Cramer "Crown" plates were also used for some photographs in the blue region.

The spectrum beyond λ 6600 and to the violet of λ 3880 has been photographed with a concave grating spectrograph recently constructed, which gives very bright spectra with excellent definition. Many photographs of the visible region were also taken with this and used chiefly to check the large-scale plates. The grating in this instrument is of 1-meter radius and 5.1×8 cm ruled surface, and was made by J. A. Anderson on the Rowland dividing engine. It is mounted, with slit and camera-box, on a rigid steel frame, inclosed in a light-tight case. As adjusted, the spectral region covered is approximately from λ 2000 to λ 8000 in the first order, a strip of film 38 cm long being used in a metal

[1] Hale, *Contributions from the Mount Wilson Solar Observatory*, No. 27; *Astrophysical Journal*, **28**, 244, 1908; King, *Carnegie Institution of Washington Publication*, No. 153, 13, 1912.

[2] *Astrophysical Journal*, **26**, 299, 1907.

holder. In the work thus far, the regular Eastman kodak film has been used. It is sensitive for strong lines to about λ 5600, with a gradual decrease from the blue to the yellow; but the sensitiveness to the red of λ 5000 is much increased by the three-dye bath, which the films take very satisfactorily. The sharp definition permits the films to be examined under a high magnification, so that a preliminary survey of the ultra-violet in the second order (dispersion 1 mm = 8.5 Å.) was made to good advantage, the first order being used in the extreme ultra-violet on account of its strength. This small spectrograph has been mounted at the end of the furnace opposite to that which is directed at the large Littrow instrument, and simultaneous photographs were frequently taken, giving a long range of spectrum for the same conditions under which a selected region was photographed by the large-scale spectrograph.

4. *Operation of the furnace.*—The furnace having been placed in position, with a quantity of iron filings in the tube, and the current turned on, an image of the interior of the tube was focused on the slit of the spectrograph employed, the focusing lens of glass or quartz being selected of such focal length as to give an image of convenient size on the slit, and at the same time fully to illuminate the grating. The fixed character of all parts of the furnace prevents any change in the illumination of the grating such as often occurs with other sources.

Furnace spectra for two and sometimes three different temperatures were usually taken on the same plate, the exposures being timed so as to give a number of prominent lines about the same intensity in each photograph. Several arc spectra were added with varying exposure times by reflecting the light from an iron arc through the same lens as was used for the furnace exposures. This gave a means of easy identification of the furnace lines together with a number of arc spectra, from which the one whose average intensity was best suited for comparison with the furnace spectra could be selected. Since the first-order spectrum of the 30-ft. (9.1 m) spectrograph gave almost 800 Å. on a single plate, it will be seen that a plate properly exposed furnished a large amount of material in which photographic differences, especially in the

matter of development, did not enter. While special weight was given to a few photographs of exceptional quality, these have been fully checked by other good plates taken with both the large and the small spectrographs, the former including a number of second-order spectra for the detection of possible blends.

Three approximate temperatures were chosen, which will be referred to as low, medium, and high. Pyrometer readings for these, taken in the manner already described, gave values of 1800° to 1900° C. for the low temperature, 2100° to 2300° C. for the medium, and 2500° to 2700° C. for the high temperature. The intensities of lines were estimated on at least three good plates, and frequently more, for each range of temperature, the results being consistent enough to enable one to place pretty definitely the position of a plate on the temperature scale by the relative intensities of certain lines especially sensitive to temperature change.

THE GENERAL PHENOMENA OF VARIATION OF SPECTRA WITH FURNACE TEMPERATURE

The arc spectrum under certain conditions, in general those given by the central region of the arc, not close to either pole, is usually taken as the standard for such substances as can be volatilized to advantage in the arc, by reason of the richness in lines and the approximate constancy of relative intensities in the spectrum thus produced. As compared with the arc spectrum of iron, the furnace spectrum shows marked differences at all stages of temperature. At the lowest temperature at which the vapor radiates, a group of lines appears which includes not only some of the strongest arc lines but also other lines which are only moderately strong and others which are quite faint in the arc. Many very strong arc lines are entirely absent. As the temperature rises, other lines appear, but by no means in the order to be expected from their arc intensities; and by taking a range of temperature such as was used in these experiments it is seen that *the lines grow in intensity with temperature at very different rates.* Part of the lines which appeared at the lowest temperature increase in intensity very slowly, some of them, in reference to the majority of the

lines, actually seeming to weaken slightly at the higher temperatures. Other lines about hold their own, while many of the representative arc lines increase very rapidly in intensity. At the highest furnace temperatures, the furnace spectrum compares closely in richness with that of the arc; in fact, experiments thus far have indicated that it is chiefly a matter of long exposure to bring out practically all of the arc lines in the furnace at high temperature. The differences in relative intensities of lines, however, are still large, with two extreme types. One class includes lines very strong in the arc which are produced faintly only with great difficulty in the furnace. Lines of the other class are strong at all furnace temperatures, and in the arc are of moderate strength, sometimes very weak. Between these extremes there is a large variety in the behavior of the furnace lines which will be made the basis of the classification to be given.

THE STRUCTURE OF FURNACE LINES

In a previous paper by the writer on the pressure-shift of furnace lines,[1] attention was called to the width of the lines in proportion to their density as possibly being connected with their sensitiveness to pressure displacement. This structure has been studied farther in the spectra taken with the furnace *in vacuo* at different temperatures.

The vapor distribution in the furnace differs materially from that in the arc. We have in the furnace tube a very gradual decrease in temperature and vapor density from the middle toward either end. At the lowest temperature for which a spectrum can be obtained, only the vapor in the central portion is hot enough to radiate. The lines are then comparatively narrow, "hard," and can be given any desired blackness in the negative by long exposure. A higher temperature results in vapor nearer the ends of the tube being made hot enough to radiate, and therefore absorb, the central vapor at the same time giving a wider line. The effect of this gradual increase in the absorptive power of the vapor as it nears the end of the tube is to soften the middle of the

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 60; *Astrophysical Journal*, **35**, 183, 1912.

spectrum line and give all variations as the temperature of the tube increases, from a hard, narrow line to complete self-reversal, provided that the rarer vapor near the ends of the tube ever becomes hot enough to radiate (and absorb) the wave-length in question. A short range of spectrum photographed at high temperature will often illustrate the steps in this process by different lines. Those lines which the hottest vapor at the center of the tube is barely able to give are narrow. Other lines not requiring so high a temperature show the gradual widening with softening of the center, through the diffuse stage which precedes reversal, and then clear reversals for such lines as are given by the vapor close to the end. While these conditions for reversal are necessary, they are not always sufficient, since the ease of reversal of iron lines increases toward the violet end of the spectrum and may vary for different lines in other respects which the present investigation has not shown.

INTENSITY SCALE

It would be highly desirable to place the grading of line intensities on a strict photometric basis; but all who have attempted this have met the fundamental difficulty that the increase of width with density as a line strengthens is very different for different lines. A scale of blackness alone (for lines in the negative) would be highly misleading. A scale made by successive photographs of an actual spectrum line whose intensity could be closely controlled and made to increase at a known rate could be used only for a class of lines showing the same behavior as regards the increase of width with density. Even this would involve large errors when the photographic scale was compared with spectrum plates differing in contrast and amount of development, and not always normally exposed. Different scales for sharp and diffuse lines would be quite necessary, and there is a wide variation in each of these classes. Further, the comparison of narrow, dense lines with widely reversed lines would seem to be beyond the reach of such a method.

These difficulties, which are general for emission spectra, are increased when furnace lines for different temperatures are com-

pared. The rapid widening and softening of a line with increased temperature, mentioned in the preceding section, makes it harder to compare with the appearance of the same line at low temperature, where it is narrow and its density is given by long exposure. When, however, the spectrum for a given temperature is considered by itself, it is found quite possible to grade the line intensities in a consistent manner when all features of a given line are considered.

Proceeding in this way in the present work, a line distinctly outlined on the plate has been graded "1," a fainter appearance being indicated by "trace." If, as was usually the case, none of the lines were exposed to reach full blackness in the negative, a grading of intensities for the various lines could be made which would be confirmed with small variations on a second examination of the plate by the same person, and with differences only slightly greater by another observer. The extension of the scale to include reversed lines was not difficult, since the portion of spectrum being studied usually offered lines in various stages preliminary to reversal whose position in the scale between sharp lines and those fully reversed was fairly distinct. The steady character of the furnace radiation and the dependence of a line's intensity on the portion of the tube where the vapor is in a condition to produce the line permit a grading of this kind to be more consistent than is often the case with the arc or spark. The lines for a different furnace temperature are graded in the same way. In the final tabulation, if a line is graded 10 for two temperatures, this does not mean that the line is exactly alike as regards width and density in the two cases, but that it bears the given relation to the lines in its own spectrum for each temperature.

The writer is well aware that personal and photographic differences enter to an undesirable degree in this method of assigning intensities, but where the aim is to tell whether at a certain stage of temperature a line appears on a normally exposed plate and to give its approximate intensity compared with other lines at the same temperature, I believe the method pursued fulfils the requirements better than photometric measures which would require constant modification by a consideration of the general appearance of the spectrum line concerned.

THE TEMPERATURE CLASSIFICATION OF IRON LINES

In pursuance of the object of indicating the relative response to temperature of different lines, both as to the stage of initial appearance and the rate of growth with increase of temperature, six classes have been formed. A reference class is required, which should be made up of a number of lines which are strong at all temperatures and show about the same rate of change. This is the group to be designated as Class I B. They are plentiful throughout the spectrum, except in the orange and red, and the photographs have been made so as to give these lines nearly the same intensity for different temperatures.

Class I A has the distinguishing characteristic of strength at all furnace temperatures after the vapor begins to radiate, and notable weakness in the arc. As compared to the reference group I B, some I A lines show a slight weakening as the temperature increases, but they remain among the stronger furnace lines at the highest temperature. In the arc, however, they are among the weaker lines, sometimes being barely visible in a normally-exposed arc spectrum. Care must be taken to distinguish these from impurity lines which show a similar behavior, the strongest lines of a foreign substance often being given with greater intensity in the furnace than in the arc between iron terminals. To make the identification of the lines of Class I A as certain as possible, all of them have been photographed in the second-order spectrum with a scale of about 0.95 Å. per mm, an arc spectrum being taken on the same plate with the ends of the furnace and arc lines overlapping. The I A lines were found to coincide exactly in every case with faint arc lines which are regularly ascribed to iron in the wave-length tables. Possibly a few of these lines appearing in the iron arc may be shown later to belong to foreign substances, a point which has not been thoroughly investigated in this laboratory, but their regular behavior in the arc and spark is that of weak iron lines, so that the phenomena offered by this important class may be regarded as in all probability real.

Class I B embraces the well-known group of "flame" lines which are given readily in the hotter flames and are relatively strong in the outer envelopes of the iron arc. In the furnace

they are strong at all temperatures, being taken as the reference group in this classification. In the arc they are usually of moderate strength, although those in the green-yellow are among the strongest arc lines. There is usually a clear distinction in the arc between these and the lines of Class I A, for which the arc conditions appear to be distinctly unfavorable.

Class II includes a large number of the stronger arc lines. In the furnace they are distinct, often strong, at the lowest temperature, but as the temperature rises, these lines strengthen much faster than the lines of Class I, and are usually found to have gone up rapidly in the interval between the highest furnace temperature and that of the arc.

Class III lines are absent or faint at the lowest temperature, appear distinctly at medium temperature, show a rapid growth at high temperature, and are strong in the arc. Their rate of growth is similar to the lines of Class II, but they require a higher initial temperature for their production.

Class IV lines appear only at the highest furnace temperature (perhaps as a trace at medium), but show a fair strength. Their arc intensity is usually relatively much greater than in the furnace.

Class V includes lines which are absent or very faint in the furnace, their strength in the arc varying greatly. Experiment has shown that it is possible to bring out many if not all lines of this class by prolonged furnace exposure at high temperature, but the difference between their arc and furnace intensities is greater than for Class IV. In the blue and violet, where impurity lines from several rich spectra are somewhat troublesome, lines entered as "trace" or "1" at high temperature may occasionally belong to foreign substances, as their weakness made them difficult to obtain on the largest-scale plates and hard to measure closely when they did appear. If any of these are not due to iron in the furnace spectrum, the classification would be altered only in a few cases by moving Class IV lines into Class V.

From the furnace material available, it is usually easy to tell in what group of the foregoing classification a line belongs. A few lines are on the border between Classes I A and I B, since

the distinction depends mainly on the arc intensity. A decision between Classes IV and V is sometimes difficult for weak arc lines, but for such lines the difference is not usually important. The distinction is clear and valuable when we have a strong arc line that may be brought out readily (Class IV) or only with great difficulty (Class V) at the highest furnace temperature.

For some purposes a grouping of lines according to the temperature at which they first appear may be useful. In that case, Classes I A, I B, and II may be regarded as low-temperature lines, Class III as belonging to medium, and Classes IV and V to high temperature. This grouping leaves out of account the rate of increase with temperature, to show which is one of the main objects of the present work.

THE DETECTION OF BLENDS

It was naturally of great importance to determine how far the observed intensity of furnace lines might be affected by blends with lines belonging to foreign substances. The most disturbing elements, whose stronger lines appeared with those of iron in the furnace, were chromium, manganese, titanium, and vanadium, the two latter being given by the Acheson graphite. Fortunately, furnace spectra were available for these elements on the same scale as for iron, so that in the case of a suspected blend, it was a simple matter to see whether the furnace spectrum of the foreign substance gave that line of sufficient strength to appear when only a small quantity of the element was present as an impurity. When the furnace line of iron was probably affected in this way, the nature of the blend is given in the "Remarks" column of Table I. In the case of some blends it was not necessary to leave the intensity of the iron line wholly doubtful. For example, a low-temperature iron line is blended with the chromium line λ 5204.680, one of the strongest in that spectrum. The chromium line, however, is one of three which are affected similarly in any given source. From a furnace plate for chromium the strength of this line relative to the other two could be seen, so that on the iron plate it was possible to tell closely how much of the blend was due to the chromium line. The large scale of the second-order spectrum

was useful in separating lines frequently blended with lower dispersion, especially for iron lines occurring in the carbon flutings which are always troublesome at high temperature.

EXPLANATION OF THE TABLE

In Table I the wave-lengths in the first column are those of Rowland. The second column gives the intensities of these lines in the arc, estimated from spectra taken on the same plate with some of the best furnace spectra. The intensities were estimated on the same plan as for the furnace spectra. The large range of intensity, together with the low values for some lines, results from the choice of an arc spectrum not very strongly exposed, so that the contrasts are distinct and the weaker lines faint. A line graded as low as 2 is a moderately strong line in a fully exposed arc spectrum. All arc lines above a certain minimum strength are included in the table, whether they appear in the furnace or not, this minimum being determined by the fact that some lines as weak as this (usually of Class I A) do appear in the furnace spectrum. The arc lines listed are strong enough to be considered in any general treatment of the arc spectrum, and their action in the furnace is of interest.

The method of grading the furnace lines has been given. For both arc and furnace, the standard spectra to the violet of λ 4700 were obtained on a Seed "27" plate (old emulsion), and those for the remainder of the spectrum on plates or films bathed with the three-dye solution. The intensities between λ 4500 and λ 4700 and in the red region are low on an absolute scale by reason of decreased sensitiveness of the plates, but arc and furnace spectra are affected alike. The letter *n* in connection with the arc intensity denotes that the line belongs to the class of "nebulous" lines which show a large width in proportion to their density, while *r* and *R* indicate respectively that the line is partially or completely reversed. The approximate values of the three temperatures are given on p. 244.

TABLE I

TEMPERATURE CLASSIFICATION OF IRON LINES

λ (Rowland)	Arc	Furnace			Class	Remarks
		High Temp.	Medium Temp.	Low Temp.		
3884.518	3	1	..	..	IV	
3885.657	5	3	..	..	IV	
3886.434	40*R*	40*R*	40*R*	25	I B	
3887.196	15	15	15	10	I B	
3888.671	20	10	9	6	II	
3888.971	3	..	..	..	V	
3890.986	2	1	..	..	IV	
3892.069	3	..	..	..	V	
3893.542	7	3	..	..	IV	
3895.803	25*r*	20*r*	20	12	I B	
3898.032	8	1	..	..	V	
3898.151	10	8?	6?	4	I B?	Blend with *V* which disturbs at medium and high temperature. Probable intensity of *V* line subtracted
3899.171	2	1	..	..	IV	
3899.850	30*R*	30*R*	25	14	I B	
3900.681	2	..	..	..	V	
3903.090	20	12?	10?	6	II	Blend with moderately strong *Cr* line
3904.052	5	2	..	..	IV	
3906.628	8	15	15	9	I B	
3906.890	2	..	..	..	V	
3908.077	4	2	..	..	IV	
3909.976	3	..	..	..	V	
3913.775	4	3	..	..	IV	
3916.879	6	Tr.	..	..	V	
3917.324	8	10	10	4	I B	
3918.464	3	2	..	..	IV ?	Not resolved on plates strong enough to give lines in furnace
3918.563	4					
3918.789	6	1	..	..	V	
3919.208	3	1	..	..	IV	
3920.410	20*r*	20*r*	20	15	I B	
3923.054	25*R*	30*R*	25	15	I B	
3925.790	4	1	..	..	IV	
3926.086	6	2	..	..	IV	
3928.075	30*R*	30*R*	25	15	I B	
3930.450	25*R*	35*R*	30	18	I B	
3932.785	4	1	..	..	IV	
3935.965	8	3?	..	..	IV?	Furnace line may be *Ba*
3937.479	3	1	..	..	IV	
3941.025	5	10?	9?	4?	I A?	May be partly *Sr* and *Co*
3942.586	6	1	..	..	V	

TABLE I—*Continued*

λ (Rowland)	Arc	Furnace			Class	Remarks
		High Temp.	Medium Temp.	Low Temp.		
3943.489	2	1	..	..	IV	
3945.033	3	Tr.	..	..	IV	
3945.260	4	Tr.	..	..	IV	
3947.142	4*n*	Tr.	..	..	IV	
3947.675	5	Tr.	..	..	V	
3948.246	6*n*	Tr.	..	..	V	
3948.925	10	..	..	..	V	
3950.102	10	8	4	Tr.	III	
3951.311	9	1	..	..	V	
3952.754	8	2	..	..	IV	
3953.303	4	Tr.	..	..	V	
3956.099	2	..	..	..	V	
3956.603	9	..	..	..	V	
3956.819	12	8	4	Tr.	III	
3957.177	4*n*	Tr.	..	..	IV?	Furnace line may be *Ca*
3963.252	6*n*	Tr.	..	..	V	
3964.663	3	..	..	..	V	
3966.212	10	10	8	2	II	
3966.778	10*n*	2	1	..	IV	
3967.570	8	1	..	..	V	
3968.114	4*n*	..	..	..	V	
3969.413	30	16	12	8	II	
3970.540	4	Tr.	..	..	IV	
3971.475	9	4	2	..	III	
3973.796	3	..	..	..	V	
3976.770	4	Tr.	..	..	IV?	Furnace line may be *Cr*
3977.891	12	9	4	1	II	
3981.917	7	12?	10?	2	I B	Close blend with *Ti* line which may predominate in furnace spectrum at medium and high temperature
3984.113	10	4	2	..	III	
3985.539	3	..	..	..	V	
3986.321	5	..	..	..	V	
3990.525	2	..	..	..	V	
3994.265	2	..	..	..	V	
3996.140	4	1	..	..	IV	
3997.115	2	..	..	..	V	
3997.547	15	7	2	..	III	
3998.205	10	4?	2?	..	III?	May be partly *Co*
4000.611	2	..	..	..	V	
4001.814	5	3	2	..	III	
4003.912	2	..	..	..	V	
4005.408	25	15	12	9	II	
4006.464	3	..	..	..	V	
4006.776	2	..	..	..	V	

TABLE I—*Continued*

λ (Rowland)	Arc	Furnace: High Temp.	Furnace: Medium Temp.	Furnace: Low Temp.	Class	Remarks
4007.429	6	2	Tr.	..	III	
4009.864	10	9?	6?	1	II?	Furnace line chiefly *Ti*
4013.964	2	..	..	..	V	
4014.677	10	3	1	..	III	
4017.308	6	2	1	..	III	
4022.018	12	6	2	..	III	
4024.881	6*n*	..	..	..	V	
4029.796	3*n*	..	..	..	V	
4032.117	4	..	..	..	V	
4032.789	4	8	6	1	III?	Arc intensity low for Class III. Well separated from *Mn* 4033.224
4040.792	4	..	..	..	V	
4044.056	5*n*	..	..	..	V	
4044.766	6	2	..	..	IV	
4045.975	60*r*	30	22	15	II	
4055.023	3	..	..	..	V	
4055.189	3	..	..	..	V	
4057.499	2	..	..	..	V	
4058.372	4*n*	..	..	..	V	Blend with *Co*
4058.915	3	..	..	..	V	
4059.872	3	..	..	..	V	
4062.599	10	4	2	..	III	
4063.759	45	20	15	12	II	
4067.139	6	2	..	..	IV	May be partly *Cr*
4067.429	4	2	..	..	IV	
4068.137	8*n*	2	..	..	IV	May be partly *Mn*
4070.930	5*n*	1	..	..	IV	
4071.908	40	18	12	10	II	
4073.921	4*n*	1	..	..	IV	
4074.947	5	1	..	..	IV	
4076.792	8*n*	2	..	..	IV	
4078.515	4	2	..	..	IV	May be partly *Ti*
4079.996	4	1	..	..	IV	
4080.368	2*n*	Tr.	..	..	IV	
4084.647	6	1	..	..	V	
4085.161	4	1	..	..	IV	
4085.467	4	1	..	..	IV	
4096.129	4	2	1	..	III	
4098.335	4*n*	1	..	..	IV	
4100.901	3	10	8	3	I A	
4104.288	3	..	..	..	V	
4107.649	12	4	2	..	III	
4109.953	9	?	..	..	V	Weak if present at all in furnace. Close to strong *V* line at 4109.905
4113.117	3*n*	..	..	..	V	

TABLE I—*Continued*

λ (Rowland)	Arc	Furnace High Temp.	Furnace Medium Temp.	Furnace Low Temp.	Class	Remarks
4114.606	5	2	Tr.	..	IV	
4118.708	15	2	1	..	IV	
4120.368	5	2	..	..	IV	
4121.963	5	1	..	..	IV	
4122.673	4	1	..	..	IV	
4126.344	3*n*	..	..	..	V	
4127.767	7	2?	..	..	IV	Furnace line partly *Ti*
4127.957	3*n*	..	..	..	V	
4132.235	25	15	15	8	II	Furnace line very slightly affected by *V* blend
4133.062	8	3	..	..	IV	
4134.840	12	2?	..	..	IV	Furnace line partly *V*
4137.156	7	Tr.	..	..	V	
4140.089	2	6	4	2	I A	
4143.572	15	5	2	..	III	
4144.038	30	15	12	10	II	
4147.836	10	10	8	2	I B	
4149.533	5*n*	..	..	..	V	
4152.343	4	10	8	2	I A	
4154.071	10*n*	2	..	..	IV	
4154.667	12	4	2	..	III	
4154.976	9*n*	1	..	..	V	
4156.970	12	4	2	..	III	
4157.948	8*n*	1	..	..	V	
4158.959	5*n*	Tr.	..	..	V	
4171.068	5	1	..	..	V	
4172.296	5	1	..	..	V	
4172.923	4	9	9	3	I A	
4173.480	2	Tr.	..	..	IV	
4174.095	2	6	6	2	I A	
4175.082	5	10	10	5	I A	
4175.806	10	3	1	..	III	
4176.739	7*n*	2	..	..	IV	Blends with high-temperature line of *Mn*. Furnace line probably *Fe*
4177.698	4	10	10	4	I A	
4181.919	15	6	3	1	III	
4182.548	4	1	..	..	IV	
4185.058	10	3	1	..	III	
4187.204	20	10	7	2	II	
4187.943	20	10	7	2	II	
4191.595	15	9	6	1	III	
4195.492	5	1	..	..	V	
4196.372	4	1	..	..	IV	
4198.494	20	10	7	1	III	
4198.800	4*n*	Tr.	..	..	V	
4199.267	20	7	3	Tr.	III	

TABLE I—*Continued*

λ (Rowland)	Arc	Furnace High Temp.	Furnace Medium Temp.	Furnace Low Temp.	Class	Remarks
4200.148	1	9	8	4	I A	
4201.089	3*n*	..	..	..	V	
4202.198	30	18	12	9	II	
4204.101	10	3	1	..	III	
4206.862	3	12	12	8	I A	
4207.291	4	Tr.	..	..	V	
4208.766	3*n*	..	..	..	V	
4210.494	15	8	4	Tr.	III	
4213.812	5	2	..	..	IV	
4215.581	2	1	..	..	IV	
4216.351	8	10	12	8	I B	
4217.720	7*n*	1	..	..	V	May be partly *Cr*
4219.516	12	2	1	..	IV	
4220.509	4	1	..	..	IV	
4222.382	12	8	4	Tr.	III	
4224.337	6*n*	1	..	..	V	
4224.673	3*n*	..	..	..	V	
4225.619	6*n*	1	..	..	V	
4226.116	3	Tr.	..	..	V	
4226.584	3	Tr.	..	..	V	
4227.606	30	5	3	..	III	
4232.887	1	8	7	3	I A	
4233.772	18	9	6	1	III	
4236.112	25	10	8	2	II	
4238.188	4	Tr.	..	..	V	
4238.970	10*n*	2	1	..	IV	
4239.890	3	1	..	..	IV	
4240.014	2	3	2	..	I B ?	Always faint. Would probably show at low temperature with long exposure
4245.422	6	3	2	1	II	
4246.251	3	..	..	..	V	
4247.591	12	3	2	..	III	
4248.384	4	Tr.	..	..	V	
4250.287	25	10	5	1	III	
4250.945	25	15	12	8	II	
4258.477	2	10	8	6	I A	Probably chiefly if not wholly due to *Fe*. A weak *Mn* line is given by Hasselberg at 4258.48 which appears faintly in *Mn* furnace but may be due to *Fe* which is present when *Mn* is used
4260.640	35	15	10	2	III	
4267.122	3	1	..	..	IV	

TABLE I—*Continued*

λ (Rowland)	Arc	Furnace High Temp.	Furnace Medium Temp.	Furnace Low Temp.	Class	Remarks
4267.985	5	1	..	..	V	
4268.915	2	Tr.	..	..	IV	
4271.325	20	10	5	2	III	
4271.934	35	20	15	12	II	
4282.565	12	10	7	3	I B	
4285.605	3	1	..	..	IV	
4291.630	4	10	10	8	I A	
4294.301	15	12	12	6	I B	
4299.410	18	9	5	2	II	
4305.614	3	Tr.	..	..	IV	
4308.081	35	20	15	10	II	
4309.541	4	1	..	..	IV	
4315.262	10	8	6	1	III	
4325.939	35	20	15	10	II	
4327.274	3	..	..	..	V	
4337.216	10	10	9	4	I B	
4351.711	3	..	..	..	V	
4352.908	9	8	8	2	I B	
4358.670	3	Tr.	..	..	IV	
4367.749	5	Tr.	..	..	V	
4368.071	2	4	3	1	I A	
4369.941	7	3	1	..	III	
4376.107	9	15	15	10	I B	Wider in proportion to density than 4415.293
4383.720	45*r*	30	20	12	II	
4388.057	3	Tr.	..	..	IV	
4388.571	4*n*	Tr.	..	..	IV	
4389.413	2	10	8	5	I A	
4391.123	4	Tr.	..	..	IV	
4404.927	30	20	15	9	II	
4407.871	5	3 ?	?	Tr.	III ?	Allowance made for blend with *V*
4408.582	6	4	?	Tr.	III ?	Blended with *V* except on one plate
4415.293	20	15	12	7	II	
4422.741	6	2	1	..	IV	
4427.482	10	12	12	9	I B	Wider in proportion to density than 4415.293
4430.785	6	4	2	1	II	
4433.390	3*n*	Tr.	..	..	IV	
4435.321	2	10	8	6	I A	Just resolved from *Ca*
4442.510	12	9	5	1	III	
4443.365	7	2	Tr.	..	IV	
4447.892	9	8	5	2	II	
4454.552	5	2	1	..	III	
4459.301	10	8	4	2	II	
4461.818	8	12	12	10	I B	
4466.727	12	12	10	6	I B	

TABLE I—*Continued*

λ (Rowland)	Arc	Furnace: High Temp.	Furnace: Medium Temp.	Furnace: Low Temp.	Class	Remarks
4469.545	5*n*	1	..	..	IV	
4476.185	10	4	2	..	III	
4482.338 4482.438	4 6	15	15	10	I B	Not fully resolved on most plates. 4482.338 seems about 4 times strength of 4482.438. In arc 4482.338 behaves as I A line and 4482.438 as I B
4484.392	4	..	..	..	V	
4489.911	3	12	12	8	I A	
4494.738	12	9	5	2	II	
4525.314	5*n*	1	..	..	IV?	Furnace line may be V
4528.798	18	12	9	3	II	
4531.327	8	10	7	4	I B	
4548.024	4	..	..	..	V	
4556.306	4*n*	..	..	..	V	
4592.840	5	8	6	2	I B	
4603.126	9	10?	9?	4?	I B?	Furnace line may be largely *Li*
4607.831	3*n*	..	..	..	V	
4611.469	5*n*	2	1	..	III	
4613.386	2*n*	..	..	..	V	
4619.468	3*n*	*	..	..	IV	
4625.227	3	*	..	..	IV	
4633.100	2	5	3	..	III?	Weak in arc for Class III
4637.685	3	*	..	..	IV	
4638.193	3	*	..	..	IV	
4647.617	6	*	1	..	IV	
4654.672 4654.800	5 5	? ..	? ..	1 ..	II? V	Not fully resolved. 4654.672 blends with fluting line at high temperature
4667.626	6	..	..	..	V	
4668.243	6	*	..	..	IV	
4679.027	7	..	..	..	V	
4691.602	6	*	..	..	IV	
4707.457	8	*	..	..	IV	
4710.471	5	*	..	..	IV	
4727.582	3*n*	*	..	..	IV	
4728.732	3*n*	*	..	..	IV	
4733.779	4	*	4	1	I B?	
4736.963	12	*	*	2?	II?	Blend with head of *C* band and intensity doubtful
4741.718	3	..	..	..	V	

* Occurrence uncertain on account of strong carbon bands at high temperature, but *Fe* lines are weak in furnace, if present at all. Provisionally placed in Class IV.

TABLE I—*Continued*

λ (Rowland)	Arc	Furnace			Class	Remarks
		High Temp.	Medium Temp.	Low Temp.		
4745.992	3*n*	..	..	..	V	
4768.595	3*n*	..	..	..	V	
4773.007	3	3	2	..	III	
4787.003	5	1	..	..	IV?	Furnace line may be *V*
4789.849	7	..	..	..	V	
4859.928	15	6	2	..	III	
4871.512	25	9	3	..	III	Furnace line may be slightly affected by *V* blend
4872.332	20	7	2	..	III	
4878.407	12	5	2	..	III	
4885.620	2	..	..	..	V	
4890.948	25	8	2	..	III	
4891.683	50	12	3	..	III	
4903.502	12	4	1	..	III	
4919.174	30	8	2	..	III	
4920.685	60	12	4	Tr.	III	
4924.956	3	..	..	..	V	
4938.997	10	3	..	..	IV	
4939.868	4	10	8	5	I B	
4946.568	4	5	..	..	IV	
4957.480	20	5	2	..	III	
4957.785	60	12	6	1	III	
4966.270	8	1	..	..	V	
4973.281	3	..	..	..	V	
4978.785	2	..	..	..	V	
4982.682	8*n*	..	..	..	V	
4983.433	5*n*	..	..	..	V	
4984.028	6*n*	..	..	..	V	
4985.432	7	Tr.	..	..	V	
4985.730	7	1	..	..	V	
4994.316	8	9	9	4	I B	
5002.044	12	1	..	..	V	
5005.896	10	..	..	..	V	
5006.306	20	4	1	..	III	
5012.252	12	10	9	8	I B	
5015.123	10	..	..	..	V	
5022.414	6	..	..	..	V	
5027.305	5*n*	..	..	..	V	
5028.308	4	..	..	..	V	
5041.255	7	7	6	3	I B	
5041.936	10	7	4	1	III	Blend with *Ca* probably has little effect on furnace intensity
5050.008	15	12	2	..	III	
5051.825	10	10	8	6	I B	
5065.207	6*n*	?	..	..	V	Concealed by *C* if present

TABLE I—*Continued*

λ (Rowland)	Arc	Furnace			Class	Remarks
		High Temp.	Medium Temp.	Low Temp.		
5068.944	10	..	..	..	V	
5074.932	10*n*	..	..	..	V	
5079.409	6	3	Tr.	..	IV	
5079.921	4	6?	4	2	I B	Blend with *C* at high temperature
5083.518	7	10?	7	4	I B	Blend with *C* at high temperature
5098.885	8	4	Tr.	..	IV	
5107.619	6	4	3	2	I B	Blend on some plates makes estimates difficult
5107.823	8	3	2	1	II	
5110.574	10	8?	10	15	I B	Remarkably strong at low temperature. Blend with *C* at high temperature
5123.899	6	9?	5	3	I B	Blend with *C* at high temperature
5125.300	6*n*	..	..	..	V	
5127.533	5	8?	4	2	I B	Blend with *C* at high temperature
5133.870	20*n*	?	..	..	V	Blend with *C*. Very weak if present
5137.558	6*n*	..	..	..	V	
5139.427	10	4?	..	..	IV	Blend with *C* at high temperature
5139.644	20	4?	Tr.	..	IV	Blend with *C* at high temperature
5143.111	6	5	4	3	I B	
5151.020	6	5	4	2	I B	
5152.087	4	6?	3	1	I B	Blend with *C* at high temperature
5162.449	10*n*	?	..	..	IV?	Blend with *C* at high temperature. Probably present, but weak
5166.454	4	8	8	10	I A	Blend with weak *Cr* line which probably has little effect
5167.678	40	12	9	7	II	
5169.069	4	8	8	10	I A	
5171.778	20	10	6	3	II	
5191.629	20	4	..	..	IV	
5192.523	30	5	Tr.	..	IV	
5195.113	10	9	5	2	I B	
5198.888	4	2	..	..	IV	
5202.439	8	4	1	..	IV	

TABLE I—*Continued*

λ (Rowland)	Arc	Furnace High Temp.	Furnace Medium Temp.	Furnace Low Temp.	Class	Remarks
5204.768	2	6?	6?	7?	I A	Blend with *Cr* 5204.680, the probable intensity of which is subtracted
5208.776	7	2	..	..	IV	
5215.353	6	2	..	..	IV	
5216.437	10	6	3	1	II	
5217.552	5	1	..	..	V	
5225.695	1	6	5	5	I A	
5227.043	15	2	..	..	IV	
5227.362	40	12	10	6	II	
5230.030	5*n*	1	..	..	V	
5233.122	40	7	2	Tr.	III	
5242.658	4	1	..	..	IV	
5247.229	1	5	6	5	I A	
5250.385	1	5	6	5	I A	
5250.817	6	3	Tr.	..	IV	
5255.121	1	8	8	6	I A	
5263.486	8	1	..	..	V	
5266.738	30	5	1	..	IV	
5269.723	60	25*R*	20	20	I B	
5270.558	30	12	10	5	II	
5273.558	4	1	..	..	IV	
5281.971	10	2	..	..	IV	
5283.802	18	3?	..	..	IV	Furnace line may be partly *Ti*
5302.480	10	1	..	..	V	
5307.541	2	3	1	..	III?	Difficult to classify. Arc intensity low for Class III
5324.373	30	6	1	..	IV	
5328.236	50	25*r*	20	20	I B	
5328.696	15	12	10	3	II	
5333.089	4	5	2	1	I B?	Strengthens rapidly for Class I B
5340.121	12	2	..	..	V	
5341.213	20	10	8	3	II	
5365.069	15*n*	Tr.	..	..	V	
5365.596	3	Tr.	..	..	V	
5367.669	20*n*	1	..	..	V	
5370.166	25*n*	1	..	..	V	
5371.734	50	25	20	20	I B	
5383.578	35*n*	2	..	..	V	
5393.375	10	3	Tr.	..	IV	
5397.344	40	20	18	18	I B	
5404.357	30*n*	2	..	..	V	
5405.989	40	20	18	18	I B	
5411.124	15*n*	Tr.	..	..	V	
5415.416	35*n*	2	..	..	V	
5424.290	45*n*	3	..	..	V	
5429.911	40	20	18	18	I B	

TABLE I—*Continued*

λ (Rowland)	Arc	Furnace: High Temp.	Furnace: Medium Temp.	Furnace: Low Temp.	Class	Remarks
5434.740	30	18	15	15	I B	
5445.259	15*n*	Tr.	..	..	V	
5447.130	40	20	18	18	I B	
5455.834	40	20	18	18	I B	
5463.494	10*n*	..	..	..	V	
5476.778	10	3	..	..	IV	
5497.735	15	15	12	5	I B	Compare with other I B lines to violet
5501.683	12	12	10	4	I B	
5507.000	18	18	15	6	I B	
5555.122	4	..	..	..	V	
5563.824	3	..	..	..	V	
5565.931	4	..	..	..	V	
5569.848	20	4?	..	..	IV	Blend with *C* at high temperature
5573.075	30	5?	..	..	IV	Blend with *C* at high temperature
5576.320	10	2?	..	..	IV	Blend with *C* at high temperature
5586.991	40	6	Tr.	..	IV	
5598.524	4	?	..	..	IV?	Blend with *C* at high temperature
5603.186	10	2?	..	..	IV	Blend with *C* at high temperature
5615.877	50	8	1	..	IV	
5624.769	10	3?	..	..	IV	Blend with *C* at high temperature. May be affected by *V* blend
5638.488	3	..	..	..	V	
5655.715	4	..	..	..	V	
5659.052	10	3	..	..	IV	
5662.744	6	1	..	..	V	
5701.772	7	5	..	..	III?	Would be expected to show at moderate temperature on strong plates
5709.601	10	3	..	..	IV	
5753.344	5	..	..	..	V	
5763.218	10	Tr.	..	..	V	
5859.809	5	..	..	..	V	
5862.582	8	..	..	..	V	
5884.028	4	..	..	..	V	
5905.895	3*n*	..	..	..	V	
5914.335	8	..	..	..	V	
5930.406	8	..	..	..	V	
5934.881	5	..	..	..	V	
5952.943	3	..	..	..	V	
5975.575	4	..	..	..	V	
5977.007	5	..	..	..	V	
5983.908	6	..	..	..	V	
5985.040	8	2	..	..	IV	

TABLE I—*Continued*

λ (Rowland)	Arc	Furnace: High Temp.	Furnace: Medium Temp.	Furnace: Low Temp.	Class	Remarks
5987.290	6	..	..	..	V	
6003.239	8	..	..	..	V	
6008.785	9	..	..	..	V	
6020.401	10*n*	..	..	..	V	
6024.281	15	..	..	..	V	
6027.274	4	..	..	..	V	
6042.315	2	..	..	..	V	
6056.227	4	..	..	..	V	
6065.709	15	7	4	..	III	
6078.710	4*n*	..	..	..	V	
6102.392	5	..	..	..	V	
6103.400	3	..	..	..	V	
6136.829	20	10	6	..	III	
6137.915	18	7	3	..	III	
6141.938	4	..	..	..	V	
6157.945	4	..	..	..	V	
6170.730	4*n*	..	..	..	V	
6173.553	3	3?	1	..	III	Blend with *C* at high temperature
6191.779	20	10	7	1	II	Weak at low temperature for Class II
6200.527	4	1	..	..	IV	
6213.644	5	4	2	..	III	
6219.494	6	5	2	..	III	
6230.943	25	12	8	..	III	Blend with *V* which probably has little effect
6232.856	5	..	..	..	V	
6246.535	15	1	..	..	V	
6252.773	20	9	5	..	III	
6254.456	6	4	2	..	III	
6256.572	4	2	1	..	III	
6265.348	6	5	3	..	III	
6280.833	2	9	7	3	I A	
6291.184	3*n*	..	..	..	V	
6298.007	5	4	2	..	III	
6301.718	15	2	Tr.	..	IV	
6302.709	6	..	..	..	V	
6318.239	10	7	3	..	III	
6322.907	5	2	1	..	III	
6335.554	10	8	4	..	III	
6337.048	12	1	..	..	V	
6344.371	2	1	Tr.	..	III	
6355.246	4	2	1	..	III	
6358.898	3	9	7	3	I A	
6380.958	3	..	..	..	V	
6393.820	15	12	8	Tr.	III	
6400.217	25	4	2	..	III	
6408.233	8	..	..	..	V	
6411.865	12	2	1	..	IV	
6420.169	5*n*	..	..	..	V	

TABLE I—*Continued*

λ (Rowland)	Arc	Furnace			Class	Remarks
		High Temp.	Medium Temp.	Low Temp.		
6421.570	10	8	5	. .	III	
6431.066	10	10	7	Tr.	III	
6462.965	5	?	?	?	?	Occurrence in furnace uncertain on account of blend with low temperature *Ca* line
6469.408	2	. .	. .	. .	V	
6475.846	3	1	. .	. .	IV	
6482.098	4	1	. .	. .	IV	
6495.213	25	15	10	2	II	
6546.479	10	7	3	. .	III	
6569.460	5	. .	. .	. .	V	
6575.270	2	. .	. .	. .	V	
6593.161	10	6	2	. .	III	
6594.121	4	3	. .	. .	IV	
6609.360	4	2	. .	. .	IV	
6633.995	7*n*	. .	. .	. .	V	
6663.701	8	4	. .	. .	IV	
6678.235	12	9	3	. .	III	
6750.407	6	3	. .	. .	IV	
6828.850	4	. .	. .	. .	V	
6841.598	4	. .	. .	. .	V	
6843.913	4	. .	. .	. .	V	
6855.419	5	. .	. .	. .	V	
6945.477	4	2	. .	. .	IV	
6979.120	3	1	. .	. .	IV	
7187.645	5	. .	. .	. .	V	These lines are really stronger than indicated, as plate loses sensitiveness rapidly in this region
7207.715	3	. .	. .	. .	V	

DISCUSSION OF THE CLASSIFICATION

An examination of Table I shows a considerable variety as to the rate of growth among lines placed in the same class; but there is little to be gained by increasing the number of classes to cover these variations. There appears in all classes to be a continuous change in the intensities of furnace lines with temperature, the several classes being distinguished by the rate of change shown by each. As the scale has been arranged in Table I, the low-temperature lines of Classes I A and I B strengthen slowly or remain nearly the same for the three temperatures. As a rule,

there is little difference between medium and high temperature for these lines. If the intensities of Class II lines had been graded so as to show approximate equality for the three temperatures, the lines of Classes I A and I B would have shown a gradual weakening as the temperature rises. This point of view, however, does not seem to me to represent the true state of affairs as well as that adopted in the arrangement of Table I, which shows the Class II lines rapidly increasing in strength while the lines of Classes I A and I B about hold their own. Class II and Class III lines usually show a regular change of intensity with temperature, but many variations occur in the rate of change for individual lines.

If we regard the arc conditions as equivalent to a higher-temperature stage than that of the furnace, it is necessary to assume that large changes may occur in the temperature interval separating the two sources, and of very different magnitudes for different lines. However, lines in the same class, when near together in the spectrum so that the intensities were estimated from the same set of plates, show a fair degree of correspondence between the rate of change in the furnace and that for the interval between furnace and arc. This is best seen for lines in Classes II and III. So that, making allowance for difficulties in forming a concordant scale of intensities for arc and furnace lines, it may be said that if each class maintains the rate of change which it shows through the temperature range of the furnace observations (about 800°), it would be expected to reach an arc intensity of the order of that observed.

To account for the behavior of Class I A lines on the basis of a continuous change with temperature, we must adopt a scale of gradation such that these lines will diminish in intensity through the furnace range if they are to reach the weakness shown in the arc, where many of them, such as $\lambda\lambda$ 4258.477, 5225.695, and 5255.121, can only be given an intensity comparable with the furnace line by exposures of the arc so long as to give very high intensities to the lines of other classes. As compared with the rest of the spectrum, the arc is certainly at a great disadvantage in producing these lines. It is an interesting question whether the

furnace would show them relatively as weak if it could be brought to the temperature of the arc. Judging from the way the lines of Class I A maintain their strength through the range of furnace temperatures which has been produced, this seems unlikely. With two or three exceptions, the lines of Class I A have not been considered in investigations on the variation of arc and spark lines, doubtless because of their faintness in these sources. Their appearance in the flame and core of the arc is important and will be described later in this paper.

The high furnace temperature required to give the red end of the spectrum is noteworthy. The low intensity of the red lines is due in part to the decreased sensitiveness of the plates for this region, but very long exposures with both the large and small spectrographs have failed to give any red lines of iron at low temperature, except traces of the strongest arc lines and the two lines $\lambda\lambda$ 6280.833 and 6358.898 belonging to Class I A. At medium and high temperatures a large number of red lines were obtained. This behavior has placed almost all of these lines in classes not lower than III. The relative ease with which the blue end of the spectrum appears shows that the emission of the line spectrum does not follow closely the laws for an incandescent solid. This was well shown in a two-hour exposure with the very bright 1-meter concave grating, a bathed film being used. The furnace temperature measured about 1800° C. The exposure was so long that the weak continuous spectrum given by the vapor (probably largely through light from the wall of the tube reflected by the vapor particles) was able to impress itself on the plate distinctly in the red and orange. Iron lines were almost entirely lacking for this region, while they were plentiful in the blue where the continuous spectrum was very weak. This behavior, however, does not present an exception to any known law of spectral distribution, since no series have been recognized among iron lines, and a resemblance to the intensity gradations for an incandescent solid can be expected only for lines thus related.

Other notable groups of strong arc lines which are given only at the higher furnace temperatures occur at λ 4860 to λ 4960 and λ 5570 to λ 5625. The "nebulous" lines, indicated by *n* in con-

nection with the arc intensity, are among the most difficult to obtain in the furnace. Some of the best examples occur between λ 5360 and λ 5460. By long exposure at high temperature it was possible to obtain all of these distinctly, so that probably similar lines throughout the spectrum can be obtained under the same conditions. They usually belong clearly in Class V.

LINES WHICH MAY BE USED AS CRITERIA OF TEMPERATURE CONDITIONS

In sources for which a difference in temperature is considered to have a large part in the observed variations in the spectrum, a comparison of the intensities of lines near together which show a very different behavior in the furnace will usually show, in a general way at least, how the temperatures of the sources are related. Many pairs and groups of lines in Table I may be used in this way, but the following (Table II, p. 268) are suggested as especially suitable. The two lines in each case show a decided difference, both as to the temperature at which they appear and the rate of their change.

Some of the pairs in Table II are especially useful in deciding whether a given source more nearly resembles the arc or the furnace. Cases in which a Class I A line is paired with a line of Class II or Class III are favorable for this purpose. The behavior of the three lines $\lambda\lambda$ 5498, 5502, 5507 as compared with the strong I B lines from λ 5456 to λ 5269 is peculiar. These three lines are weak at low temperature but increase rapidly in the interval covered by the three furnace temperatures. The other I B lines in this region are strong at low temperature and increase slowly, but show a much greater increase in the interval between furnace and arc than is shown by the group of three, which would seem to belong in a class between Classes I B and II. If it were not for their relatively low intensity in the arc, the three lines would belong clearly in Class II. λ 5333.089 shows a very similar behavior in furnace and arc.

PRELIMINARY ACCOUNT OF THE ULTRA-VIOLET FURNACE SPECTRUM

A set of furnace photographs of the ultra-violet for which the iron arc spectrum reaches as far as λ 2300 has been made with the

1-meter concave grating already described. In most respects these are very satisfactory, but as a concave grating spectrograph of 15 ft. radius is under construction, it will be desirable to supplement

TABLE II

NEIGHBORING LINES WHICH ARE DIFFERENTLY AFFECTED IN THE FURNACE

λ	Class	λ	Class
3950.102	III	4459.301	II
3951.311	V	4461.818	I B
3966.212	II	4489.911	I A
3966.778	IV	4494.738	II
4100.901	I A	4528.798	II
4107.649	III	4531.327	I B
4177.698	I A	4919.174	III
4181.919	III	4939.868	I B
4199.267	III	5050.008	III
4202.198	II	5051.825	I B
4206.862	I A	5166.454	I A
4210.494	III	5171.778	II
4216.351	I B	5225.695	I A
4219.516	IV	5233.122	III
4232.887	I A	5341.213	II
4233.772	III	5371.734	I B
4250.287	III	6246.535	V
4250.945	II	6252.773	III
4271.325	III	6280.833	I A
4271.934	II	6301.718	IV
4260.640	III	6335.554	III
4308.081	II	6337.048	V
4291.630	I A	6355.246	III
4294.301	I B	6358.898	I A
4415.293	II	6393.820	III
4427.482	I B	6411.865	IV

them by large-scale photographs covering at least the portion where lines are most numerous in order to reduce the uncertainties as to blends and show more clearly the character of the lines. As the small spectrograph is adjusted, the second order can be followed from λ 4000 to about λ 2800, from which point the greater strength

of the first order is an advantage for the very short wave-lengths. The scale in the second order being 8.5 Å. per mm and the definition very sharp, the spectrum can be examined to good advantage under the microscope of a measuring machine with fairly high power. The resolving power of the grating is sufficient to separate sharp lines at λ 2800 which are 0.15 Å. apart. While postponing the detailed publication of the ultra-violet spectrum, a few of the leading features may be given here.

At high temperature, the line of shortest wave-length thus far photographed has been λ 2463. At medium temperature, prolonged exposures have yielded lines beginning at λ 2706; while the lower furnace temperature has thus far failed to show any lines of wave-length less than λ 3440. These limits are interesting as showing the gradual extension into the ultra-violet with increasing temperature. As absolute limits for a given temperature, they would probably be modified somewhat by a quartz prism apparatus or by a grating reflecting the short waves more strongly. Photographic emulsions more sensitive to this region would also give an extension if the furnace vapor does not absolutely cease to radiate at the limits observed. At the highest temperatures I have suspected that absorption by the vapor in the furnace tube may act in shortening the ultra-violet spectrum. The generation of vapors of all kinds, including those from carbon and the tube impurities is then very vigorous and the extension toward shorter wave-lengths has varied for different runs of the furnace made at approximately the same high temperature. For medium and low temperatures the results in this regard have been more consistent. This feature makes more difficult the comparison of temperatures between the furnace and any other source, based on the extension into the ultra-violet.

The classification adopted in Table I would have a different meaning if applied to the extreme ultra-violet, since there is an actual discontinuance the spectrum of at certain points according to the temperature employed. Thus the lines below λ 2700 would go into Classes IV and V, with a large addition of Class V lines which are given only in the arc for this region. From λ 2700 to λ 3440 numerous Class III lines would enter the list, while from

λ 3440 into and through the visible region all of the classes would appear.

It was found when a graphite plug was placed in the middle of the furnace tube and the furnace heated to different temperatures that the extension of the continuous spectrum into the ultra-violet for each temperature varied in a way closely corresponding to that of the line spectrum; but when such continuous spectra were taken on the same film at low, medium, and high temperatures each of them reached at least 300 Å. farther than it has been possible to obtain lines at the same temperature, even with much longer exposures than were used for the continuous spectra. The furnace tube with a plug at its center obviously approximates black-body conditions, and the inference is that the radiation of a black body is stronger than the emission of a line spectrum at the same temperature. Some experiments in which the furnace was charged with metallic calcium containing as impurities several other elements of low melting-points showed the same relation. As far as experiments have proceeded, the furnace never gives a bright line as far to the violet as the continuous spectrum from a plugged tube will easily extend at the same temperature.

A COMPARISON OF THE INTENSITY VARIATION SHOWN BY THE CORE AND THE FLAME OF THE IRON ARC

In the earlier work done at this observatory on the spectra of sun-spots,[1] the relative intensities of lines given by the core and by the outer envelope of the arc were compared and ascribed in the main to temperature differences. Some experiments at that time with a furnace of Moissan type in which a carbon tube containing iron was heated by an arc playing over its outer wall gave spectra resembling that of the outer arc vapors. The material obtained in the present work for a variety of furnace temperatures offered a means of testing the validity of the hypothesis as to the relative temperatures of arc vapors. A series of large-scale photographs was made with the image of the arc focused so as to give its cross-section on a long slit. The arc could be made to burn

[1] Hale, Adams, and Gale, *Contributions from the Mount Wilson Solar Observatory*, No. 11; *Astrophysical Journal*, **24**, 185, 1906.

very steadily, the greenish core showing at the middle of the slit, while the flame vapors, gradually decreasing in luminosity, spread toward the end. The spectrum photographed records the intensity of each line as given by the vapor in a horizontal plane through the arc, about midway between the poles. The arc lines then show the following characteristics:

1. The line is always most intense at its middle, where the radiation is given by two thicknesses of flame vapor, together with that of the core. From this point the line narrows, but at a rate which differs for different lines. The "short and long line" classification of Lockyer is based on this characteristic.

2. The ability of the cooler vapors of the flame to give a certain line is registered by the strength of the extensions from the center. This varies greatly for different lines and in general the distance from the center to which a line extends and the rate at which it strengthens from the end toward the center resemble, respectively, the stage of temperature at which a line appears in the electric furnace and the rapidity of its increase in strength with temperature.

This relation is shown in Fig. 1, the lower part of which represents by the shaded rectangles the relative strengths of the several classes at low, medium, and high temperature, while the upper portion gives sketches of typical lines of each class showing one-half of the line when the cross-section of the arc is projected on the slit in the manner described. The broken horizontal lines in the upper portion inclose parts of the spectrum line which may be considered as appearing in the core (reinforced by the superposed flame in the line of sight) and in the inner and outer portions of the flame vapors respectively. Photographs which give lines similar to those in this figure have been made by placing fine wires across the slit at the proper intervals.

Taking the several classes in turn, the I A lines are always weak in the arc, but they run some distance into the flame without much diminution of intensity. Judging from their appearance, very little of their strength is given by the central core of the arc. They do not, however, appear in the outermost vapors of the flame, as do the lines of Classes I B and II, the arc differing materially from the furnace in this respect. The furnace sometimes

shows a I A line very similar at all temperatures to a neighboring I B line, while the latter is far stronger in the arc. Examples are $\lambda\lambda$ 4291.630 and 4294.301, also $\lambda\lambda$ 4489.911 and 4466.727.

Lines of Class I B and the stronger ones of Class II run farthest into the flame vapors but show a decided contrast in the way in

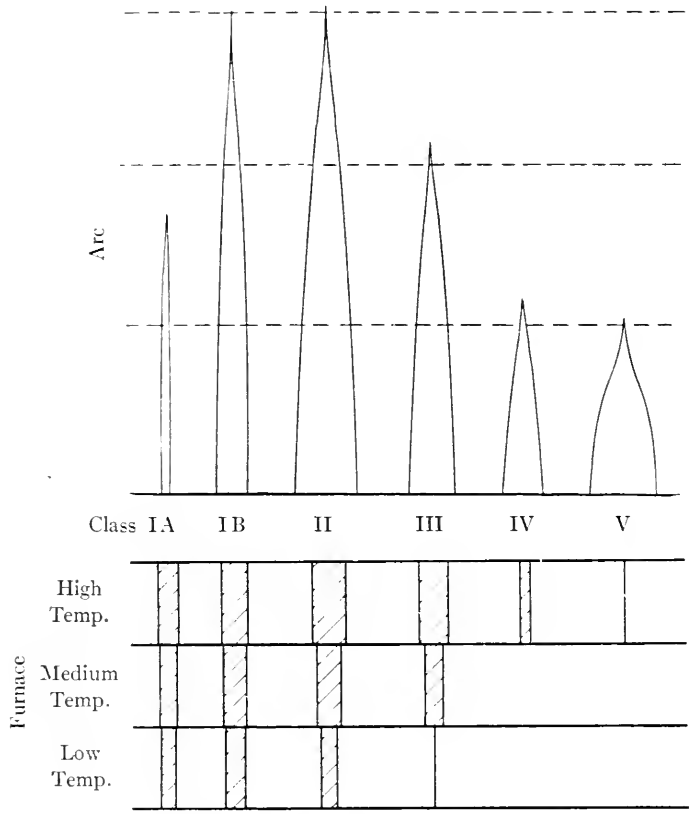


FIG. 1.—Comparison of relative intensities of furnace lines with the intensities of lines produced by different parts of the arc.

which they thicken as they approach the core. Adjacent lines of Classes I B and II may extend the same distance into the flame, but the Class II lines will often be two or three times as strong as the other at the core. $\lambda\lambda$ 4415.293 and 4427.482 are good examples and there are many similar ones. This behavior in the arc

corresponds closely with that in the furnace, both being low-temperature lines as regards initial appearance, but the I B lines being *relatively* strong at low temperature.

Class III lines do not extend as far into the flame as those of Classes I B and II, but strengthen more rapidly toward the core. This condition becomes more pronounced for Classes IV and V, strong lines of the latter being very wide in the core and showing little extension into the flame. Good examples are $\lambda\lambda$ 5383.578, 5415.416, and others in that region.

Taken as a whole, it is found that an iron-arc spectrum produced by a plane grating in this way shows that the lines divide into classes according to their rate of growth in intensity as we pass from flame to core, in much the same way as they have been classified as to their rate of strengthening at various furnace temperatures. This agreement is only qualitative. There are many differences in detail for lines other than those of Class I A, and only the furnace is available if the spectrum resulting from a measured temperature is desired. However, the furnace results indicate that the changes in relative intensity of lines observed in passing from the outer vapors to the core of the arc are due largely to temperature differences.

If the melting- and boiling-points of iron are taken as 1500° C. and 2450° C. respectively, it is evident from the furnace results that iron vapor can be made to radiate much below the boiling-point. Since the highest furnace temperature is above the boiling-point of iron, the arc should give a spectrum similar to that of the furnace if the metal were simply boiling in the arc. We find, however, that to produce the arc spectrum would require the equivalent of a much higher temperature than is given by the furnace. Experimental evidence is lacking as to the exact nature of this difference between furnace and arc conditions, but it would seem that it is to be attributed mainly to the high electronic speeds given by the arc discharge.

Another point of interest is the fact that there appears to be little reason to ascribe the high intensity of Class I B and Class II lines in the flame of the arc to chemical action, especially to the increased oxidation to be expected in the flame vapors. In the

furnace such chemical action as there is may be expected to increase with the temperature, and it is the low-temperature spectrum, presumably with minimum chemical action, which resembles most closely that given by the outer vapors of the arc. In fact, the material at hand indicates that relative changes in the spectrum do not result in any large degree from increase of oxidation. In the recent investigation of pressure effects[1] a set of furnace photographs was made for the blue and green-yellow regions of iron at atmospheric pressure and in compressed air up to 24 atmospheres. The brightness of the lines greatly increased, the radiation becoming stronger from end to end of the tube, as was evidenced by the general widening and the larger number of reversals; but no selective action, such as the bringing-out of some lines and the suppression of others, was to be seen, the nearest approach to the latter effect being for some high-temperature lines which did not reverse but were rendered very broad and hazy by pressure, so that unless the plates were strongly exposed such lines appeared greatly weakened. At atmospheric pressure, the spectrum is so closely similar to that in a partial vacuum, as regards relative intensity of lines, that the probability of oxidation playing an important part in the modification of a line spectrum, aside from general increase of intensity, seems very small.

COMPARISON WITH THE SPARK SPECTRUM

The evidence offered by the furnace is in agreement with the general phenomena of the arc and spark to the effect that the lines which are stronger in the spark than in the arc require a very high temperature or other abnormally violent excitation to bring them out. With one exception, all of the enhanced lines of iron listed by Lockyer[2] are entirely lacking in the furnace spectra. Only λ 3935.965 appears at high temperature, and this may be a barium line agreeing closely in wave-length which would be expected to show in the furnace. Most of the enhanced lines of iron are very faint in the arc, but the condition as to the weakness or absence of

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 60; *Astrophysical Journal*, **35**, 183, 1912.

[2] "Tables of Wave-Lengths of Enhanced Lines," Solar Physics Committee, 1906.

such lines in the furnace spectrum holds for other elements such as titanium, many of whose enhanced lines are strong in the arc.

A comparison with iron spark spectra made in this laboratory shows that the lines of Class I A, while present in the condensed spark, are produced with somewhat greater difficulty than in the arc, as would be expected from the general relations of arc and spark spectra. The connection of temperature changes in line intensities with those resulting from the use of self-induction with the spark will be made the subject of a later study.

COMPARISON WITH THE FLAME SPECTRUM

The investigations of Hemsalech and de Watteville have furnished material sufficient to establish the general character of flame spectra in reference to that of the furnace, though complete lists of lines for some of the flames have not been published. The flame lines given by de Watteville[1] as occurring in the outer vapors and those appearing at the boundary of the cone of a special air-coalgas flame correspond closely to the furnace spectra at low and medium temperatures respectively. The flame spectrum obtained by de Watteville extends farther into the ultra-violet than I have been able to photograph with the furnace, but his use of a quartz prism spectrograph for the flame may largely account for this, as comparisons of the extension into the ultra-violet are valid only when the same optical system is used. The possible absorption of short waves by the furnace vapors previously referred to may also affect this comparison with the flame. The list of flame lines does not include those which I have placed in Class I A for the furnace, with the exception of λ 4489.911. This would be remarkable were it not that de Watteville clearly omitted lines of this character which were relatively weak in the self-induction spark used by him for identification of the iron lines.

In an improved form of apparatus in which the iron vapor was generated by an arc in a closed chamber and then mixed with the gas which passed to the flame, Hemsalech and de Watteville[2]

[1] *Philosophical Transactions*, **204** A, 139, 1904.

[2] *Comptes rendus*, **146**, 859, 1908.

observed that an air-coalgas flame gave next to the cone a supplementary spectrum which did not appear in the hotter flames produced by combinations of oxygen with coalgas, air with hydrogen, and oxygen with hydrogen when used in the same apparatus. In a later paper[1] they noted that this "supplementary spectrum" contained a large number of lines in the ultra-violet which are given most strongly near the poles of the iron arc, many of them being relatively stronger in the spark than in the arc. These lines extend from λ 2374 to λ 3936.

Some of these "polar" lines appear in the high temperature furnace, but most of them are below λ 2700, in which region the furnace gives only the stronger arc lines. Recently the source named by the writer the "tube-arc" has been operated. The furnace tube is allowed to burn through with a high vaporization of carbon and the arc thus formed around the periphery has been shown[2] in the case of titanium, to give the enhanced lines strongly predominating. The tube-arc spectrum of iron reaches a shorter wave-length than the furnace spectrum, lines as far as λ 2327 having been photographed. Most of the polar lines noted by Hemsalech and de Watteville in the flame spectrum occur in the tube-arc, but I have not as yet been able to identify with certainty any of the enhanced lines of iron in the visible region. It thus appears that the polar lines in the ultra-violet do not require as strong excitation as do the enhanced lines of the visible spectrum, which is borne out by the fact that the former class of lines appear through the central region of the arc, though strongest close to the poles.

As to why the supplementary spectrum containing the polar lines did not appear in the hotter flames, I would venture to suggest that the condition noted by Hemsalech and de Watteville in a later paper[3] may furnish the explanation, namely, that in the more intense flames the strong current of gas carries the particles of iron vapor through the region of highest temperature too rapidly for the temperature to exert its full effect, while the more quiet

[1] *Comptes rendus*, **146**, 1389, 1908.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 65; *Astrophysical Journal*, **37**, 119, 1913.

[3] *Comptes rendus*, **150**, 329, 1910.

conditions of the Bunsen flame may permit such temperature as exists to be fully felt. In other words, iron vapor introduced into the oxyhydrogen flame by this method may not be heated to the same degree as a solid body would be on which the flame is allowed to play.

The oxyhydrogen flame spectrum of iron, of which 118 lines from λ 2483 to λ 5372 are given by Hemsalech and de Watteville,[1] must be classed as a low-temperature spectrum. Without exception, the lines in the visible portion belong in Class I B or Class II, whose strength at low temperature is characteristic. Many other lines should have appeared if the effective temperature of this flame had been as high as that for the medium-temperature furnace spectrum of Table I.

A study by the same authors[2] of the iron spectrum given by the oxyacetylene flame showed that in addition to the lines of the oxyhydrogen flame a set of lines occurs which are characteristic of the cone of the Bunsen flame, though not so strong as in the latter. The furnace spectrum shows that nearly all of these lines are in either Class III or Class II. If in the latter class, they show a rapid increase from low to medium furnace temperature. The strong Class III lines near λ 4900 are given in this set. This serves to place the oxyacetylene flame as corresponding to the furnace at the lower limit of the medium temperature. A conclusion equivalent to this was reached by Hemsalech and de Watteville by a comparison with the furnace results then at their disposal.

The several flames may thus be placed in the following order of their effective temperatures in producing radiation from iron vapor, the temperatures given being those read for the furnace when it gives in the visible region a spectrum similar in the lines present and in general relative intensities to that of the corresponding flame spectrum.

Flame	Furnace Temperature
Air-coalgas (outer vapors)	Below 1800° C.
Oxyhydrogen.	About 1800°
Oxygen-coalgas / Oxyacetylene	About 2000° to 2100°
Air-coalgas (cone)	About 2200°

[1] *Ibid.*, **146**, 962, 1908. [2] *Ibid.*, **150**, 329, 1910.

Judging from the furnace spectra, the flame temperature need not be higher than 2200° to produce the spectrum shown by the cone of the Bunsen flame and it may be appreciably lower. Furnace temperatures of 2400° and over not only gave a spectrum much richer in lines than is given by the Bunsen cone, but those lines which show a rapid growth with temperature are relatively much stronger. The lines classed as I A in the furnace spectra would be of value as an index to these low temperatures, but they were omitted by Hemsalech and de Watteville.

The difficulty remains to reconcile the above gradation of flames in the order of their spectroscopic efficiency with their relative temperatures as measured by melting-point tests. It seems at least plausible, however, that the explanation involves the mechanics of the flame gases, rather than the real thermal condition of each flame.

There is not enough material at hand to test definitely whether a certain flame yields the same spectrum as that of the furnace at equal temperature, though in the visible spectrum at least the evidence points that way. The list of flame lines by de Watteville[1] contains no lines between λ 3900 and λ 5600 which do not occur in the furnace. The medium-temperature furnace spectrum (2100°–2300°) contains more lines than the flame, the high-temperature furnace shows many more. Practically the same condition exists to about λ 2700; while farther in the ultra-violet, each source shows a number of lines apparently not given by the other. A further investigation of the extreme ultra-violet is required to show whether the use of different spectrographs and possibly the presence of foreign substances in furnace or flame (the identification of lines in this region often being uncertain) may be responsible in any large degree for this difference.

THE QUESTION OF TEMPERATURE RADIATION

The later work with the furnace has given no ground to modify the view expressed by the writer at the close of a paper[2] published

[1] *Op. cit.*

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 35; *Astrophysical Journal*, **29**, 190, 1909.

over three years ago, in which it was pointed out that the furnace is not suited for crucial experiments as to the mechanism of the radiation, but that temperature is obviously the agent which sets this mechanism in operation and controls the observed variations in the furnace spectrum.

The view appears to be gaining ground among spectroscopists that the emission of a line spectrum by temperature alone is possible and that quite probably it is given by the tube furnace, though definite proof of this cannot be claimed to have been offered. Opinions along this line have been clearly expressed in recent writings by Kayser[1] and by Schuster.[2] The vital question is whether a part of the translatory energy of the molecules due to heat can be changed to the "internal energy" which gives the emission of light. In this paper I wish to draw attention to several characteristics of furnace phenomena which bear more or less directly on the general problem.

1. There is unquestionably strong ionization from the carbon tube at the temperatures employed. Recent experiments by Harker have shown this to be the case at atmospheric pressure, while its existence at low pressure is well known.

2. Chemical reactions take place which are probably mainly of two kinds—combination with oxygen, the total exclusion of which is all but impossible even with the vacuum furnace, and the formation of iron carbide. The residue of iron is always found sticking fast to the interior of the tube and appears to eat into the graphite, this being perhaps in part due to the porosity of the latter.

3. The furnace radiation does not depend upon the existence of a potential difference. When the current is broken, the high-temperature lines gradually disappear, while some of the low-temperature iron lines have been observed 5 minutes, and the D lines of sodium 16 minutes after breaking the current. There is every reason to believe that if the tube could be as efficiently heated by a flame or other non-electrical source of heat, the radiation from the inclosed vapors would be the same as observed in these experiments.

[1] *Zeitschrift für wissenschaftliche Photographie*, **8**, 151, 1910.

[2] Article "Spectroscopy," *Encyclopaedia Britannica*, 11th ed.

4. The increased oxidation due to the presence of air at, or much above, atmospheric pressure appears to have no distinct effect upon the spectrum for a given temperature aside from a general widening of the lines.

5. Many experiments have shown that when the furnace tube is obstructed by a plug or other mass of solid matter so as to approach black-body conditions, the weaker lines are entirely concealed by the continuous spectrum and the stronger lines thrown into absorption, indicating that the black-body radiation is stronger than that of the metallic vapor for any given wave-length.

SUMMARY

The ground covered in this investigation may be summarized as follows:

1. The relative intensities of iron lines in the visible spectrum have been given for three furnace temperatures and for the arc, with a division into six classes based on the temperature at which a line appears in the furnace and its rate of growth with increase of temperature.

2. The changes in relative intensities of lines observed in passing from the furnace to the arc may in general be accounted for by a difference in conditions equivalent to a large temperature difference, though one class of furnace lines presents difficulties in establishing a clear relation of this kind.

3. The red end of the iron spectrum appears to require high temperature for its production as compared to the blue region, and the distribution of lines through the visible spectrum bears little resemblance to the intensity gradation observed for the spectrum of an incandescent solid.

4. A preliminary examination of the ultra-violet showed this region to be very rich in lines given by the furnace. The extension of the line spectrum toward shorter wave-length with increase of temperature was observed, it being found that lower limits were reached for a given temperature by the spectrum of an incandescent solid.

5. A list is given of pairs of lines whose members belong in different classes as regard response to temperature changes and

may be used in estimating the relative temperatures of light sources.

6. The increase in intensity of lines from the outer vapors into the core of an iron arc was found usually to resemble the rate of growth shown by the same lines with rising furnace temperature. This renders it unlikely that chemical reactions in the outer vapors affect the relative intensity of arc lines in any large degree.

7. The enhanced lines of iron in the visible region have not been observed in the furnace spectrum.

8. A comparison with published data on flame spectra has shown how the various flames are probably related as to effective temperatures in producing radiation. Except perhaps in the ultra-violet, furnace spectra at low and medium temperatures are found to be very similar to those of the several flames.

9. While presenting no definite proof that temperature radiation in a strict sense takes place, the position of temperature as the exciting and regulating agent in furnace phenomena seems to be clear.

Miss Sheldon of the Computing Division has rendered frequent assistance in this work, especially in the examination of plates for the ultra-violet spectrum.

Mount Wilson Solar Observatory
October 17, 1912

ON THE MEASUREMENT OF THE ZEEMAN EFFECT

By G. F. C. SEARLE

In the *Astrophysical Journal* for November 1911 (**34**, 312) J. E. Purvis drew attention to the discrepancies between the values of the Zeeman effect obtained by him in the case of certain lines in the spectrum of chromium and the values obtained by Miller, by Hartmann, and by Babcock. On reducing to a common value of 23,850 gausses the magnetic forces which the four observers stated they had employed, Purvis found the following results for the separation of the triplet arising from the line λ 4646.39:

VALUES OF $\frac{d\lambda}{\lambda^2} \cdot 10^8$ FOR 23,850 GAUSSES

Purvis	Miller	Hartmann	Babcock
+0.92 0 −0.92	+1.31 0 −1.31	+1.21 0 −1.21	+1.42 0 −1.42

Similar results were found for other triplets.

Purvis says of his own work: "The field-strength I used was 39,980 C.G.S. units, and there can be no doubt of its accuracy."

In the *Astrophysical Journal* for April 1912 (**35**, 213) A. Cotton discusses the discrepancies to which Purvis had called attention and concludes that the magnetic field in the gap of his magnet, which Purvis assumed to be 39,980 gausses for an exciting current of 20 amperes, was not, in fact, so great. From a comparison of Purvis' results for the Zeeman effect with those of other observers, Cotton deduces that the field used by Purvis was approximately 30,000 gausses, instead of 39,980 gausses.

The value of 39,980 gausses assigned by Purvis for the magnetic field due to a current of 20 amperes in the coils of his magnet, when the trunkated conical pole-pieces are terminated by disks 7 mm in diameter and the gap is 4 mm wide, is taken from the results of a series of measurements made by myself, with assistance,

in 1903. The magnet was supplied by W. G. Pye & Co. of Cambridge, England, and my tests were made within a few days after its delivery by the makers. The results of some of those tests are shown in Fig. 1 by the curve marked (2) and numerical values are given in Table I. The curve marked (1) represents results obtained in 1903 with pole-pieces 0.3 cm in diameter and a gap of 0.15 cm. The testing coil used for curve (2) had a larger diameter than that used for curve (1).

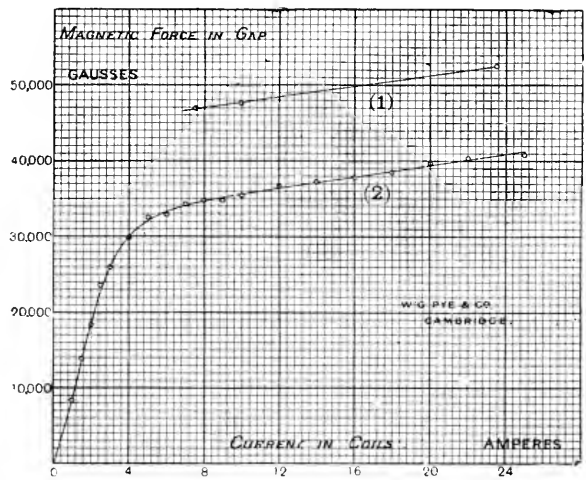


FIG. 1.—Field-strength for different values of the current

The determinations of 1903 were made by means of a ballistic galvanometer and a small coil which was suddenly jerked out of the magnetic field. Suitable mechanical arrangements were made to insure that the coil was properly placed between the pole-pieces. A standard of mutual induction from the Laboratory of Engineering was used to calibrate the ballistic galvanometer, and the current in the primary winding of this standard was measured by a good ampere-meter from the Cavendish Laboratory. I verified by tests that the value of the mutual induction marked on the standard was, at any rate, nearly correct. The small coil had a single layer of very fine wire; the diameter of the bobbin was carefully measured and a proper allowance was made for the

thickness of the wire. The current exciting the magnet was measured either by a good ampere-meter from the Cavendish Laboratory or by an instrument belonging to the Chemical Laboratory, which was compared with the Cavendish Laboratory instrument. I cannot now say which plan was adopted. Since the field increases very slowly as the current increases in the neighborhood of 20 amperes, moderate accuracy in the measurement of the exciting current suffices.

Those who have made similar measurements on large electromagnets with small pole-pieces and a narrow gap will understand that, although the tests are easily described, the experimental difficulties in the way of obtaining very precise results are considerable. All I can say at the present time is that I took all the precautions which I thought were necessary and that I should have felt disappointed if my results for a gap of 4 mm and for pole-pieces 7 mm in diameter with 20 amperes of exciting current were in error by as much as 5 per cent.

Purvis has throughout his work relied on my measurements of 1903. Since that date, however, one of the coils of the magnet has been damaged by overheating and has been rewound with a slightly smaller number of turns. The change in the number of turns is not sufficient of itself to account for the differences in the values of the magnetic force found in 1903 and in the measurements of 1912 which are described below. Defects may have developed in the other coils also, but they have not forced themselves upon the attention of those who have used the magnet.

In view of the criticisms of Cotton, I agreed, at the request of Purvis, to make a new set of measurements by the ballistic method. These were made in October 1912; the measurements were not so extensive as those of 1903, but are sufficient to settle the point at issue. The result is to show that the current of 20 amperes, which Purvis trusted to give him 39,980 gausses, at the date of the recent measurements only gave 28,700 gausses.

In the measurements of 1912, the ampere-meters were tested by aid of a standard resistance and a certified cadmium cell. The standard of mutual induction was one constructed at the Cavendish Laboratory under my supervision. The form was that

recommended by Searle and Airey,[1] and the coefficient of mutual induction (M) was calculated with great care by the formulae given by Searle and Airey from measurements made by myself. This standard was compared with one from the Engineering Laboratory and the value of M for the Cavendish Laboratory standard deduced from the value of M marked on the Engineering Laboratory standard was found to be in good agreement with that calculated from the formulae.

The bobbin of the coil used in measuring the magnetic field had a diameter of 0.500 cm and was wound with 15 turns of silk-covered wire 0.013 cm in diameter. The effective area was thus $\pi \times 15 (0.2565)^2$ or 3.1004 sq. cm. The coil was wound on an ivory bobbin. The coil was connected in series with the secondary coil of the induction standard and with a moving coil ballistic galvanometer. The coil was suddenly withdrawn from the gap of the magnet and the ballistic effect was observed. By comparison with the ballistic effect due to the reversal of a measured current through the primary coil of the induction standard, the value of the magnetic field was obtained.

The two limbs of the electromagnet were adjusted so that the pole-pieces were as nearly as possible symmetrically placed with regard to each other. The gap was set by aid of a metal distance piece 0.4 cm thick, so that it measured 0.4 cm in width when the magnet was excited with 20 amperes. The attraction between the poles with an exciting current of 10 amperes is sufficient to diminish the width of the gap by about 0.017 cm.

TABLE I

Exciting Current Amperes	Magnetic Force 1912 Gausses	Magnetic Force 1903 Gausses
4	23,900	29,800
6	24,700	33,600
10	25,800	35,400
15	27,800	37,300
18	28,600	38,400
20	28,700	39,700

Table I gives the values of the magnetic force found in the present measurements (1912). For comparison I have added the values found in 1903 for the same exciting currents, as read off from the chart which I made in 1903.

[1] *The Electrician*, December 8, 1905.

There is a slight discrepancy between the value 39,700 read off from the chart and the value 39,980 used by Purvis, but this may be neglected in the present discussion.

The magnet used by Purvis is shown in Fig. 2. It is constructed of a special mild steel and consists of a massive base carrying two limbs on which the coils are wound. Slots in the base allow the width of the gap to be adjusted. The limbs have a cross-section

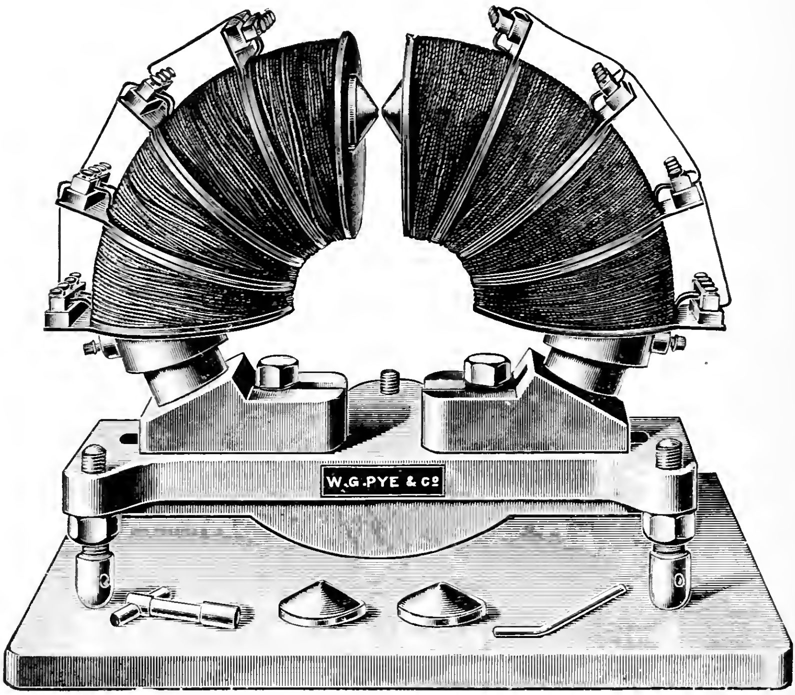


Fig. 2.—Magnet used by Purvis

of 50 sq. cm and the central line of each limb is part of a circle of 20 cm radius. The ends of the limbs are made truly plane and the bases of the conical pole-pieces are ground to fit them. The cones have angles of 120°. Each limb carries four coils, each originally wound with 280 turns of cotton-covered wire, 0.25 cm in diameter; the total resistance of the eight coils in series was about 3 ohms. The coils carry 20 amperes continuously without undue heating.

The magnetic forces obtained in 1903 with the pole-pieces mentioned above and with other pole-pieces exceeded the theoretical

maximum values deduced on the assumption that the pole-pieces were uniformly magnetized to a saturation intensity of magnetization of about 1600 C.G.S. It seemed probable that the magnetic force in the gap was due not only to the surface magnetism on the surface of the pole-pieces, but also to a volume distribution within the pole-pieces due to the non-uniformity of the magnetization which must occur in practice, the amount of magnetism per unit volume being:

$$\frac{dI_1}{dx}+\frac{dI_2}{dy}+\frac{dI_3}{dz},$$

where I_1, I_2, I_3 are the x, y, z components of the magnetization. Thus the fact that the values observed in 1903 were greater than the theory of uniform magnetization allows was not conclusive evidence that those values were erroneous. The circumstance led me, however, to investigate the magnetic properties of the steel of which the magnet was formed. Messrs. W. G. Pye & Co. supplied me with a carefully turned ring of the metal and I tested it for permeability (μ). The ring was wound with wire until no more wire could be wound on. As the magnetizing force reached values which are not often attained by the use of electric currents alone, it may be of interest to give the results.[1]

TABLE II

Magnetic Force Gausses	Magnetic Induction Maxwells per sq. cm	Permeability μ	Intensity of Magnetization I
1	950	950	75.5
2	5,190	2595	413
3	7,950	2650	633
4	9,680	2420	770
5	10,700	2140	852
7	12,160	1737	968
10	13,380	1338	1065
20	14,870	744	1182
30	15,600	520	1240
50	16,430	328.6	1305
100	17,460	174.6	1382
200	18,850	94.25	1485
400	20,380	50.95	1590
600	20,960	34.93	1621
800	21,400	26.75	1640

CAVENDISH LABORATORY
CAMBRIDGE, ENGLAND

[1] These results were communicated two or three years after the 1903 tests to Professor Andrew Jamieson for a book on magnetism he was then writing, and a copy was preserved by Messrs. W. G. Pye & Co.

MINOR CONTRIBUTIONS AND NOTES

NOTE ON THE SPECTRUM OF THE ALUMINIUM ARC

In July 1909 Paschen[1] published measurements of the principal series in the aluminium spectrum. He gave the wave-lengths of four doublets corresponding in his series formula to $m=3$, 4, 5, and 6. The last-mentioned doublet falls within one of the aluminium bands and, as it is faint, is difficult to obtain and measure. Paschen puts a question mark after his determination but, as he gives only 0.06 Å.U. as the limit of error, it may be supposed that the uncertainty lies not so much in the measurement as in the identity of the line.

When Professor Hicks[2] applied his formula to this case, using the wave-lengths of the lines $m=3$, 4, and 5 to calculate the constants, he found that the calculated value for $m=6$ differed considerably from the measured value. By calculation he found $P_2(5)=5107.81$, $P_1(5)=5108.22$, while Paschen gives 5105.57 with the note "in a band, very uncertain, probably double." At his suggestion I therefore undertook to test this question, using a spectrograph with two calcite prisms.

The first photographs taken were of the aluminium arc in air at low pressures (from 2 to 20 cm of mercury), using an apparatus designed by Professor Hicks. It was thought that the principal series would appear better with the arc under low pressure. The apparatus used is shown in section in Fig. 1. The body of it consists of two cylindrical iron tubes, C, D, fitted at right angles. The arc is formed at the point of intersection of the axes of the two tubes. At the ends of the horizontal tube D quartz lenses were fixed, by sealing wax, so that a convergent or a parallel beam of light could be obtained at pleasure. This arrangement also

[1] *Annalen der Physik* (4), **29**, 625.

[2] *Phil. Trans. Roy. Soc.*, **212** A, 57; *ibid.*, **212** A, 33.

allowed the arc to be easily observed. The aluminium pole pieces were held in the specially shaped ends of the two iron rods, A, B. The rod A was screwed through a disk of iron in the tube C, and could be adjusted by the screw. The rod B passes up a glass tube slightly greater in diameter, which is fastened by sealing wax into a fiber disk, itself firmly sealing-waxed onto the lower end of the tube C. This and the screw on A had mercury seals. A wide piece of glass tube E was fitted as shown onto a rubber stopper fixed to the iron rod B. This tube served the purpose of a mercury reservoir. On exhausting the interior of the apparatus, through a small side tube not shown in the diagram, the mercury rose in the space between the rod B and the glass tube, thus serving as a manometer. The arc was struck by raising the rod B to touch the fixed rod A. The length of the arc was adjusted by means of the screw F. This adjustment was necessary owing to the expansion of the rods and the pole pieces.

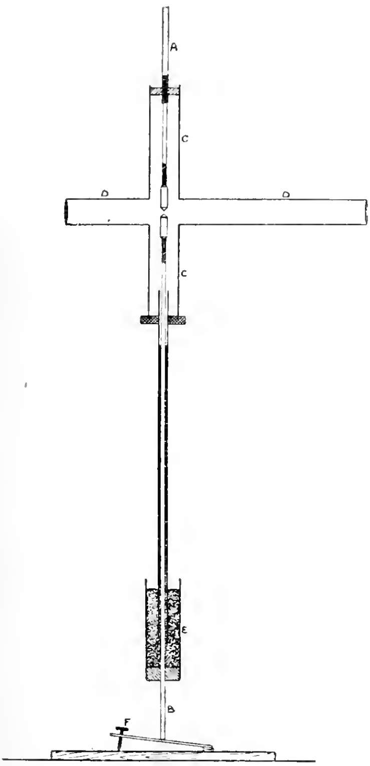


FIG. 1

On the first photographs no trace of the doublet could be seen and it was thought that the band spectrum possibly obscured it. Previous work on this band spectrum had not quite resolved the doubt as to whether it was due to the aluminium itself or to the oxide. The effect of forming the arc in nitrogen between poles of pure aluminium, free from oxide, was therefore tried. Although it was not found possible thus to eliminate the band spectrum entirely, the latter was much less intense with the arc in nitrogen than in air. For this reason all other photographs were taken with the arc in

nitrogen. On these last photographs the doublet corresponding to $m=6$ was still invisible, but on some of them the doublets corresponding to $m=4$, 5 could be seen quite plainly.

It was then decided to vary the pressure and to observe carefully the changes in intensity of these doublets. The conditions which would be such as to make these most intense would probably at the same time be the best for showing any trace of $m=6$.

The doublets corresponding to $m=4$, 5 were found to increase in intensity with the pressure, and were therefore at their best with the arc under atmospheric pressure. (The apparatus used, as will have been observed, was available only for pressures equal to or less than atmospheric. It was later found possible, by lengthening

FIG. 2.—Aluminium arc at 10 cm pressure

the outer tube E, to increase the pressure up to nearly two atmospheres, but no further appreciable increase in intensity was observed.)

By using the arc in nitrogen at atmospheric pressure, a photograph was obtained showing what was most probably the required doublet, $m=6$. The dispersion was not sufficient to separate the components of the doublet, which thus appeared as a single line. The line was faint but quite measurable, its wave-length being 5107.5, a result agreeing as closely as could be expected with that predicted.

Apart from this result some other new and rather remarkable lines were observed on the photographs of the aluminium arc at low pressures. These formed three groups, two of which were much more prominent than the third, in the neighborhood of $\lambda=4250$ Å.U. These groups are not seen with the arc at atmospheric pressure except under prolonged exposures, and then only very faintly. With the arc at a pressure of about 10 cm they come out

extremely well. They are shown in Fig. 2. The individual lines in the groups do not vary much in intensity, and the heads, where several lines run together, stand out very clearly, except in the least prominent of the three groups. The groups show equally well with the arc in air, nitrogen, or coal-gas. The wave-lengths of as many lines as possible in the groups were measured approximately, using the iron spectrum as a standard, and are given below:

First Group	Intensity	Second Group	Intensity	Third Group	Intensity
4379.4	2	4287.4	9	4253.0	5
4375.5	2	4283.9	9	4251.1	5
4371.7	1	4280.4	9	4249.5	5
4365.4	2	4277.7	9	4248.1	5
4363.7	2	4275.2	9	4246.5	5
4361.7	1	4272.4	9	Several running to a head at 4241.25	9
Several running to a head at 4360.9	2	4271.0	7		
		4269.3	7		
		4267.5	5		
		4266.0	5		
		Several running to a head at 4260.05	10		

A. B. MANNING

THE UNIVERSITY, SHEFFIELD

NOTE.—The work described above was done in 1910, but was not sent to press until October 1912. I find that the groups of lines in the aluminium arc under low pressures have been measured recently by Miss Howson.[1] Miss Howson, using a Rowland grating, was able to obtain a much greater dispersion than was used in the above work, and measured many more lines of the groups.

ON THE EARTH LIGHT COMPRISED IN THE BRIGHTNESS OF THE MIDNIGHT SKY

In the *Astrophysical Journal*, May 1912, we find an interesting paper by Mr. Humphreys concerning which I should like to say a few words.

Mr. Humphreys explains that the light of the midnight sky seems to be composed of two parts, one reaching us directly from the stars, the other resulting from processes in the atmosphere, and

[1] *Astrophysical Journal*, **36**, 286, 1912.

that this latter part, termed earth light, is probably due, wholly or in part, to a permanent aurora. Mr. Humphreys then points out another possible cause of this earth light; the source is the continual bombardment of the outer atmosphere by materials of meteoric origin, i.e., by all particles picked up by the earth in its orbital motion.

I have shown[1] that, because of the number of shooting stars visible to the naked eye and the telescope, and the probable size of the particles producing this phenomenon, the reflection and scattering of sunlight by these meteoric particles ought not to be neglected. This should explain, at least in some measure, the difference that exists between the brightness of the midnight sky and the total light of all the stars together.

I did not take any notice of the proper light of the shooting stars, which Mr. Humphreys considered, and I feel obliged to give a reason for it.

Mr. Humphreys, starting from the estimated and generally accepted amount of meteoric material received by the earth, calculates the brightness that must result from it for the sky during night-time. But is it not possible that there is here perhaps a sort of syllogistic circle?

We really do not know the mass of meteoric material received by the earth, but only the brightness of the luminous processes produced by the shooting stars. Starting from that brightness, several learned men, more particularly Schiaparelli, have estimated the mass of the meteorites and have found it of the order of a gram. But the particles forming shooting stars have never been weighed. Stanislas Meunier even supposes that those particles have nothing to do with the meteorites picked up from the ground and coming from the aeroliths and that they are only gaseous bubbles.

The value of the mass which we have adopted may be very inaccurate. It is possible that the extrapolation of Stefan's law at a temperature as high as that of the stars is entirely wrong and that, in the case of shooting stars, the enormous energy produced by their arrival in the atmosphere is kept in the form of electricity, as the Marquis de Mauroy supposed it to be. In short, the

[1] *Bulletin astronomique*, January 1912.

relation between the mass of the shooting stars and their light may be very different from that generally admitted. But in any case, that mass should not be used to estimate the brightness of the midnight sky that must come from it.

The brightness of the sky due to the shooting stars should be estimated directly.

Moreover, if we compare the light of the shooting stars, visible to the naked eye, with that of all the stars together, it seems to be very feeble. It would be very interesting to know if the proportion is the same for the telescopic shooting stars. In any case if we limit ourselves to the shooting stars that are large enough to be seen individually, it appears that their total light can only be very faint compared to the total light of the fixed stars.

This remark does not at all lessen the interesting hypothesis given by Mr. Humphreys, for this theory may be applied to the particles that are far too small to produce visible streaks, the number and mass of which are quite unknown to us. It is possible, too, that the arrival of these particles in the atmosphere is quite sufficient to produce abundant ionization.

I only wish to explain that it is rather difficult to apply this theory to the shooting stars giving individual streaks, the only ones we know experimentally, and the only ones to which my calculations on the absorption and the scattering of the light through the meteorites may be applied.

P. SALET

OBSERVATOIRE DE PARIS

Reviews

Leçons sur les hypothèses cosmogoniques. By H. Poincaré. Rédigées par Henri Vergne. Paris, 1911. Pp. 294.

Researches on the Evolution of the Stellar Systems. Vol. II. By T. J. J. See. Lynn, Mass., 1910. Pp. 735.

Poincaré's last astronomical gift, his lectures on the hypotheses of cosmogony, are a fine testimony to his vigor as well as to the objectiveness and kindness of his mental disposition. The literature on cosmogony is very heterogeneous and in part impenetrable in consequence of prolixity, pretension, confusion, and general lack of mathematical control. Poincaré has gathered from this wild garden quite a multitude of fine fruits, a collection of ideas, of which perhaps everyone is likely to touch some part or other of the making of the worlds. He has not destroyed these ideas by criticism, but laid bare the core and even represented many of them in a clearer way than their authors. We must not find fault with him, when he takes notice chiefly of French authors with whom he is naturally most familiar, and in doing so overlooks some others. He quotes Kant, Laplace, Faye, du Ligondès, Bélot, See, Darwin, Lockyer, Schuster, Arrhenius. But he does not take notice, for instance, of the planetesimal hypothesis of Chamberlain and Moulton and of Emden's book on gaseous spheres (Gaskugeln).

The theories on cosmogony of the last decade are generally adverse to the nebular hypothesis of Laplace. Poincaré is no partisan of this opposition; he thinks, on the contrary, that the Laplacian hypothesis still, after some comparatively insignificant adjustments, is the best. The principal objection to the Laplacian hypothesis consists in the statement that the moment of momentum of a homogeneous sphere of the sun's mass, which has the diameter and period of revolution of Neptune's orbit, would be many times larger than the moment of momentum of the present solar system. To get rid of the objection, one has to suppose that the Laplacian nebula had from the beginning a very strong central condensation of almost the sun's mass, and that only about one-thirtieth of the whole mass expanded like an atmosphere around the massy central nucleus. When this fundamental supposition is introduced, it is possible to explain on the lines of Roche's deductions

and with casual application of tidal friction the origin of the solar system without undue difficulties.

The Kantian hypothesis takes as origin of the solar system a swarm of meteorites. The collisions of the meteorites produce in the end the formation of the sun and the planets and the regularity of their orbits. The Kantian hypothesis reappears in a form which is free from its original ill-treatment of the principles of mechanics, as well in the work of Mr. du Ligondès as in the planetesimal hypothesis of Chamberlain and Moulton. There seems to be no doubt that frequent collisions, which are accompanied by loss of energy, are actually able to mould a regular system from the primitive chaos of meteorites. But Poincaré finds a difficulty in the following consideration: Besides the collisions approaches occur between the meteorites, which give rise to deflections of their paths in consequence of their mutual attraction by gravitation. These approaches act like collisions without loss of energy. They are like the collisions of the common kinetic theory of gases. Therefore, if the approaches alone occurred and the collisions proper were excluded or negligible, the whole swarm would in the course of time become a sphere in a kind of adiabatic equilibrium. When a planetary system is to originate from the chaos, it is necessary, on the contrary, that the effect of the collisions proper, which are accompanied by loss of energy, preponderate and the gravitational deflections are comparatively insignificant. The approaches naturally are much more frequent than the collisions. Therefore Poincaré seems to fear the preponderation of their total effect. But G. H. Darwin has already shown how small the effect of the single approach is. The writer has convinced himself that the total effect of the approaches is also small compared with the effect of the collisions as long as the single elements of the swarm of meteorites have not become as large as planets. Therefore this scruple can be put aside.

The hypothesis of See also belongs to the group of modernized Kantian conceptions. After the central sun and the principal planetary nuclei arose from the swarm of meteorites, the totality of the remaining small masses forms a kind of resisting medium in which the larger bodies move. A resisting medium of density increasing toward the center diminishes, as Mr. See proves, the axes and excentricities of the orbits. In this way the smallness of the excentricity of the planetary orbits is explained. As to the satellites, Mr. See takes them as former small planets, which have been captured by the larger planets with the help of the resisting medium.

After all, the sympathies of the writer are with the modernized Kantian hypothesis. The planetary system shows a mixture of chance and order. The Laplacian hypothesis seems to call for a perfect regularity, whereas with the Kantian hypothesis the partly accidental appearances of the present system may be traced to the accidental constitution of the primitive meteorites.

Most of the modern cosmogonies venture even to enter into the former history of the chaos from which our solar system originated. In doing so, they are inspired by the charm which emanates from the photographs of spiral nebulae. The origin of spiral nebulae Mr. See explains by the approach and the gravitational uniting of two diffuse swarms of matter. Chamberlain and Moulton calculate the consequences of the approach of two suns; Mr. Arrhenius suggests a real collision of two suns, which, according to him, first gives rise to a nova, from which afterward a spiral nebula originates. For an explanation of the formation of spiral nebulae perhaps one should take more into account the fact brought forward by Mr. E. v. d. Pahlen, that the finest spirals of the sky agree remarkably well with logarithmic spirals (*Astron. Nachrichten*, No. 4503).

To all these authors the regular figure of the spiral nebulae seems the first indication of the regularity of a planetary system; the brighter patches of light in the nebulae appear to them as the nuclei of future planets. According to their common opinion, the solar system has originated from a spiral nebula.

To the writer it seems not very credible that the solar system has ever been a spiral nebula. It is well known that the spiral nebulae as observed by us have a diameter at least a hundred or a thousand times larger than the diameter of Neptune's orbit. If they had a mass of the order of the sun's mass, there would follow a density of the order of 10^{-15} to 10^{-18} of the density of water. A small density gives no difficulty in itself, but it is hard to suppose that matter of such tenuity should give a continuous spectrum with absorption lines, as it is shown according to Scheiner by the nebula in *Andromeda*, and, according to Fath, by some other spirals. Therefore it seems to follow that not only the diameter, but also the mass of spiral nebulae, is of quite another order of magnitude than those of the solar system, which implies that the solar system cannot have been a spiral nebula.

With his fascinating imaginative power Mr. Arrhenius conceives even a cyclic course of the world. Spiral nebulae become solar systems and extinguished solar systems are again by collision changed into spiral

nebulae. But a cyclic process is incompatible with Clausius' famous theorem of the caloric death (*Wärmetod*) of the universe. In fact Mr. Arrhenius supposes that this theorem does not hold, that the "deterioration" of energy, which takes place in the suns by the transformation of mechanic into thermal energy, is balanced by a corresponding "amelioration" of energy in the spiral nebulae. The nebulae lose indeed the quickest molecules, which escape the control of their gravitation. By this process the nebulae become cooler. Those escaped molecules which reach a certain sufficiently distant part of space, all move in the same direction. This is the transformation of irregular thermal agitation into regular motion, which is necessary to overthrow Carnot's principle. Nevertheless Poincaré—although he does not express himself very clearly on this point—seems *not* to believe that the caloric death is much detained by the foregoing mechanism. The writer shares Poincaré's opinion. Firstly, it is not proved that under reasonable suppositions the amelioration of energy in the nebulae is anything like equal in amount to its deterioration in the suns. Secondly, the molecules escaped from different nebulae will mix everywhere, and even without collisions constitute again an irregular motion. On the suggestion of Mr. Bestelmeyer the writer made some time ago the calculation given in the note below, which proves that the mean kinetic energy, i.e., the temperature of the mixture of escaped molecules, is *less* than the temperature of the nebulae from which they escaped. Therefore not even regarding temperature, the escape of molecules constitutes an exception to Carnot's principle. Up till now no really effective rescue from Carnot's principle seems to have been found. Nevertheless it cannot be pretended that a cyclic course of the world is impossible within exceedingly long periods, 10^{50} years, let us say. One must remember the various other reasons as exposed by Seeliger in his note, "Über die Anwendung der Naturgesetze auf das Universum" (München: *Sitzungsberichte der Bayr. Akad.*, 1909), which forbid the unrestricted application of Carnot's principle.

NOTE.—Suppose the distribution of velocities v of the molecules at a point near the outward limit of a nebula to be given by Maxwell's law (a^2 a constant):

$$e^{-a^2v^2}v^2dv$$

The mean kinetic energy or temperature T at this point will be:

$$T=\frac{1}{2}\frac{\int_0^\infty e^{-a^2v^2}v^4dv}{\int_0^\infty e^{-a^2v^2}v^2dv}$$

When the potential of gravitation at the same point is equal to k^2, all the molecules for which $v>k$ will escape. The final velocity of these molecules becomes: $w_1=\sqrt{v^2-k^2}$. Their final mean kinetic energy will be:

$$T'=\frac{1}{2}\frac{\int\limits_k^\infty e^{-a^2v^2}w_1^2v^2dv}{\int\limits_k^\infty e^{-a^2v^2}v^2dv}$$

or, by introducing w as independent variable:

$$T'=\frac{1}{2}\frac{\int\limits_0^\infty e^{-a^2w^2}w^3\sqrt{w^2+k^2}dw}{\int\limits_0^\infty e^{-a^2w^2}w\sqrt{w^2+k^2}dw}$$

By subtracting the temperatures T and T' and multiplying by the denominator, we get:

$$2(T-T')\int\limits_0^\infty e^{-a^2v^2}v^2dv\cdot\int\limits_0^\infty e^{-a^2w^2}w\sqrt{w^2+k^2}dw=\int\limits_0^\infty\int\limits_0^\infty e^{-a^2(v^2+w^2)}v^2w\sqrt{w^2+k^2}(v^2-w^2)dv\,dw$$

Writing the double integral again with the notations v and w interchanged and taking the mean of both expressions, we get:

$$\frac{1}{2}\int\limits_0^\infty\int\limits_0^\infty e^{-a^2(v^2+w^2)}[v^2w\sqrt{w^2+k^2}(v^2-w^2)+w^2v\sqrt{v^2+k^2}(w^2-v^2)]dv\,dw$$

$$=\frac{1}{2}\int\limits_0^\infty\int\limits_0^\infty e^{-a^2(v^2+w^2)}v^2w^2(v^2-w^2)\left(\sqrt{1+\frac{k^2}{w^2}}-\sqrt{1+\frac{k^2}{v^2}}\right)dv\,dw$$

which is clearly positive. Therefore $T>T'$ Q.E.D.

K. SCHWARZSCHILD

ASTROPHYSICALISCHES OBSERVATORIUM
POTSDAM, GER.

The Structure of the Atmosphere in Clear Weather. (A Study of Soundings with Pilot Balloons.) By C. J. P. CAVE. Cambridge University Press, 1912. 4to, pp. 144.

In this book are published in detail the data obtained in 200 pilot balloon soundings. The highest points to which the balloons were observed in these soundings varied from 1.0 to 18.5 km, and their mean height was 4.4 km. The data are also summarized, the classification being made for the most part with reference to the wind data themselves as defined in the introduction and further in chap. i. The five types of structure considered are: (*a*) wind in the upper air steady

with no increase in velocity with height; (*b*) wind in the upper air increasing sometimes to several times the gradient value but remaining more or less steady in direction; (*c*) wind in the upper air decreasing in velocity; (*d*) reversals or great changes in wind direction in the upper air; (*e*) wind in the upper air blowing away from centers of low pressure. Class (*e*) is subdivided into (e_1) upper winds between west and north, and (e_2) upper winds between south and west. Under the heading "The Wind in the Stratosphere" are discussed 12 soundings in which the balloons were observed to pass through the base of the "stratosphere." Eleven soundings are unclassified. Type (*b*) was observed 47 times, (*e*) 37, (*d*) 34, (*a*) 32, and (*c*) 27.

The classification used has the merit of being suggested by the data themselves. It is indefinite, as noted on p. 35, in that the height to which a sounding extends has possibly too much to do with its classification, e.g., if the sounding of April 13, 1907, had extended to 3.5 km only, instead of to 6, it might have been put into class (*a*) instead of class (*b*). This leads to the remark that one might read the book with a better understanding if the stratum referred to as the "upper air" and frequently used in defining the types of structure of the atmosphere had itself been clearly defined in chap. i or in the introduction.

One concludes in reading the book that the limits of this stratum vary with the height of the observation, and to a considerable extent with the type of structure. Sixty-five per cent of all observations fall below 4.4 km. In the different types 53, 79, 89, 68, and 51 per cent, respectively, of the observations fall below this average height of all soundings.

In view of these percentages, a classification based on the behavior of the wind in the first 5 or 6 km, suggested on p. 69, would not be so generally useful in these data as one based on the behavior of the wind in the first 3 or 4 km. In a continental climate, such a classification as the one used would probably be fairly definite if the lower 3 to 5 km of the atmosphere, depending on the season, were considered. This height could apparently be reduced by 1 to $1\frac{1}{2}$ km for insular climates.

The classification used has the further merit of bearing a fairly definite relation to surface pressure and temperature distribution. In the careful, systematic discussion of these relations in connection with the summaries of data and results in chaps. v and ix, a number of conclusions of much importance to meteorology are reached. The types of structure (*d*) and (*e*) appear of especial interest to the forecaster as well as to the dynamic meteorologist.

In type (*d*) the upper current is usually southerly and potentially, if not actually, warmer than the surface current, usually northerly—a condition nearly always accompanied by rain, frequently by thunderstorms. In the discussion of this type the direct statement of the conclusion that "the old idea of one current flowing close over another, and producing waves . . . . receives no support . . . . from actual observations" is a welcome one. This conclusion has been reached by other students of data, but the "old idea" still persists and is frequently used in the explanation of some cloud and other phenomena.

The conclusion pointed to by the observations of type (*e*), i.e., that a low-pressure area follows the direction of an upper wind blowing away from its center, is an important one from both the practical and the theoretical points of view. The same conclusion is indicated by the data of other observatories. The suggestion of a possible way in which the ascending air in the low-pressure area finds its way to the surface again in the high-pressure area is an interesting one.

Every student of dynamic meteorology sees the importance of free air observations to as great altitudes as may be reached; but, in the above and in other conclusions indicated in this "Study of Soundings with Pilot Balloons," are practical suggestions which should commend regular upper-air observation to the various weather services as a valuable aid to the forecaster in his work.

Chaps. ii, iii, and iv are devoted to discussions of methods of observing, the accuracy and checking of the data obtained, and the rate of ascent of rubber balloons—matters of great interest to the experimental meteorologist, possibly of less to the general reader.

Changes in the wind during the day and during consecutive days are considered in chap. vi. The wind in the "stratosphere" and the wind near the earth's surface receive attention in chaps. vii and viii, respectively. It is shown that in nearly two-thirds of the soundings the relation between the increase in velocity and the altitude up to 1 km is linear. Light variable winds are found in the "stratosphere," while in the region immediately below it are found the highest wind velocities observed.

In chap. vii or in an additional chapter the wind observations between the 1- and about the 4-km levels might profitably have been considered. The data from the soundings show a peculiarity in the lower as well as in the upper region of inverted temperatures or "stratosphere." All soundings considered, 87 per cent show a decrease in velocity with altitude somewhere between 1 and 4 km, 4 per cent show neither increase nor decrease for one or more kilometers in the same region, and only

9 per cent show an increase in velocity with altitude at all levels explored. Of those soundings that do not show a decrease in wind velocity with altitude, none reaches the 4-km level. Considering all soundings in the type of structure (*b*) that reach 5 km, it is found that in every case there is, somewhere between the 1- and 4-km levels, a decrease in wind velocity with altitude. The means of the wind velocities at the different levels in these soundings show no increase in velocity with altitude between the 1- and 2-km levels, a slight increase between the 2- and 2.5-km levels, and then no increase until the 4-km level is reached. This type of structure from its definition is the one least likely to show the phenomenon in question. These changes in wind velocity in the lower region of inverted temperatures are usually accompanied by greater or less changes in wind direction and also by changes in the other meteorological elements. A more or less regular increase in wind velocity with altitude is usually found between these two regions in which inversions of temperature are most frequently observed.

The discussion of the data is aptly illustrated with 47 figures and the tabular data themselves are supplemented with 35 pairs of diagrams showing the relations of wind velocity and direction to height in selected, typical soundings. These diagrams are accompanied by weather charts showing surface conditions.

The task of collecting these data and preparing them for publication comprises experimental work in the field in all seasons; the solution of some 8,000 triangles in connection with the reduction of the observations; the study of the data, their arrangement, illustration, and discussion; and finally the more mechanical work of proofreading and printing. In reading the book one is convinced that this task has been carefully performed and that the conclusions reached are well founded.

William R. Blair

Mt. Weather Observatory
Bluemont, Va.

Lehrbuch der sphärischen Astronomie. By L. de Ball. Leipzig: Wilhelm Engelmann, 1912. 4to, pp. xv+387. \$5.00.

According to the introduction, this work is intended, on the one hand, to serve as a book of reference for practical astronomers, and, on the other, to give to students the possibility of acquiring the knowledge absolutely necessary for a successful practical solution of the problems which arise in the realm of spherical astronomy.

As a prerequisite for the reading of the book the author mentions a

knowledge of the elements of analytical geometry, differential and integral calculus, and analytical mechanics. Series, interpolations, least squares, spherical trigonometry, and co-ordinates are treated in the first four chapters of the book. The fifth chapter contains a long and comprehensive treatment of the rotation of the earth. This is followed by chapters dealing with the old familiar subjects precession, nutation, aberration, refraction, parallax, proper motion, occultations, eclipses, etc.

A chapter on the formation of a fundamental star catalogue was evidently introduced as an afterthought, as it is numbered XIV*a*.

Brief sections are devoted to star streams and to the proper motion of our solar system.

Photographic methods and the fundamental equations connected therewith are utterly ignored.

In the purely mathematical part of the book a few numerical examples are given, but in the astronomical chapters illustrative examples are wholly lacking. The book contains comparatively little that is new, either in the line of material or in the method of presentation.

FREDERICK SLOCUM

Stereoskopbilder vom Sternhimmel. 1. Serie. Von PROFESSOR MAX WOLF. 4. Auflage. Leipzig: Verlag von Johann Ambrosius Barth. M. 5.

With the exception of pictures of the sun and moon astronomical photographs always look flat. From the nature of the case this must be so for the distances are so great that we can have no effect of perspective. But by properly combining photographs taken at suitable intervals of time the stereoscope introduces a perspective and gives an apparent solidity that must be charming to everyone who is interested in celestial objects.

This series of twelve pictures by Professor Max Wolf are very satisfactory both in choice of subject and in execution. They are as follows: (1) "A Variable Star"; (2) "A Planet with Moons"; (3) "A Planetoid"; (4) "A Meteor"; (5) "Perrine's Comet"; (6) "Perrine's Comet"; (7) "Perrine's Comet"; (8) "Proper Motion of a Fixed Star"; (9) "The Nebula of *Andromeda*"; (10) "The Great Nebula of *Orion*"; (11) "Surface of the Moon"; (12) "Surface of the Moon." The picture of *Saturn*, showing two of its moons, is particularly interesting.

We would suggest that these pictures might well find their way into the classroom, as few students will fail to be delighted with them.

W. D. MACMILLAN

Rocks and Their Origins. By GRENVILLE A. J. COLE. Cambridge: Cambridge University Press; New York: Putnam, 1912. 12mo, pp. vi+175. Figs. 20. $0.40.

This work, published as one of the long series of "Cambridge Manuals of Science and Literature," is a really excellent little book. Though a geological subject, it is intended primarily for those who are not specialists in geology, and as such necessarily presents the subject-matter in a simple, direct manner. But while declaredly a brief non-technical discussion, addressed to a general scientific audience, the character and treatment of the material selected are such that the more specialized student feels instinctively the complete command of the subject which alone makes possible such a succinct statement. There is so much meat condensed in these brief paragraphs that anyone interested in the subject finds himself continually calling for more and wishing that the treatment were fuller.

R. T. C.

The Work of Rain and Rivers. By T. G. BONNEY. Cambridge: Cambridge University Press; New York: Putnam, 1912. Pp. 144. Figs. 19. $0.40.

Of the various physiographic agents which fashion the face of the land, the most potent and universal is running water. How effective it is as a sculpturing agent is well told in this little volume. In five very readable and instructive essays the author has pictured some of the simpler phases of stream erosion, not by discussing the processes in text-book style, nor by elaborating principles primarily, but by describing and interpreting numerous concrete illustrations. The charm of the book lies in the abundant use of interesting examples from various parts of the world, though perhaps most largely drawn from the Alps and the British Isles. The last chapter, entitled, "Learning the Lesson," relates in graphic style how many centuries of philosophers came and went before the work of rivers came to be intelligently understood.

R. T. C.

NOTICE

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention is given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

Articles written in any language may be accepted for publication, but unless a wish to the contrary is expressed by the author, they usually will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right unless the author requests that the reverse procedure be followed.

Accuracy in the proof is gained by having manuscripts typewritten, provided the author carefully examines the sheets and eliminates any errors introduced by the stenographer. It is suggested that the author should retain a carbon or tissue copy of the manuscript, as it is generally necessary to keep the original manuscript at the editorial office until the article is printed.

All drawings should be carefully made with India ink on stiff paper, usually each on a separate sheet, on about double the scale of the engraving desired. Lettering of diagrams will be done in type around the margins of the cut where feasible. Otherwise printed letters should be put in lightly with pencil, to be later impressed with type at the editorial office, or should be pasted on the drawing where required.

Where unusual expense is involved in the publication of an article, either on account of length, tabular matter, or illustrations, arrangements are made whereby the expense is shared by the author or by the institution he represents.

Authors will please carefully follow the style of this JOURNAL in regard to footnotes and references to journals and society publications.

Authors are particularly requested to employ uniformly the metric units of length and mass; the English equivalents may be added if desired.

If a request is sent *with the manuscript*, one hundred reprint copies of each paper, bound in covers, will be furnished free of charge to the author. Additional copies may be obtained at cost price. No reprints can be sent unless a request for them is received before the JOURNAL goes to press.

The editors do not hold themselves responsible for opinions expressed by contributors.

The ASTROPHYSICAL JOURNAL is published during each month except February and August. The annual subscription price is $5.00; postage on foreign subscriptions, 62 cents additional. Business communications should be addressed to the *University of Chicago Press, Chicago, Ill.*

All papers for publication and correspondence relating to contributions should be addressed to *Editors of the* ASTROPHYSICAL JOURNAL, the *University of Chicago, Chicago, Illinois, U.S.A.*

3o6

THE

ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME XXXVII — JUNE 1913 — NUMBER 5

STUDIES OF THE NOCTURNAL RADIATION TO SPACE

By ANDERS ÅNGSTRÖM

I. HISTORICAL SURVEY

The first observations relating to the problem of the earth's radiation to space are due to Wilson,[1] Wells,[2] Six,[3] Pouillet,[4] and Melloni,[5] the observations falling between the years 1780 and 1850. These observers have investigated the nocturnal cooling of bodies exposed to the sky, a cooling that evidently is not only due to radiation but is also influenced by conduction and convection of heat through the atmosphere. Melloni draws the conclusion from his experiments that this cooling is for the most part due to the radiation of heat to space.

In the year 1885, J. Maurer[6] published a paper dealing with the cooling and radiation of the atmosphere during the night. From thermometrical observations of the atmosphere's cooling, Maurer deduces a value $\sigma=0.007 \cdot 10^{-4}$ (cm³ min.) for the radiation coefficient of the air and from this a value for the radiation of the whole atmosphere: $0.39 \frac{\text{cal.}}{\text{cm}^2 \text{ min.}}$ at 0°. This value is obtained

[1] *Edinburgh Phil. Trans.*, 1, 153.

[2] *Ann. de chimie et de physique*, 3, Ger. Vol. 5.

[3] Six, *Posthumous Works*. Canterbury, 1794.

[4] Pouillet, *Eléments de physique*, 1844.

[5] *Annales de chimie et de physique*, 1848.

[6] *Met. Zeitschrift*, 1886.

on the assumption that the atmosphere is homogeneous, having a height of $8 \cdot 10^5$ cm and by application of the formula

$$R = \frac{S}{a}(1 - e^{-ax}) ,$$

where a is the absorption coefficient deduced from the radiation coefficient by the application of Kirchhoff's law. I will make some remarks on the theoretical foundation of Maurer's value for the atmospheric radiation in the theoretical part of this paper. In the year 1887, Maurer[1] made measurements on the nocturnal radiation to space from a blackened body and obtained a value of 0.13 $\frac{\text{cal.}}{\text{cm}^2 \text{ min.}}$ for this radiation (June; temp. 15°–18°).

Pernter[2] has measured the nocturnal radiation at Sonnblick (alt. 3095 m) and simultaneously at Rauris (alt. 900 m) and obtained the values 0.201 and 0.151 respectively. Pernter made his observations with an actinometer of the Violle type.

In the year 1897, Homén[3] published an important paper bearing the title, *Der tägliche Wärmeumsatz im Boden und die Wärmestrahlung zwischen Himmel und Erde.* He observed the radiation with an Ångström actinometer, constructed for measuring the insolation but here modified for the purpose of measuring the radiation from earth (irradiation). Homén draws from his observations on the radiation between earth and space the following conclusions:

1. If the sky is clear, there will always be a positive radiation from earth to space, even in the middle of the day.

2. If the sky is cloudy, there will always in the daytime be a radiation from sky to earth.

3. In the night-time the radiation for clear as well as for cloudy sky always has the direction, from earth to sky. Homén made also some measurements on the radiation to different parts of the sky and found that this radiation decreases rapidly when the zenith

[1] *Sitzungsber. d. Berl. Akad.*, 1887.

[2] *Sitzungsber. d. Wiener Akad.*, **47**, 1562, 1888.

[3] Homén, *Der tägliche Wärmeumsatz*, etc. Leipzig, 1897.

angle approaches the value of 90°. His values on the nocturnal radiation for clear sky vary between 0.13 and 0.22 $\frac{\text{cal.}}{\text{cm}^2 \text{ min.}}$.

Exner[1] (1903) made observations on the radiation to space at Sonnblick (alt. 3106 m) and used for this purpose a modified compensation pyrheliometer of K. Ångström's type. The radiation was measured for a limited part of the sky and a correction applied with regard to the distribution of radiation to the different zones found by Homén. Exner found a mean value for the radiation of 0.21 and in agreement with former investigations of Maurer and Homén a relatively constant value for the radiation during the night. He observed a slight maximum in the radiation, one to two hours before sunrise.

In the year 1905, K. Ångström[2] published a description of a new instrument for determining the nocturnal radiation to space. This instrument is founded upon the principle of electric compensation, and as it has been used in the work here published, I will in the following give a short description with some remarks on the advantages and sources of error in the use of this instrument. K. Ångström found a radiation varying between 0.13 and 0.18 for clear sky, the observations being made at Upsala between May 22 and November 7, 1904.

The latest contribution to the question we are dealing with, which I have been able to find, is a paper in *Nuovo Cimento* by A. Lo Surdo.[3] Lo Surdo gives a critical survey of the methods that have been used and discusses some observations of his own, made at Naples in September 1908 with the Ångström nocturnal compensation instrument. Contrary to Homén, he finds even when the sky is clear a positive radiation from sky to earth in the daytime. He observes the variation in the radiation during a clear and specially favorable night and finds a pronounced maximum about two hours before sunrise. Such a maximum had already been observed by Exner on the top of Sonnblick under different

[1] *Met. Zeitschrift* (1903), p. 409.

[2] *Nova Acta Reg. Soc. Scient. Upsal.*, Ser. IV, Vol. 1, No. 2.

[3] *Nuovo Cimento*, 1908.

atmospheric conditions. The following table gives a condensed survey of the results obtained by different observers:

Observer	Date	Place	Temperature	Height	Mean Value
Maurer..........	June 13–18, 1887	Zürich	15°–18°	500	0.128
Pernter..........	Feb. 29, 1888	Sonnblick	8°	3095	.201
Pernter..........	Feb. 29, 1888	Rauris	......	900	.151
Homén..........	Aug. 1896	Lojosee	......	....	.17
Exner...........	1902	Sonnblick	......	3106	.19
Exner...........	July 1, 1902	Sonnblick	......	3106	.268 (max.)
K. Ångström	May–Nov. 1904	Upsala	0°–10°	200	.155
Lo Surdo........	Sept. 5–6, 1908	Naples	20°–30°	30	.182
A. Ångström.....	July 10–Sept. 10, 1912	Algeria	20°	1160	0.174

The problem of the radiation from earth to space is comparable in importance to the insolation problem for the knowledge of the climatic conditions at a certain place. While the insolation problem has been subjected to a very thorough study, so that we know relatively well the dependence of the insolation on different atmospheric conditions, this cannot be said of the irradiation, of whose dependence on other meteorological conditions we are able to draw very few conclusions from the more sporadic observations made.

What seems to be wanted here are continuous and systematic investigations made simultaneously at different places, the observations so made put in connection with the *height above sea-level, the temperature of the earth and the air, and the composition of the air in regard to water-vapor, ozone, and other constituents.*

The observations here published, made at Bassour, Algeria, at a height of 1160 m, may be regarded as a preliminary contribution to the complicated question of the influence of the constitution of the air on the radiation to space.

That it has been possible to draw some more general conclusions in spite of the short time (July 10–September 10), under which these observations have been carried out, may be due largely to the excellent atmospheric conditions, which permitted me to make observations almost every night under clear sky.

In this first paper I will give an account of some measurements showing *the influence of the water-vapor on the nocturnal radiation;*

in a following paper some observations at different heights and for different zenith angles will be communicated.

To the Smithsonian Institution, which has favored my investigations through a grant from the Hodgkins Fund, and to Mr. C. G. Abbot, director of the Astrophysical Observatory at Washington, in whose solar expedition I have had the honor to take part, and whose experienced counsels have been of great value to this work, I wish to express my most sincere thanks.

II. THEORETICAL CONSIDERATIONS

I shall in the following use the term "effective radiation" for the radiation determined by the loss of heat through radiation from unit area of black surface during one minute. In the case of the nocturnal radiation this effective radiation must be regarded as a sum of several terms: (1) the radiation from the surface to space (E_c), for a black body determined by Stefan's radiation law; (2) the radiation from the atmosphere to the surface (E_a), to which comes the sum of the radiations from planetary bodies (E_s), a radiation source that by Poisson is indicated by the term "sidereal heat." If I is the effective radiation we shall evidently have:

$$I = E_c - E_a - E_s .$$

For the special case where the temperature of the surface is constant and the same is assumed to be the case for the sidereal radiation, we can write

$$I = K - E_a ,$$

where K is a constant. Under these circumstances the variations in the effective radiation are dependent on the atmosphere's radiation only, and the problem is identical with the problem of the radiation from a gaseous body that here is a mixture of several different components.

As is well known from thorough investigations, a gaseous body has no continuous spectrum, but is characterized by a selective radiation, the radiation being relatively strong at certain points of the spectrum and often inappreciable in points lying between. The law for the distribution of energy is generally very complicated

and different for different gases. The intensity is further dependent upon the thickness, density, and temperature of the radiating layer.

Suppose that we consider the intensity of the radiation for a special wave-length λ from a uniform gaseous layer of the thickness R and the temperature T toward a little elementary surface $d\tau$. We shall, to begin with, consider only the radiation that comes in from an elementary radiation cone, perpendicular to $d\tau$, which at unit distance from $d\tau$ has a cross-section equal to $d\Omega$. One can easily deduce:

$$I_\lambda = \int_0^R \epsilon_\lambda e^{-ar} dr d\Omega d\tau = \epsilon_\lambda d\Omega d\tau \int_0^R e^{-a_\lambda r} dr \,,$$

which gives

$$I_\lambda = \frac{\epsilon_\lambda}{a_\lambda} d\Omega d\tau (1 - e^{-a_\lambda R}) \,, \tag{1}$$

where ϵ_λ is the emission coefficient and a_λ the absorption coefficient for the wave-length λ.

Evidently:

$$\lim_{R=\infty} I_\lambda = \frac{\epsilon_\lambda}{a_\lambda} d\Omega d\tau = E_\lambda d\Omega d\tau \,, \tag{2}$$

where E_λ is the radiation from a black body for the wave-length λ. It follows from this that in all cases where one can assume a_λ independent of the temperature, ϵ_λ must be the same function of the temperature as E_λ multiplied by a constant, that is:

$$\epsilon_\lambda = C\lambda^{-5} \frac{1}{e^{\frac{c_1}{\lambda T}} - 1} \,.$$

If now the gas has many selective radiation bands, we may write instead of (1):

$$I = \sum \frac{\epsilon_\lambda}{a_\lambda} (1 - e^{-a_\lambda R}) d\Omega d\tau \,. \tag{3}$$

If R is taken so great that the product $a_\lambda R$ has a very great value for all wave-lengths, the expression (3) will become

$$\lim_{a_\lambda R=\infty} \Sigma E_\lambda = \rho T^4 \,, \tag{4}$$

which is Stefan's radiation law for a black body.

If $a_\lambda R$ cannot be regarded as infinitely great for all wave-lengths, the radiation I will be a more complicated function of T, expressed through the general expression (3). The less the difference is between the radiation from the gas and the radiation from a black body at the same temperature, so much more accurately will the formula (4) express the relation between radiation and temperature.

Dr. Trabert[1] draws from his observations on the nocturnal cooling of the atmosphere the conclusion that the radiation from unit mass of air is simply proportional to the absolute temperature. If this should be true, it can be explained only through a variation of a_λ with the temperature. Later Paschen[2] and Very[3] have measured in the laboratory the radiation from air layers at different temperatures and found a much more rapid increase with rising temperature.

The relation

$$I=\sum\frac{\epsilon_\lambda}{a_\lambda}(1-e^{-a_\lambda R})d\Omega$$

represents the general expression for the radiation within the radiation cone $d\Omega$ perpendicular to the unit of surface. Maurer,[4] in his calculation of the atmospheric radiation, starts with the more simple expression:

$$I=\frac{\epsilon}{a}(1-e^{-aR}),$$

where he puts R equal to the height of the reduced atmosphere and a equal to the absorption coefficient for unit volume. This is evidently an approximation that is open to criticism and must give a value of the atmospheric radiation that is too great. In the first place, it is not permissible to regard R as the height of the reduced atmosphere, and this for two reasons: first, because the radiation depends chiefly upon the existence of water-vapor and carbondioxide in the atmosphere, whose density decreases rapidly with the altitude; and secondly, because we here have to deal with a radiation that comes in from all sides, R being variable with

[1] *Denkschriften der Wien Akad.*, **59**.

[2] *Wied. Ann.*, **50**, 1893.

[3] Very, *Atmospheric Radiation*. Washington, 1900.

[4] *Loc. cit.*

the zenith angle. But even if we assign to R a mean value with regard to these circumstances, Maurer's formula will be true only for the case of one single emission band and is, for more complicated cases, incapable of giving an idea of the real conditions.

If, with Maurer, we regard the atmosphere as homogeneous and of uniform temperature, having a certain height h, we must, considering that R is a function of the zenith angle, write (1) in the following form:

$$I_\lambda = -\frac{\epsilon_\lambda}{a_\lambda}\int d\Omega\left(1-e^{-a_\lambda\frac{h}{\cos\phi}}\right)\cos\phi ,$$

where the integration shall be taken over the hemisphere representing the space. Now we have:

$$d\Omega = d\phi d\psi \sin\phi ,$$

and therefore:

$$I_\lambda = -\frac{\epsilon_\lambda}{a_\lambda}\int_0^{2\pi} d\psi \int_0^{\frac{\pi}{2}}\left(1-e^{-a_\lambda\frac{h}{\cos\phi}}\right)\sin\phi\cos\phi\, d\phi .$$

This expression can easily be transformed to

$$I_\lambda = \frac{\epsilon_\lambda}{a_\lambda}\left[\pi - \rho^2\int_\rho^\infty \frac{e^{-x}}{x^3}dx\int_0^{2\pi} d\psi\right],$$

where $\rho = a_\lambda h$. As $h \doteq 0$, this expression approaches zero, when $h \doteq \infty$, I_λ approaches the value $\pi\frac{\epsilon_\lambda}{a_\lambda}$ which is equal to the radiation of a black body at the same temperature. We have in fact:

$$\lim_{\rho=\infty}\rho^2\int_\rho^\infty \frac{c^{-x}}{x^3}dx = \lim_{\rho=\infty}\frac{\frac{c^{-\rho}}{\rho^3}}{\frac{1}{2}\cdot\frac{1}{\rho^3}} = \lim_{\rho=\infty}\frac{c^{-\rho}}{2} = 0 ,$$

and in the same way:

$$\lim_{\rho=0}\rho^2\int_\rho^\infty \frac{c^{-x}}{x^3}dx = \tfrac{1}{2} .$$

I have found, as will be seen from the experimental results, that the influence of the water-vapor pressure (p) on the atmospheric radiation at constant temperature can, between certain limits of p, be expressed by a formula:

$$E_a = K - Ce^{-\gamma p} ,$$

where K, C, and γ are constants.

This relation can be deduced from (3) if we make certain assumptions in regard to the absorption of the gaseous layer. If we collect all terms in the expression (3) for which R is so great that a further increase in R will alter the radiation for the considered wave-length only by a negligible quantity, and for the other terms make the approximation:

$$\Sigma_e^{-a_\lambda R} = e^{-a_m R_m} ,$$

we may write the radiation formula in the form:

$$E_a = K - Ce^{-a_m R_m} ,$$

where R_m may be taken, not as the height of the considered homogeneous atmosphere, but as a mean value in regard to the radiation from all sides. Under the assumption that only the water-vapor pressure is varying and that the effect of the radiation of other atmospheric constituents as carbondioxide, ozone, etc., is included in the constant K, we may put R proportional to the water-vapor pressure at the earth's surface.[1] We arrive in this way at the above formula:

$$E_a = K - Ce^{-\gamma p} .$$

For the effective radiation to space this gives:

$$I = K_1 + Ce^{-\gamma p} .$$

This expression, founded as it is on assumptions that are approximately true only within certain limits ($p_1 < p < p_2$), is, however, in its general form in good agreement with the observations that I have made and that are described in a later part of this paper. That the formula in its relatively simple form can be applied to the atmospheric radiation as a function of the water-vapor pressure has its explanation in the form of the radiation-curve of water-vapor, as derived from the investigations of Rubens, Aschkinass

[1] Hann, *Meteorologie*, pp. 224–226.

and others on its absorption.[1] It may be seen from these observations that the radiation from the atmosphere, probably for all wave-lengths excluding those between the limits 8 μ and 11 μ, is almost like that of a black body. This circumstance explains why the radiation may be treated as if the variations were due to one emission band only, the radiation between 8 μ and 11 μ being the only part which is dependent on the thickness of the radiating layer (for $R > R_0$). I may from this elementary theoretical consideration pass to a description of the observations and the apparatus used.

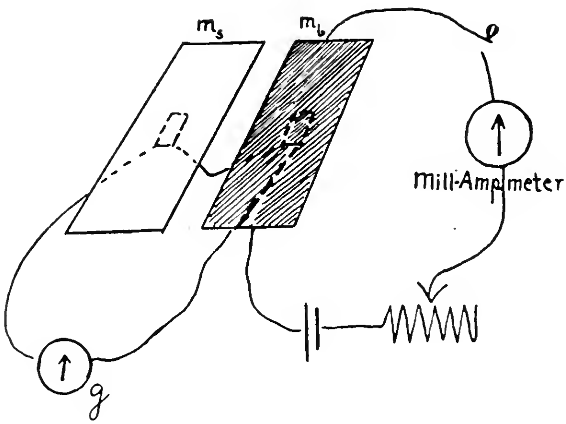


FIG. 1

III. INSTRUMENTS

I used for the following observations two nocturnal compensation instruments of the type described by K. Ångström[2] in a paper in 1905. Without going into details for which I refer to the original paper, it may be advantageous to give here a short description of the instrument.

Founded on the same principle of electric compensation used in the Ångström pyrheliometer, the instrument has the general form indicated in Fig. 1. M_S and M_C are two thin metal strips, of which one is blackened, the other bright. On the backs of the

[1] See also Very, *Atmospheric Radiation*, p. 122.

[2] *Nova Acta Reg. Soc. Scient. Upsal.*, Ser. IV, Vol. 1, No. 2.

metal strips are fastened the two contact points of a thermo-junction, connected with a sensitive galvanometer g. If the strips are shadowed by a screen of uniform temperature, the thermo-junctions have approximately the same temperature and we can read a certain zero position on the galvanometer g. If the screen is removed and the strips are exposed to the sky, a radiation will take place, which is greater for the black strip than for the bright one, and there will be a deflection on the galvanometer due to the temperature difference between the two strips. In order to regain the zero position of the galvanometer, we may produce the heat lost through radiation by sending an electric current through the black strip. Theoretical considerations, as well as experiments made, show that the radiation is proportional to the square of the current used, that is,

$$I = ki^2,$$

where k is a constant depending upon the dimensions, resistances, and radiating power of the strips. As the radiating power of the strips is difficult to compute, the constant k is determined from experiment with a known radiation. The strips are exposed to radiate to a half-sphere of known temperature T, and the constant is determined by the relation:

$$ki^2 = \sigma(T^4 - T_1^4).$$

The advantage of this construction over the form used, for instance by Exner and Homén, where the effect of conduction and convection also are eliminated, lies in the possibility of measuring the radiation to the whole sky and not only a limited part, which is the case when one of the strips must be shadowed. It must always be regarded as a dangerous approximation to calculate the radiation to the whole sky from the radiation to a limited part, assuming a certain standard distribution of radiation.

On the other hand, the value k here is dependent on the accuracy with which the radiation constant σ is determined. Further, since the emissive power of the strips, which is different for different wave-lengths, enters into the constant k, this constant can be applied only for cases where the radiation is approximately of the same wave-length as in the experiment from which k is

computed. In the night-time, this may be considered the case. But the instrument cannot without further arrangement be used for determining the radiation during the day, where the diffused radiation from the sky of short wave-length enters as an important factor.

The constants of my instruments, made by G. Rose, Upsala, were determined at the Physical Institute at Upsala. In order that all the instruments that are sent out may have constants founded on the same value of σ, Kurlbaum's value has been used. In my case, where the variations of the radiation are of the greatest interest, the absolute value of σ is not of such great importance. It may, however, be remarked that since the values for σ, obtained by later investigators, are somewhat larger than Kurlbaum's value, the observations here given are probably too small.

IV. OBSERVATIONS

The observations given in this part were made at Bassour, Algeria, July 10–September 10, 1912, at a height of 1160 m above sea-level.

In Table I are given the effective radiation R, the date, the time of day, the humidity, the barometric pressure B, and the temperature t. The temperature fall between the time of radiation observation in the evening and the time of sunrise is indicated by Δt.

The observations of Table I are now put together in Table II in such a way that the mean value of the determinations, when the pressure of the water-vapor falls between two certain limits indicated in the table, is calculated (R_m): these mean values are plotted in a curve, Fig. 2, which gives the probable relation between water-vapor pressure and radiation. Tables I and II show that the temperature of the radiating surface has been almost constant for the different series and ought not, therefore, to have had any influence upon the variation in the magnitude of the radiation.

The value of a single observation often deviates from the supposed curve by several per cent. This should not astonish us.

The observations of Abbot and Fowle[1] show that the amount of water-vapor contained in the air can be only roughly estimated from the water-vapor pressure at the earth's surface. Since the

TABLE I

Date	Time	B	Temperature	Δt	p	R
July 10.....	$7^h 40^m$	664.4	19.1	...	3.86	0.191
11.....	7 40	663.6	24.1	...	9.42	.156
12.....	7 45	662.9	25.4	...	6.60	.171
18.....	8 30	663.1	20.1	1.8	9.32	.166
19.....	8 10	662.6	23.3	6.3	8.54	.163
20.....	8 0	661.9	21.5	6.4	7.08	.166
22.....	9 30	664.0	17.2	0.6	5.66	.211
23.....	9 35	663.5	20.0	5.6	7.80	.169
24.....	8 25	.....	19.5	5.7	8.36	.159
25.....	8 35	664.9	18.8	−0.5	8.25	.138
29.....	8 35	665.1	18.0	1.8	9.16	.139
30.....	10 25	666.7	21.0	3.4	7.14	.187
31.....	8 35	664.7	22.6	...	4.14	.169
Aug. 1.....	9 45	662.3	23.8	4.2	4.40	.201
2.....	8 55	662.9	20.3	2.4	7.54	.171
3.....	9 05	.....	24.2	...	8.96	.173
4.....	8 50	663.5	21.2	3.2	6.60	.175
5.....	7 55	663.2	21.4	3.7	9.88	.162
6.....	8 50	.....	23.6	3.3	5.89	.173
10.....	8 50	665.7	25.0	3.3	9.98	.178
11.....	8 20	666.9	22.8	2.7	10.20	.158
13.....	9 0	662.7	19.5	1.5	8.86	.171
14.....	10 0	662.6	18.6	0.0	11.90	.147
15.....	8 30	665.4	20.6	−1.4	8.61	.179
20.....	10 10	667.7	18.9	1.7	13.24	.145
21.....	8 0	669.8	20.8	4.6	6.45	.201
22.....	8 40	667.9	17.9	2.7	7.44	.173
23.....	9 0	665.7	20.8	0.5	3.84	.192
24.....	8 45	663.4	22.0	3.2	5.46	.175
26.....	8 45	.....	21.5	...	3.80	.217
27.....	9 5	.....	21.5	...	8.48	.188
29.....	8 50	665.1	24.4	...	8.36	.190
30.....	9 15	665.6	20.3	4.4	7.10	.157
Sept. 3	8 35	664.3	13.8	4.2	10.40	.138
4.....	8 5	666.7	11.1	...	4.98	.169
5.....	9 50	664.0	20.8	2.1	4.57	.205
6.....	9 30	661.5	20.0	2.4	3 99	.220
8.....	9 0	666.7	15.7	−1.0	6.80	0.177

mechanical conditions of the atmosphere are complicated on account of many disturbances, we cannot expect a general rule without considerable deviations. As a result of interest, it may here be mentioned that my comparison of the radiation measure-

[1] *Annals of the Astrophysical Observatory of the Smithsonian Institution*, **2**, 131, 1908.

ments with regard to the corresponding transmission power of the sky, for visible rays, as given by pyrheliometer measurements with colored screen, has shown me that the influence of the "haziness" upon the nocturnal radiation is negligible in comparison with

TABLE II

p	3.50–4.50			4.50–5.50			5.50–6.50		
	t	p	R	t	p	R	t	p	R
	19.1	3.86	0.191	22.0	5.46	0.175	17.2	5.66	0.211
	22.6	4.14	.169	11.1	4.98	0.169	23.6	5.89	0.173
	23.8	4.40	.201	20.8	4.57	0.205	20.8	6.45	0.201
	20.8	3.84	.192	....	....	....	....	....	....
	21.5	3.80	.217	....	....	....	....	....	....
	20.0	3.99	0.220	....	....	....	....	....	....
Means........	21.3	4.00	**0.198**	18.0	5.00	**0.183**	20.5	6.00	**0.195**

p	6.50–7.50			7.50–8.50			8.50–9.50		
	t	p	R	t	p	R	t	p	R
	25.4	6.60	0.171	20.0	7.80	0.169	24.1	9.42	0.156
	21.5	7.08	.166	19.5	8.36	.159	20.1	9.32	.166
	21.0	7.14	.187	18.8	8.25	.138	23.3	8.54	.163
	21.2	6.60	.175	20.3	7.54	.171	18.0	9.16	.139
	17.9	7.44	.173	21.5	8.48	.188	24.2	8.96	.173
	20.3	7.10	.157	24.4	8.36	0.190	19.5	8.86	.171
	15.7	6.80	0.177	....	....	....	20.6	8.61	0.179
Means........	20.4	6.98	**0.173**	20.7	8.13	**0.169**	21.4	8.98	**0.164**

p	9.50–10.50			11.90–13.24					
	t	p	R	t	p	R			
	21.4	9.88	0.162	18.6	11.90	0.147	....	....	....
	25.0	9.98	.178	18.9	13.24	0.145	....	....	....
	22.8	10.20	.158	....	....	....	....	....	....
	138.	10.40	0.138	....	....	....	....	....	....
Means........	20.8	10.12	**0.159**	18.8	12.57	**0.146**			

other influences. The mean value of the irradiation corresponding to great haziness differs only within the limits of error (1 per cent) from the mean value corresponding to a more clear sky. As long as the diffusing particles are small and no clouds

are formed, the diffusing particles seem not to have the faculty to absorb or radiate the long waves, with which we here are dealing. This opinion is supported also by the statement of Very,[1] that "The experiments with dust-laden air have indicated that the addition of a small amount of solid matter, diffused through a large volume of air, does not change the radiating power of the latter

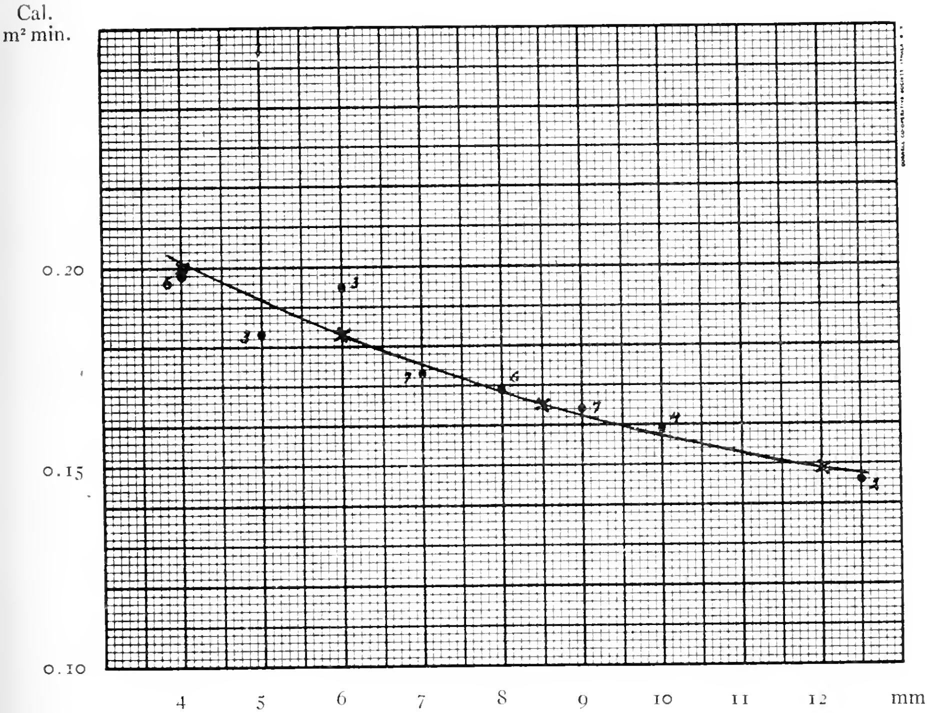


FIG. 2

. . . = observed curve; XXX = calculated curve. Numbers indicate numbers of observations for every mean value.

perceptibly." The results of these experiments are in full agreement with my observations of the radiation from the sky.

The experimental curve is well satisfied through the following relation:

$$R=0.109+0.134 \cdot e^{-0.10p}$$

which in its general form has been founded on theoretical considerations.

[1] Very, *Atmospheric Radiation.* Washington, 1900.

The question is now to what extent this relation, as it is obtained for a certain atmospheric pressure, a certain height above sea-level, and a certain temperature of the earth can be regarded as satisfying more general conditions. Only thorough investigations can answer this question. It may, however, be assumed that the atmospheric radiation, at least for low altitudes, is practically independent of the total atmospheric pressure. The effect of the temperature conditions of the earth and the atmosphere on the radiation may be much larger. Our first step, if we wished to take account of the influence of the temperature, should be to substitute for p the density of the water-vapor as a function of the temperature, and to express the radiation from the atmosphere as a whole by a law similar to Stefan's radiation law.

From the result obtained, we can draw no conclusions about the radiation for the case when p is less than about 4 mm. If the pressure is less than 4 mm, many of the terms that we have regarded as constant in the expression (3) will begin to vary with the density of the radiating layer and hence the curve (Fig. 2) will increase much more rapidly for low pressures than would be expected from the behavior of the curve in the investigated interval.

For the case when p approaches very great values, the formula seems to indicate that the radiation approaches a value of about $0.11 \frac{\text{cal.}}{\text{cm}^2\ \text{min.}}$, which shows that the water-vapor even in very thick layers is almost transparent for certain wave-lengths. Here, however, we must consider that the conditions are complicated through the temperature fall in the radiating atmospheric layer. In great concentration of the water-vapor the temperature fall will be of little importance, the air layers that influence the radiation being relatively thin and lying near the earth's surface. If the concentration is small, however, the radiation from the earth will penetrate to colder air layers, or, what is the same, the radiation from colder layers will penetrate to the earth, and the assumption that the atmosphere has a constant temperature will be more or less incorrect.

SUMMARY

My observations have shown:

1. That the influence of the density of the water-vapor on the atmospheric radiation can, between certain limits, be expressed through an exponential formula.

A change in the water-vapor pressure from 12 to 4 mm will, under the conditions I have defined in the paper, increase the mean radiation from the earth's surface about 35 per cent. On the other hand, the absorption power for sun radiation will only vary between about 5 and 15 per cent for the mentioned pressure-interval. The importance of the humidity for the climate is clear from this consideration and from the fact that the loss of heat from the earth's surface is largely due to radiation.

2. That a slight haziness, as indicated by changes in the transmissive power of the atmosphere for visible rays (clouds not formed), seems to have no appreciable influence upon the radiation of the atmosphere.

CORNELL UNIVERSITY

December 1912

RADIAL MOTION IN SUN-SPOTS[1]

I. THE DISTRIBUTION OF VELOCITIES IN THE SOLAR VORTEX

By CHARLES E. ST. JOHN

In the spring of 1909 Mr. Evershed announced the remarkable discovery of the displacement of the Fraunhofer lines in the penumbrae of sun-spots. In brief, he found that practically all the lines of the reversing layer are shifted and that the stronger lines appear less affected than the weaker. He advanced the hypothesis that the displacements are due to the radial movement outward and tangential to the solar surface of the gases of the reversing layer. From his observations he concluded that the motion is accelerated outward but ceases abruptly at the limits of the penumbrae, that the movement is confined to the reversing layer, that the gases of the higher chromosphere do not share in the movement, but, on the contrary, show a tendency to move inward, and that the maximum velocity indicated by the majority of the lines is 1.57 km per second.[2]

In the summer of 1910 I began an extended investigation of the subject, using the spectrograph of the 60-foot tower telescope. By means of a simple occulting device, the spectra of the edges of the penumbrae directed toward the center and the limb of the sun, respectively, were obtained juxtaposed upon the photographic plate; and in what follows these edges will be referred to simply as the center and limb edges. The two diagrams show the details of the arrangement and the manner of using it. In position 1 of the occulting device the limb edge of the penumbra is tangent to the edge of the occulting bar at the point directly over the slit (*a*), which is parallel to the radius of the solar image passing through the spot. After making an exposure, the occulting bar is moved until the fiducial line (*c*) is directly over the slit, and

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 69.

[2] *Kodaikanal Observatory Bulletin*, No. XV; *Kodaikanal Observatory Memoirs*, 1, Pt. 1.

the image of the sun is moved parallel to the slit until the center edge of the penumbra is tangent to the central line at the point (*d*). The occulting bar is then moved into position 2, and the second exposure made. In the procedure described, the spectrum of the spot was not photographed, but by placing the center edge of the penumbra tangent to the edge of the occulting bar in its first position and by a similar change in the second position, the spectra of the limb edge and the center edge of the penumbra are side by side along the center of the plate with a spectrum of the spot on either side. This has the advantage that the distance of the point

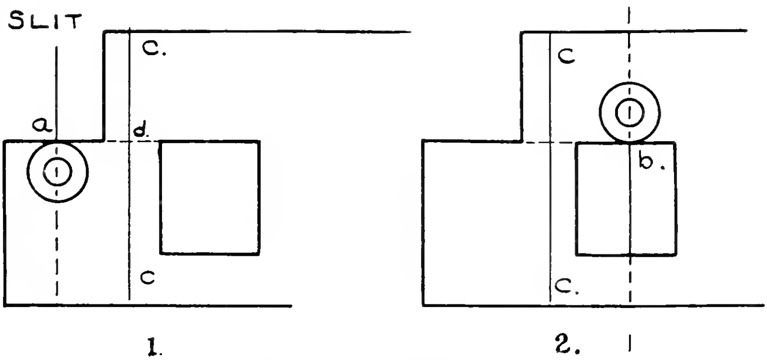


FIG. 1.—Occul ing device

measured from the center of the spot can be determined, and the whole course of the displaced line can be examined; but for purposes of measurement, the first plan was found more satisfactory. The displacement was doubled in either case, as on one edge of the penumbra the lines are shifted to the red, and on the other to the violet. The positions of the spots on a solar image 170 mm in diameter varied from 10 to 45 mm from the limb. The usual course of the line on such plates shows no sharp break and the displacement does not suddenly cease at the periphery of the penumbra, but the line gradually returns to its normal course. This differs from Mr. Evershed's observation, and seems to remove one great difficulty in explaining the displacement as due to motion.

There was a question whether the slight shifting of the solar image would not of itself introduce displacements caused by the change in the illumination of the grating. This was tested by

measurements of the atmospheric lines wherever they appeared upon the plate, and the absence of any effect due to this cause or to instrumental displacements completely justified the method. On 54 plates the mean displacement of the atmospheric lines was +0.0003 Å.

For measurement, the plate was first adjusted so that the edges of the spectra where they were opposed moved parallel to the screw. The cross-hair of the microscope was fixed parallel to the normal direction of the lines by setting upon the two parts of the line on each side of the division and far enough from the penumbra to be undisturbed by the radial motion. The field of the microscope was sufficiently flat to allow of this. Measurements were made upon the points of the line that were equally distant from the horizontal cross-hair, which was central with respect to the two spectra, and these distances were equal for all lines. The plates in the violet and green were taken in the third-order spectrum and those in the yellow and the red in the second order, the dispersion in the two cases being 1 mm=0.56 Å. and 1 mm=0.86 Å. It was found that for such lines as D_1 and D_2 and H_α, the second order gained in accuracy over the third as much as it lost in dispersion, and the requisite intensity of the spectra was obtained by much shorter exposure time.

Observations were obtained upon the following regular spots which are identified by the Greenwich numbers. The diameters of the umbrae and penumbrae are given for the central meridian on an image 170 mm in diameter.

Spot No.	C.M. G.M.T.	Umbra	Penumbra	Spot No.	C.M. G.M.T.	Umbra	Penumbra
		mm	mm			mm	mm
6847..	1910 May 18.6	1.1	2.3	6942..	1911 Apr. 27.7	0.9	2.5
6864..	July 6.3	1.7	4.0	6944..	Apr. 28.7	1.1	3.0
6874..	Aug. 9.5	1.7	4.0	6945..	Apr. 28.9	0.5	1.5
6880..	Sept. 5.8	1.2	3.0	6951..	May 30.8	1.0	2.5
6886..	Oct. 26.9	0.9	2.5	6972..	Nov. 27.1	0.9	2.0
6940..	1911 Apr. 9.3	1.0	3.0				
				Mean		1.1	3.0

In laying out the program for what was expected to be a preliminary survey, it was not at first clear what lines would prove of the greatest interest. An examination of the blue region studied

by Mr. Evershed showed at once that the list should embrace a large number of lines of as diverse character as possible. The lines were selected on the following bases:

a) As many elements as possible to be included, in fact, 27 substances are represented in the list.

b) Lines known to originate in the higher levels of the chromosphere such as H, K, H_α, and the strongest lines of magnesium, aluminium, and iron.

c) Elements of exceptionally high atomic weight, and rare elements, particularly barium, lanthanum, cerium, lead, ytterbium, neodymium, and niobium.

d) Lines covering a wide range of intensities for each element.

e) A very large number of lines of all intensities of some one element represented by many lines, such as iron.

f) Lines of peculiar interest, such as the enhanced lines, and lines belonging to different pressure groups.

The program has included 506 lines which fulfil the conditions indicated in sufficient numbers to permit statistical treatment, and they are distributed as follows: violet 268, blue-green 164, yellow-red 74.

In the first three columns of Table I are the wave-lengths, identifications, and intensities, as given in Rowland's table. The fourth column contains the absolute displacements in Ångströms reduced to the limb; the fifth column, the number of plates; and the sixth column, the mean deviations. These are not a measure of the accuracy of the measurements, but include the deviation from plate to plate, which is the larger element involved. The displacements vary rapidly with the distance from the edge of the penumbra, and, with the utmost possible care in guiding, the slit cannot be set and rigorously held upon the same point with respect to the edge of the penumbra, so that plates taken in quick succession give different absolute displacements, but for each plate (45 cm long) the relative displacements of the various lines upon the plate are in the proper relation. If the position of the slit is changed by a fifth of a millimeter, the point of observation on the sun for the mean positions of the spots is shifted 2000 km toward or away from the axis of the spot, and for such changes in distance

the change in velocity is large. The seventh column under Δ' requires a word of explanation. As has just been said, the absolute values for different plates vary. If, therefore, upon any given plate it has for any reason been impossible to measure the displacement of a line appearing upon the other plates, the omission of this line from the plate under consideration might appreciably affect the final mean if the plate as a whole showed displacements differing greatly from the mean of the other plates. The values under Δ' are obtained in the following way: when the displacement of a line in the list has not been measured on a given plate because of some defect or lack of intensity or definition, an interpolated value for the displacement of the line on this plate has been used in determining the final mean. The interpolation was carried out as follows: considering only the lines common to all the plates, the ratio was found between the displacements on the given plate and the mean displacements on the other plates. This ratio is the operator by which the required displacement is found from the mean displacement of the same line on the other plates. The procedure not only eliminated the irregularities introduced by the omission of the line from one or more plates, but equalized the weights of the observations for neighboring lines. The mean values are the ones used in the discussion. As a matter of fact, the greater number of the displacements under Δ' agree with those under Δ. Changes occur in the case of 97 out of 506 lines. In 63 cases the correction is ± 0.001 Å., in 26 cases, ± 0.002 Å., and in 8 cases, ± 0.003 Å. The adjustments have a random distribution and produce no systematic effect. In the eighth column the displacements are reduced to λ 5000; and in the ninth column, under character and group, are given the grouping of the iron and titanium lines suggested by Gale and Adams,[1] with the addition of group *e* and the subdivision of group *d* suggested in a paper by Miss Ware and myself on the iron standards,[2] also the enhanced lines, and some probable blends.

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 58; *Astrophysical Journal*, **35**, 10–47, 1912.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 61; *Astrophysical Journal*, **36**, 14, 1912.

TABLE I

RELATIVE DISPLACEMENTS OF THE FRAUNHOFER LINES AT THE LIMB AND CENTER EDGES OF THE PENUMBRAE OF ECCENTRICALLY LOCATED SPOTS

λ	Element	Intensity	Δ	No. of Plates	Mean Deviation	Δ′	Δ′ Reduced to λ 5000	Character or Group
3624.258...	*Ca*	3	+0.012	3	4	+0.012	+0.016	
3627.953...	*Co*	4	+0.016	3	3	+0.016	+0.022	
3628.967...	*La*	2	+0.019	3	11	+0.019	+0.026	
3629.877...	*Mn*	1	+0.017	3	5	+0.017	+0.023	
3630.045...	*Ni*	1	+0.024	3	3	+0.024	+0.033	
3631.124...	*Ca*	2	+0.016	3	4	+0.016	+0.022	
3639.943...	*Cr*	2	+0.017	3	4	+0.017	+0.023	
3646.335...	*Ti*	1	+0.012	3	3	+0.012	+0.016	
3649.137...	*Cr*	1	+0.014	3	2	+0.014	+0.019	
3652.691...	*Co*	3	+0.013	3	4	+0.013	+0.018	
3653.637...	*Ti*	5	+0.008	3	3	+0.008	+0.011	
3659.663...	*Fe*	5	+0.014	3	6	+0.014	+0.019	
3662.096...	*Ni*	3	+0.015	5	4	+0.015	+0.021	
3670.566...	*Ni*	5	+0.011	5	2	+0.011	+0.015	
3686.926...	*Cr*	1	+0.016	5	6	+0.016	+0.022	
3687.234...	*Fe*	3	+0.010	5	3	+0.010	+0.014	
3687.610...	*Fe*	6	+0.007	3	1	+0.007	+0.008	
3688.210...	*V*	1	+0.018	4	5	+0.018	+0.024	
3690.599...	*Fe*	2	+0.017	5	2	+0.017	+0.023	
3692.364...	*V*	1	+0.017	5	4	+0.017	+0.023	
3692.790...	*Fe*	2	+0.020	5	4	+0.020	+0.027	
3693.616...	*Co*	1	+0.021	5	9	+0.021	+0.028	
3694.344...	*Yt*	3	+0.020	5	6	+0.020	+0.027	
3694.576...	*La*	1	+0.027	5	3	+0.027	+0.036	
3695.194...	*Fe*	5	+0.013	5	1	+0.013	+0.017	
3697.567...	*Fe*	5	+0.012	4	5	+0.012	+0.016	
3698.744...	*Fe*	4	+0.014	5	3	+0.014	+0.019	
3699.283...	*Fe*	3	+0.019	5	4	+0.019	+0.026	
3701.234...	*Fe*	8	+0.007	4	6	+0.007	+0.009	
3702.170...	*Fe*	4	+0.019	4	2	+0.019	+0.025	
3702.639...	*Fe*	4	+0.017	5	4	+0.017	+0.023	
3704.603...	*Fe*	4	+0.016	5	3	+0.016	+0.021	
3704.840...	*V*	1	+0.016	5	3	+0.016	+0.022	
3706.363...	*Fe*	3	+0.018	3	4	+0.017	+0.023	
3707.186...	*Fe*	5	+0.014	5	7	+0.014	+0.019	
3707.600...	*Co*	2	+0.005	5	5	+0.005	+0.007	
3707.702...	*Ti*	2	+0.013	5	5	+0.013	+0.017	
3708.964...	*Co*	1	+0.015	5	5	+0.015	+0.020	
3709.389...	*Fe*	8	+0.002	4	5	+0.002	+0.003	*b*
3710.431...	*Y*	3	+0.015	5	6	+0.016	+0.020	
3711.364...	*Fe*	4	+0.014	5	4	+0.014	+0.019	
3711.552...	*Fe*	3	+0.015	5	1	+0.015	+0.020	
3716.591...	*Fe*	7	+0.012	4	2	+0.012	+0.016	
3717.539...	*Ti*	2	+0.017	5	3	+0.017	+0.023	
3718.554...	*Fe*	4	+0.015	5	1	+0.015	+0.020	
3720.084...	*Fe*	40	+0.003	3	1	+0.003	+0.004	
3724.970...	*Ni*	1	+0.018	5	8	+0.018	+0.024	
3879.716...	*CN*	1	+0.015	9	7	+0.015	+0.020	
3879.824...	*CN*	0	+0.021	7	5	+0.021	+0.028	
3880.105...	*CN*	1	+0.012	7	6	+0.012	+0.015	

TABLE I—*Continued*

λ	Element	Intensity	Δ	No. of Plates	Mean Deviation	Δ′	Δ′ Reduced to λ 5000	Character or Group
3880.205...	*CN*	0	+0.017	5	6	+0.017	+0.022	
3882.267...	*CN*	1	+0.014	7	4	+0.014	+0.019	
3895.119...	*Co*	3	+0.012	9	5	+0.012	+0.015	
3895.377...	*Ti*	2	+0.019	7	5	+0.020	+0.025	
3895.583...	*Mn*	3	+0.008	7	6	+0.008	+0.010	
3895.803...	*Fe*	7	+0.004	9	8	+0.004	+0.005	*a*
3897.596...	*Fe*	2	+0.009	9	6	+0.009	+0.012	
3898.032...	*Fe*	3	+0.007	9	8	+0.007	+0.009	
3898.531...	*Mn*	2	+0.014	9	7	+0.014	+0.018	
3899.171...	*Fe*	3	+0.013	9	5	+0.013	+0.017	
3901.114...	*Ti*	1	+0.017	8	8	+0.017	+0.021	*a*
3902.399...	*V*	3	+0.012	9	6	+0.012	+0.015	
3904.926...	*Ti*	3	+0.013	9	7	+0.013	+0.017	*a*
3905.660...	*Si*	12	0.000	9	5	0.000	0.000	Enh.-*Cr?*
3906.438...	*Co*	2	+0.010	9	6	+0.010	+0.013	
3906.628...	*Fe*	10	+0.003	9	5	+0.003	+0.004	*a*
3908.900...	*Cr*	4	+0.011	9	8	+0.011	+0.014	
3911.963...	*Sc*	2	+0.013	8	4	+0.013	+0.017	
3913.123...	*Ni*	2	+0.016	8	6	+0.016	+0.020	
3916.879...	*Fe*	5	+0.009	9	5	+0.009	+0.011	
3933.825...	K_3		−0.050	9	16	−0.050	−0.064	
3933.825...	K_2		−0.036	9	14	−0.036	−0.046	
3941.637...	*Cr*	3	+0.014	11	6	+0.014	+0.018	
3941.878...	*Co*	2	+0.015	11	7	+0.015	+0.019	
3944.160...	*Al*	15	+0.001	12	6	+0.001	+0.001	
3947.142...	*Fe*	3	+0.011	12	4	+0.011	+0.014	
3947.522...	?	2	+0.014	4	7	+0.014	+0.018	
3947.675...	*Fe*	4	+0.009	5	3	+0.009	+0.011	
3947.918...	*Ti*	2	+0.011	11	7	+0.013	+0.016	*a*
3948.818...	*Ti*	4	+0.009	14	5	+0.009	+0.011	*a*
3948.925...	*Fe*	4	+0.009	14	7	+0.009	+0.011	
3949.039...	*Ca*	1	+0.017	11	6	+0.018	+0.022	
3950.102...	*Fe*	15	+0.010	14	5	+0.010	+0.013	
3950.497...	*Y*	5	+0.013	14	5	+0.013	+0.016	
3956.603...	*Fe*	4	+0.010	16	5	+0.010	+0.013	
3956.879...	*Fe*	6	+0.009	15	5	+0.009	+0.011	
3958.073...	*Co*	2	+0.017	15	5	+0.018	+0.021	
3958.877...	*Fe*	1	+0.022	7	5	+0.021	+0.026	
3960.422...	*Fe*	4	+0.014	16	5	+0.014	+0.017	
3961.674...	*Al*	20	0.000	16	6	0.000	0.000	
3962.995...	*Ti*	3	+0.012	16	5	+0.012	+0.015	*a*
3963.831...	*Co*	3	+0.010	16	5	+0.010	+0.013	
3968.825...	H_3		−0.049	15	15	−0.049	−0.062	
3968.825...	H_2		−0.033	14	21	−0.033	−0.042	
3969.413...	*Fe*	10	+0.004	16	8	+0.004	+0.005	
3971.475...	*Fe*	5	+0.008	16	4	+0.008	+0.010	
3974.774...	*Ni*	2	+0.015	8	5	+0.014	+0.018	
3977.891...	*Fe*	6	+0.010	17	5	+0.010	+0.013	*b*
3980.779...	*Fe*	1	+0.014	9	6	+0.014	+0.018	
3981.248...	*Fe*	2	+0.017	10	5	+0.017	+0.021	
3981.917...	*Ti*	4	+0.013	9	5	+0.013	+0.016	*a*
3982.745...	*Y*	3	+0.013	9	5	+0.013	+0.016	
3984.294...	*Mn*	2	+0.021	9	7	+0.021	+0.026	

TABLE I—*Continued*

λ	Element	Intensity	Δ	No. of Plates	Mean Deviation	Δ′	Δ′ Reduced to λ 5000	Character or Group
3984.479...	*Cr*	2	+0.024	9	7	+0.024	+0.030	
3986.321...	*Fe*	3	+0.011	8	3	+0.011	+0.014	
3989.912...	*Ti*	4	+0.008	9	6	+0.008	+0.010	*a*
3990.011...	*Fe*	3	+0.012	11	7	+0.012	+0.015	
3990.525...	*Fe*	2	+0.016	11	4	+0.016	+0.020	
3991.580...	*Fe*	1	+0.020	11	6	+0.020	+0.025	
3993.246...	*Fe*	2	+0.026	10	8	+0.025	+0.031	
3994.265...	*Fe*	4	+0.015	11	8	+0.015	+0.019	
3994.828...	*Nd?*	2	+0.018	11	7	+0.018	+0.022	
3995.463...	*Co*	5	+0.011	11	5	+0.011	+0.014	
3995.899...	*La*	1 N	+0.014	11	9	+0.014	+0.018	
3996.140...	*Fe*	3	+0.015	12	8	+0.015	+0.019	
3997.115...	*Fe*	2	+0.019	12	9	+0.019	+0.024	
3997.547...	*Fe*	4	+0.012	12	6	+0.012	+0.015	*b*
3998.053...	*Co*	4	+0.017	12	10	+0.017	+0.021	
3998.205...	*Fe*	4	+0.014	12	7	+0.014	+0.018	
3998.790...	*Ti*	4	+0.012	12	5	+0.012	+0.015	*a*
4000.403...	*Fe*	2	+0.023	12	11	+0.023	+0.029	
4000.611...	*Fe*	2	+0.016	12	7	+0.016	+0.020	
4001.595...	*Cr*	1	+0.022	11	8	+0.021	+0.026	
4001.814...	*Fe*	3	+0.016	12	9	+0.016	+0.020	
4006.304...	*Ni*	1	+0.024	12	10	+0.024	+0.030	
4006.464...	*Fe*	2	+0.020	12	9	+0.020	+0.025	
4006.776...	*Fe*	3	+0.015	11	6	+0.017	+0.021	
4010.927...	*Fe*	2	+0.022	12	9	+0.022	+0.028	
4011.080...	*Fe*	2	+0.026	12	8	+0.026	+0.022	
4013.964...	*Fe*	5	+0.020	12	9	+0.020	+0.025	
4016.574...	*Fe*	2	+0.022	12	10	+0.022	+0.027	
4018.420...	*Fe*	3	+0.022	12	11	+0.022	+0.027	
4020.547...	*Sc*	1	+0.014	11	8	+0.014	+0.017	
4021.057...	*Co*	3	+0.018	12	7	+0.018	+0.022	
4023.834...	*Sc*	2	+0.026	12	7	+0.026	+0.031	
4024.726...	*Ti*	3	+0.016	12	7	+0.016	+0.019	*a*
4035.752...	*Co*	2	+0.015	10	4	+0.016	+0.019	
4055.701...	*Mn*	6	+0.018	12	9	+0.018	+0.021	Enh.-*Fe?*
4059.081...	*Mn*	3	+0.018	12	6	+0.018	+0.021	
4063.759...	*Fe*	20	0.000	12	10	0.000	0.000	*b*
4066.524...	*Co*	2	+0.020	12	8	+0.020	+0.024	
4070.431...	*Mn*	3	+0.023	12	9	+0.023	+0.028	
4077.885...	*Sr*	8	−0.002	12	7	−0.002	−0.002	Enh.
4078.631...	*Ti*	3	+0.018	10	7	+0.017	+0.021	*a*
4079.570...	*Mn*	3	+0.018	12	9	+0.018	+0.022	
4083.095...	*V–Mn*	4	+0.020	12	10	+0.020	+0.024	
4086.861...	*La*	1	+0.026	12	12	+0.026	+0.031	
4095.094...	*Ca?*	4	+0.020	12	9	+0.020	+0.024	
4099.941...	*V*	2	+0.025	12	5	+0.025	+0.030	
4102.000...	*Hδ*	40	−0.004	11	14	−0.004	−0.005	
4103.097...	*Si–Mn*	5	+0.017	12	7	+0.017	+0.020	
4109.609...	*Nd?*	1	+0.016	12	6	+0.016	+0.019	
4110.691...	*Co*	4	+0.016	11	7	+0.016	+0.019	
4111.509...	*Co*	1	+0.028	12	11	+0.028	+0.034	
4116.859...	*Nd?*	1	+0.027	9	17	+0.025	+0.030	
4118.934...	*Co*	4	+0.013	12	5	+0.013	+0.016	

TABLE I—*Continued*

λ	Element	Intensity	Δ	No. of Plates	Mean Deviation	Δ'	Δ' Reduced to λ 5000	Character or Group
4123.384...	*La*	2	+0.024	11	14	+0.023	+0.028	
4129.337...	*Ce–*	3	+0.022	12	7	+0.022	+0.026	
4130.804...	*Ba*	2	+0.027	11	10	+0.027	+0.033	Enh.
4131.271...	*Mn*	1	+0.027	11	11	+0.026	+0.031	
4132.100...	*V*	2	+0.011	10	8	+0.011	+0.013	
4132.235...	*Fe*	10	+0.002	9	7	+0.002	+0.002	*b*
4133.755...	*Fe*	2	+0.022	12	9	+0.022	+0.026	
4134.840...	*Fe*	5	+0.014	11	5	+0.014	+0.017	*b*
4136.678...	*Fe*	4	+0.023	11	7	+0.023	+0.028	
4137.156...	*Fe*	6	+0.016	12	8	+0.016	+0.019	
4137.809...	*Ce*	1	+0.014	12	10	+0.014	+0.017	
4142.330...	*Cr*	2	+0.026	12	11	+0.026	+0.031	
4142.624...	*Cr*	2	+0.027	4	12	+0.026	+0.031	
4147.645...	*Mn*	1	+0.023	12	9	+0.023	+0.028	
4149.360...	*Zr*	2	+0.016	12	9	+0.016	+0.019	
4150.608...	*Co*	1	+0.017	9	6	+0.017	+0.020	
4157.948...	*Fe*	5	+0.015	14	9	+0.015	+0.018	
4161.961...	*Sr*	1	+0.025	13	10	+0.025	+0.030	
4167.884...	*CN*	1	+0.026	9	8	+0.027	+0.032	
4168.133...	*Ni*	2	+0.020	14	10	+0.020	+0.024	
4179.025...	*Fe*	3	+0.018	17	9	+0.018	+0.022	
4180.970...	*CN*	2	+0.017	16	10	+0.017	+0.020	
4183.480...	*Zr*	1	+0.022	8	12	+0.024	+0.029	
4183.619...	*Z*	2	+0.024	12	10	+0.024	+0.029	
4184.472...	*Ti*	2	+0.022	16	11	+0.022	+0.026	
4195.006...	*Cr*	1	+0.020	7	7	+0.020	+0.024	
4196.699...	*La*	2	+0.023	16	9	+0.024	+0.029	
4196.837...	*Fe*	1	+0.020	16	10	+0.020	+0.024	
4197.257...	*CN*	2	+0.021	16	10	+0.021	+0.025	
4204.163...	*La*	4	+0.020	8	9	+0.020	+0.025	
4216.136...	*CN*	1	+0.022	13	8	+0.022	+0.026	
4225.020...	*Cr*	2	+0.021	13	10	+0.021	+0.025	
4226.904...	*Ca*	20	−0.002	11	7	−0.002	−0.002	
4227.606...	*Fe*	4	+0.010	11	10	+0.010	+0.012	
4232.111...	*Nb?*	1	+0.029	10	11	+0.030	+0.035	
4233.328...	*Mn, Fe*	4	+0.018	10	10	+0.017	+0.020	
4233.772...	*Fe*	6	+0.012	10	10	+0.012	+0.014	*d*
4236.112...	*Fe*	8	+0.003	11	7	+0.003	+0.003	*d*
4236.429...	*Ni*	1	+0.024	10	14	+0.024	+0.028	
4238.970...	*Fe*	5	+0.015	10	6	+0.015	+0.018	
4240.872...	*Cr*	1	+0.022	9	6	+0.022	+0.026	
4246.996...	*Sc*	5	+0.013	8	12	+0.013	+0.015	
4250.287...	*Fe*	8	+0.014	7	7	+0.014	+0.016	*c*
4253.157...	*Mn*	1	+0.027	6	6	+0.028	+0.033	
4254.505...	*Cr*	8	+0.008	7	6	+0.008	+0.009	
4257.815...	*Mn*	2	+0.022	6	11	+0.023	+0.027	
4258.477...	*Fe*	2	+0.022	7	12	+0.022	+0.026	
4260.640...	*Fe*	10	+0.003	5	4	+0.003	+0.003	*c*
4262.142...		1	+0.024	6	12	+0.023	+0.027	
4266.080...	*Mn*	2	+0.024	7	11	+0.024	+0.028	
4271.325...	*Fe*	6	+0.009	7	4	+0.009	+0.010	
4271.934...	*Fe*	15	−0.005	7	4	−0.005	−0.006	*b*
4273.643...	*Zr*	2N	+0.021	7	9	+0.021	+0.024	

TABLE I—*Continued*

λ	Element	Intensity	Δ	No. of Plates	Mean Deviation	Δ′	Δ′ Reduced to λ 5000	Character or Group
4274.746...	*Ti*	2	+0.016	7	10	+0.016	+0.019	
4274.958...	*Cr*	7	+0.002	7	9	+0.002	+0.002	
4277.544...	*Cr*	0	+0.023	4	10	+0.026	+0.030	
4281.257...	*Mn*	2	+0.018	7	8	+0.018	+0.021	
4282.565...	*Ti*	5	+0.012	7	7	+0.012	+0.014	
4283.169...	*Ca*	4	+0.017	7	9	+0.017	+0.020	
4284.382...	*Cr*	2	+0.023	7	10	+0.023	+0.027	Enh.
4284.838...	*Ni*	1	+0.030	7	13	+0.030	+0.035	
4287.158...	*La*	2	+0.017	6	13	+0.019	+0.022	
4287.566...	*Ti*	1	+0.026	7	8	+0.026	+0.031	*a*
4289.237...	*Ti*	2	+0.022	7	7	+0.022	+0.026	*a*
4289.525...	*Ca*	4	+0.015	7	7	+0.015	+0.018	
4290.377...	*Ti*	2	+0.013	7	10	+0.013	+0.015	Enh.
4291.630...	*Fe*	2	+0.015	7	8	+0.015	+0.018	*a*
4294.936...	*Zr*	2	+0.018	7	8	+0.018	+0.021	
4296.044...	*Ni*	1	+0.020	7	10	+0.020	+0.023	
4299.149...	*Ca*	3	+0.015	7	8	+0.015	+0.018	
4299.803...	*Ti*	2	+0.016	7	7	+0.016	+0.019	*a*
4300.211...	*Ti*	3	+0.007	7	4	+0.007	+0.008	Enh.
4300.732...	*Ti*	2	+0.018	7	11	+0.018	+0.021	*a*
4302.085...	*Ti*	2	+0.016	7	6	+0.016	+0.019	Enh.
4302.353...	*Fe*	2	+0.019	5	8	+0.019	+0.022	
4302.692...	*Ca*	4	+0.011	7	3	+0.011	+0.013	
4304.882...	*Zr*	0	+0.020	4	9	+0.020	+0.023	
4306.078...	*Ti*	4	+0.015	7	5	+0.015	+0.015	*a*
4313.034...	*Ti*	3	+0.010	6	6	+0.011	+0.013	Enh.
4314.248...	*Sc*	3	+0.016	7	7	+0.016	+0.018	
4314.964...	*Ti*	1	+0.026	5	8	+0.025	+0.029	*d?*
4315.138...	*Ti*	3	+0.009	6	5	+0.010	+0.011	
4320.907...	*Sc*	3	+0.017	7	9	+0.017	+0.020	
4325.152...	*Sc*	4	+0.012	7	7	+0.012	+0.014	
4325.939...	*Fe*	8	+0.006	7	7	+0.006	+0.007	*b*
4328.080...	*Fe*	2	+0.018	7	14	+0.018	+0.021	
4331.811...	*Ni*	2	+0.021	7	10	+0.021	+0.024	
4333.925...	*La*	1 N	+0.029	5	10	+0.026	+0.030	
4337.216...	*Fe*	5	+0.015	7	5	+0.015	+0.017	*b*
4338.084...	*Ti*	4	+0.014	7	5	+0.014	+0.016	Enh.
4338.430...	*Fe*	1	+0.025	6	10	+0.025	+0.029	
4340.634...	*Hγ*	20	−0.028	4	12	−0.029	−0.033	
4344.670...	*Cr*	4	+0.016	7	4	+0.016	+0.018	
4352.083...	*Mg*	5	+0.013	7	5	+0.013	+0.015	
4352.908...	*Fe*	4	+0.015	7	8	+0.015	+0.017	*b*
4359.784...	*Cr*	3	+0.018	7	11	+0.018	+0.021	
4359.907...	*Zr*	0	+0.016	4	9	+0.019	+0.022	
4369.941...	*Fe*	4	+0.018	6	9	+0.018	+0.021	*b*
4374.981...	*Zr*	0	+0.012	4	7	+0.015	+0.017	
4376.107...	*Fe*	6	+0.014	7	9	+0.014	+0.016	*a*
4377.948...	*Fe*	1	+0.030	7	12	+0.030	+0.034	
4379.396...	*V*	4	+0.016	7	8	+0.016	+0.018	
4380.326...	*Co*	2 N	+0.031	7	17	+0.031	+0.035	
4383.720...	*Fe*	15	−0.003	5	18	−0.003	−0.003	*b*
4385.548...	*Fe*	2	+0.021	7	12	+0.021	+0.024	Enh.
4387.220...	*Pb*	1 N	+0.026	7	10	+0.026	+0.030	Enh.

TABLE I—*Continued*

λ	Element	Intensity	Δ	No. of Plates	Mean Deviation	Δ′	Δ′ Reduced to λ 5000	Character or Group
4390.149...	*V*	2	+0.020	7	10	+0.020	+0.023	
4400.555...	*Sc*	3	+0.016	7	9	+0.016	+0.018	
4400.738...	*V*	1	+0.021	7	8	+0.021	+0.024	
4404.927...	*Fe*	10	+0.003	4	8	+0.003	+0.003	*b*
4408.364...	*V*	2	+0.027	6	8	+0.027	+0.030	
4410.683...	*Ni*	2	+0.028	6	11	+0.026	+0.029	
4634.254...	*Cr*	2	+0.033	10	10	+0.032	+0.034	
4636.027...	*Fe*	2	+0.035	14	14	+0.035	+0.038	
4637.685...	*Fe*	5	+0.028	17	9	+0.028	+0.030	
4638.193...	*Fe*	4	+0.026	16	8	+0.027	+0.029	
4643.645...	*Fe*	4	+0.030	17	12	+0.030	+0.032	
4646.347...	*Cr*	5	+0.018	18	6	+0.018	+0.019	
4648.835...	*Ni*	4	+0.026	17	7	+0.026	+0.028	
4651.461...	*Cr*	4	+0.024	17	8	+0.024	+0.026	
4652.343...	*Cr*	5	+0.023	17	6	+0.023	+0.025	
4656.644...	*Ti*	3	+0.034	3	19	+0.034	+0.036	
4662.149...	*Fe?*	1	+0.030	3	11	+0.030	+0.032	
4667.626...	*Fe*	4	+0.012	3	6	+0.013	+0.013	
4667.768...	*Ti*	3	+0.027	17	8	+0.027	+0.029	*a*
4669.354...	*Fe*	3	+0.035	3	12	+0.035	+0.037	
4675.294...	*Ti*	1 N	+0.041	14	10	+0.040	+0.043	*a*
4678.347...	*Cd*	3 N	+0.039	16	7	+0.038	+0.041	
4679.027...	*Fe*	6	+0.023	18	10	+0.023	+0.025	
4679.409...	*Ni*	2	+0.038	12	11	+0.037	+0.039	Enh.
4680.317...	*Zn*	1	+0.028	10	8	+0.025	+0.027	
4682.088...	*Ti*	3	+0.029	18	8	+0.029	+0.031	
4683.745...	*Fe*	3	+0.029	3	5	+0.029	+0.031	
4685.452...	*Ca*	2 N	+0.034	18	10	+0.034	+0.036	
4686.395...	*Ni*	3	+0.035	17	10	+0.035	+0.037	
4688.357...	*Fe*	2	+0.036	3	7	+0.036	+0.038	
4701.714...	*Ni*	1	+0.038	17	9	+0.038	+0.040	
4703.177...	*Mg*	10	+0.010	17	11	+0.010	+0.011	
4703.994...	*Ni*	3	+0.035	17	13	+0.035	+0.037	
4705.131...	*Fe*	4	+0.038	3	4	+0.038	+0.040	
4708.196...	*Cr*	2	+0.033	38	33	+0.033	+0.035	
4708.846...	*Ti*	2	+0.028	15	13	+0.031	+0.033	Enh.
4709.896...	*Mn*	2	+0.031	18	9	+0.031	+0.033	
4715.046...	*Ni*	4	+0.026	18	10	+0.026	+0.028	
4718.601...	*Cr*	3	+0.040	18	12	+0.040	+0.042	
4722.342...	*Zn*	3	+0.036	18	12	+0.036	+0.038	
4728.732...	*Fe*	4	+0.037	3	5	+0.037	+0.039	
4730.897...	*Cr*	1	+0.039	15	9	+0.038	+0.040	
4731.984...	*Ni*	1	+0.030	3	4	+0.030	+0.032	
4732.640...	*Ni*	1	+0.038	3	4	+0.038	+0.040	
4733.779...	*Fe*	4	+0.027	18	16	+0.027	+0.028	
4736.031...	*Fe*	3	+0.032	18	10	+0.032	+0.034	
4737.540...	*Cr*	2	+0.034	16	10	+0.034	+0.036	
4739.291...	*Mn*	3	+0.038	3	13	+0.038	+0.040	
4741.718...	*Fe*	3	+0.034	18	12	+0.034	+0.036	
4745.992...	*Fe*	4	+0.034	16	14	+0.033	+0.035	
4752.289...	*Ni*	2	+0.041	16	16	+0.040	+0.042	
4752.613...	*Ni*	3	+0.034	18	10	+0.034	+0.036	
4754.225...	*Mn*	7	+0.022	18	10	+0.022	+0.023	

TABLE I—*Continued*

λ	Element	Intensity	Δ	No. of Plates	Mean Deviation	Δ′	Δ′ Reduced to λ 5000	Character or Group
4756.300...	*Cr*	2	+0.026	18	13	+0.026	+0.027	
4756.705...	*Ni*	3	+0.033	18	12	+0.033	+0.035	
4757.771...	*Fe*	2	+0.038	3	13	+0.038	+0.040	
4758.308...	*Ti*	1	+0.034	18	10	+0.034	+0.036	*a*
4759.463...	*Ti*	2	+0.034	3	18	+0.034	+0.036	*a*
4761.718...	*Mn*	3	+0.032	17	11	+0.031	+0.032	
4762.567...	*Mn*	5	+0.027	18	10	+0.027	+0.028	
4762.820...	*Ni*	1	+0.039	3	2	+0.039	+0.040	
4766.050...	*Mn*	3	+0.026	18	8	+0.026	+0.027	
4766.621...	*Mn*	3	+0.028	18	10	+0.028	+0.029	
4773.007...	*Fe*	4	+0.028	3	11	+0.028	+0.029	
4779.634...	*Fe*	1	+0.041	3	7	+0.041	+0.043	
4780.169...	*Co*	2	+0.032	18	11	+0.032	+0.034	Enh.-*Ti?*
4783.613...	*Mn*	6	+0.023	17	12	+0.025	+0.026	
4786.727...	*Ni*	3	+0.021	3	11	+0.022	+0.023	
4787.003...	*Fe*	2	+0.025	18	9	+0.025	+0.026	
4788.952...	*Fe*	3	+0.039	3	15	+0.039	+0.040	
4789 528...	*Cr*	2	+0.033	17	10	+0.034	+0.035	
4789.849...	*Fe*	3	+0.029	18	14	+0.029	+0.030	
4793.045...	*Co*	1	+0.040	12	14	+0.040	+0.042	
4799.598...	*Fe*	1	+0.048	3	21	+0.048	+0.050	
4801.213...	*Cr*	1	+0.047	3	9	+0.047	+0.049	
4803.072...	*Fe*	2	+0.023	3	14	+0.023	+0.024	
4807.179...	*Ni*	2	+0.035	17	8	+0.034	+0.035	
4810.724...	*Zn*	3	+0.035	17	13	+0.035	+0.036	
4813.661...	*Co*	1	+0.038	17	18	+0.038	+0.039	
4820.593...	*Ti*	1	+0.039	13	15	+0.038	+0.039	*a*
4823.697...	*Mn*	5	+0.019	17	12	+0.019	+0.020	
4829.214...	*Ni*	3	+0.036	17	12	+0.036	+0.037	
5123.899...	*Fe*	3	+0.022	16	13	+0.022	+0.021	
5129.336...	*Ti?*	3	+0.019	17	8	+0.020	+0.019	Enh.
5129.546...	*Ni*	2	+0.026	17	10	+0.026	+0.026	
5129.805...	*Fe*	1	+0.035	10	9	+0.033	+0.032	
5131.642...	*Fe*	2	+0.024	20	10	+0.024	+0.023	
5131.942...	*Ni*	1	+0.030	15	12	+0.029	+0.028	
5137.250...	*Ni*	3	+0.021	20	8	+0.021	+0.020	
5137.558...	*Fe*	3	+0.020	20	8	+0.020	+0.019	
5141.918...	*Fe*	3	+0.023	29	9	+0.023	+0.022	
5142.693...	*Fe*	4d	+0.020	28	9	+0.021	+0.020	
5145.371...	*Fe*	1	+0.028	27	10	+0.028	+0.027	
5145.636...	*Ti*	0	+0.026	21	14	+0.027	+0.026	*a*
5146.659...	*Ni*–	3	+0.024	29	9	+0.024	+0.023	
5147.652...	*Ti*	0	+0.029	24	12	+0.028	+0.027	*a*
5148.222...	*Fe*	2	+0.023	29	11	+0.023	+0.022	
5148.410...	*Fe*	3	+0.021	28	8	+0.021	+0.020	
5151.020...	*Fe*	4	+0.016	27	9	+0.016	+0.016	
5152.087...	*Fe*	3	+0.022	27	10	+0.023	+0.022	
5152.361...	*Ti*	0	+0.029	25	10	+0.031	+0.030	*a*
5155.303...	*Ni*	1	+0.030	28	12	+0.030	+0.029	
5155.935...	*Ni*	2	+0.026	30	10	+0.026	+0.025	
5159.231...	*Fe*	2	+0.023	30	9	+0.023	+0.022	
5162.449...	*Fe, C*	5	+0.019	29	9	+0.019	+0.019	
5165.588...	*Fe*	2	+0.020	30	10	+0.020	+0.019	

TABLE I—*Continued*

λ	Element	Intensity	Δ	No. of Plates	Mean Deviation	Δ′	Δ′ Reduced to λ 5000	Character or Group
5168.832...	*Ni*	1	+0.026	30	9	+0.026	+0.025	
5171.778...	*Fe*	6	+0.013	29	9	+0.014	+0.013	
5172.856...	*Mg*	20	−0.010	30	12	−0.010	−0.010	
5173.917...	*Ti*	2	+0.019	30	7	+0.019	+0.018	*a*
5176.735...	*Ni*	1	+0.028	30	8	+0.028	+0.027	
5180.233...	*Fe*	1	+0.028	29	11	+0.028	+0.027	
5183.791...	*Mg*	30	−0.013	29	9	−0.013	−0.012	
5186.073...	*Ti*	2	+0.021	30	11	+0.021	+0.020	
5188.079...	*Fe*	1	+0.027	29	10	+0.028	+0.027	
5188.863...	*Ti*	2	+0.015	30	8	+0.015	+0.014	Enh.
5189.018...	*Ca*	3	+0.021	29	10	+0.021	+0.020	
5191.629...	*Fe*	4	+0.014	29	9	+0.014	+0.013	
5192.523...	*Fe*	5	+0.012	30	10	+0.012	+0.012	
5193.139...	*Ti*	2	+0.022	29	9	+0.021	+0.020	*a*
5195.113...	*Fe*	4	+0.017	29	9	+0.017	+0.016	
5195.647...	*Fe*	2	+0.022	29	9	+0.022	+0.021	
5196.227...	*Fe*	1	+0.023	30	9	+0.023	+0.022	
5198.888...	*Fe*	3	+0.022	30	9	+0.022	+0.021	
5210.555...	*Ti*	3	+0.016	30	11	+0.016	+0.015	*a*
5217.552...	*Fe*	3	+0.018	30	9	+0.018	+0.017	
5218.085...	*Fe*	0	+0.029	22	9	+0.028	+0.027	
5218.369...	*Fe*	1	+0.027	29	9	+0.027	+0.026	
5219.875...	*Ti*	0	+0.028	15	8	+0.028	+0.027	*a*
5220.358...	*Ni*	0	+0.023	14	8	+0.023	+0.022	
5221.928...	*Cr*	0	+0.028	17	9	+0.029	+0.028	
5223.351...	*Fe*	0	+0.031	17	9	+0.031	+0.030	
5224.471...	*Ti*	0	+0.028	29	11	+0.028	+0.027	
5225.695...	*Fe*	2	+0.024	30	10	+0.024	+0.023	
5226.707...	*Ti*−	2	+0.020	30	8	+0.020	+0.019	Enh.
5230.030...	*Fe*	4	+0.021	30	10	+0.021	+0.020	
5233.122...	*Fe*	7	+0.007	25	8	+0.007	+0.007	*d*
5235.557...	*Fe*	1	+0.022	29	7	+0.022	+0.021	
5242.658...	*Fe*	2	+0.020	30	10	+0.020	+0.019	
5243.946...	*Fe*	1	+0.029	30	9	+0.029	+0.028	
5247.229...	*Fe*	1	+0.025	29	10	+0.025	+0.024	
5247.737...	*Cr*	2	+0.023	30	10	+0.023	+0.022	
5250.385...	*Fe*	2	+0.029	28	10	+0.028	+0.027	
5250.817...	*Fe*	3	+0.021	30	8	+0.021	+0.020	
5253.633...	*Fe*	2	+0.024	30	12	+0.024	+0.023	
5262.419...	*Ca*	3	+0.020	16	7	+0.020	+0.019	
5263.486...	*Fe*	4	+0.021	30	9	+0.021	+0.020	
5265.729...	*Ca*	3	+0.018	27	7	+0.018	+0.017	
5266.738...	*Fe*	6	+0.013	29	7	+0.013	+0.012	*d*
5281.971...	*Fe*	5	+0.019	26	8	+0.019	+0.018	
5283.802...	*Fe*	6	+0.013	26	10	+0.013	+0.012	
5284.281...	*Ti*	1	+0.026	26	8	+0.026	+0.025	
5288.705...	*Fe*	2	+0.025	25	8	+0.025	+0.024	
5296.872...	*Cr*	3	+0.021	22	10	+0.022	+0.021	
5297.555...	*Cr*	2	+0.021	23	9	+0.021	+0.020	
5298.194...	*Cr*	1	+0.021	22	10	+0.021	+0.020	
5298.455...	*Cr*	4	+0.016	22	9	+0.016	+0.015	
5298.672...	*Ti*	0	+0.023	18	7	+0.022	+0.021	*d?*
5300.929...	*Cr*	2	+0.025	22	10	+0.025	+0.024	

TABLE I—*Continued*

λ	Element	Intensity	Δ	No. of Plates	Mean Deviation	Δ′	Δ′ Reduced to λ 5000	Character or Group
5302.480...	*Fe*	5	+0.015	22	11	+0.015	+0.014	
5307.541...	*Fe*	3	+0.022	22	8	+0.022	+0.021	
5324.373...	*Fe*	7	+0.006	21	11	+0.006	+0.006	*d*
5328.236...	*Fe*	8?	+0.007	21	9	+0.007	+0.007	*a*
5335.050...	*Co*	1	+0.026	12	8	+0.026	+0.024	
5340.121...	*Fe*	6	+0.019	22	9	+0.019	+0.018	*d*
5342.890...	*Co*	1	+0.028	13	12	+0.029	+0.027	
5345.991...	*Cr*	5	+0.015	19	10	+0.015	+0.014	
5348.511...	*Cr*	4	+0.017	19	10	+0.017	+0.016	
5349.653...	*Ca*	4	+0.019	19	7	+0.019	+0.018	
5349.928...	*Fe*	1	+0.028	16	14	+0.027	+0.025	
5598.524...	*Fe*	1	+0.019	12	13	+0.019	+0.016	*c*
5598.711...	*Ca*	4	+0.021	12	4	+0.021	+0.018	
5615.520...	*Fe*	2	+0.025	12	6	+0.025	+0.022	
5615.877...	*Fe*	6	+0.011	12	7	+0.011	+0.009	d_1
5624.245...	*Fe*	1	+0.027	4	6	+0.027	+0.023	
5638.488...	*Fe*	3	+0.029	12	8	+0.029	+0.025	d_1
5659.052...	*Fe*	4	+0.013	10	7	+0.011	+0.009	d_1
5682.869...	*Na*	5	+0.028	8	8	+0.028	+0.025	
5688.436...	*Na*	6	+0.026	8	15	+0.026	+0.023	
5805.441...	*Ni*	4	+0.038	26	9	+0.036	+0.031	
5806.950...	*Fe*	5	+0.035	27	14	+0.035	+0.030	
5831.821...	*Ni*	1	+0.032	19	10	+0.034	+0.029	
5838.592...	*Fe*	1	+0.036	12	14	+0.037	+0.032	
5847.221...	*Ni*	1	+0.034	21	11	+0.036	+0.031	
5848.342...	*Fe*	3	+0.035	27	14	+0.035	+0.030	
5853.902...	*Ba*	5?	+0.029	30	10	+0.029	+0.025	
5857.674...	*Ca*	8	+0.025	34	11	+0.025	+0.022	
5857.976...	*Ni*	3	+0.031	32	12	+0.030	+0.026	
5862.582...	*Fe*	6	+0.029	34	10	+0.029	+0.025	
5866.675...	*Ti*	3	+0.035	34	12	+0.035	+0.030	*a*
5890.186...	*Na*	30	−0.008	34	10	−0.008	−0.007	
5893.079...	*Ni*	4	+0.031	34	12	+0.031	+0.027	
5896.155...	*Na*	20	−0.006	34	12	−0.006	−0.005	
5899.518...	*Ti*	1	+0.041	18	9	+0.039	+0.033	*a*
5905.895...	*Fe*	4	+0.025	8	11	+0.023	+0.020	
5910.197...	*Fe*	1	+0.040	15	10	+0.037	+0.032	
5930.406...	*Fe*	6	+0.034	8	12	+0.034	+0.029	
5934.881...	*Fe*	5	+0.035	8	12	+0.035	+0.030	
5948.765...	*Si*	6	+0.037	24	11	+0.036	+0.031	
5949.566...	*Fe*	1	+0.033	19	14	+0.033	+0.028	
5953.386...	*Ti*	1	+0.038	26	16	+0.038	+0.032	*a*
5956.923...	*Fe*	4	+0.035	25	13	+0.035	+0.029	
5975.575...	*Fe*	3	+0.033	21	8	+0.033	+0.027	*b*
5977.007...	*Fe*	4	+0.028	4	6	+0.028	+0.022	*d*
5978.768...	*Ti*	1	+0.037	17	12	+0.037	+0.031	*a*
5983.908...	*Fe*	5	+0.028	4	14	+0.028	+0.023	*d*
5985.040...	*Fe*	6	+0.024	4	10	+0.024	+0.020	*c*
5987.290...	*Fe*	5	+0.016	4	3	+0.016	+0.013	*c*
6003.239...	*Fe*	6	+0.024	4	19	+0.024	+0.020	*d*
6007.540...	*Ni*	1	+0.033	3	14	+0.035	+0.030	
6008.186...	*Fe*	4	+0.027	3	10	+0.030	+0.025	*e*
6008.785...	*Fe*	6	+0.022	4	12	+0.022	+0.018	*d*

TABLE I—*Continued*

λ	Element	Intensity	Δ	No. of Plates	Mean Deviation	Δ′	Δ′ Reduced to λ 5000	Character or Group
6013.715...	*Mn*	6	+0.026	3	11	+0.028	+0.023	*d*
6016.861...	*Mn*	6	+0.029	4	11	+0.029	+0.024	*d*
6022.016...	*Mn*	6	+0.022	4	6	+0.022	+0.018	*d*
6027.274...	*Fe*	4	+0.021	3	8	+0.023	+0.019	*b*
6042.315...	*Fe*	3	+0.029	4	9	+0.029	+0.024	Enh.
6056.227...	*Fe*	5	+0.030	4	10	+0.030	+0.025	*e*
6065.709...	*Fe*	7	+0.018	3	3	+0.016	+0.013	*b*
6393.820...	*Fe*	7	+0.017	25	10	+0.017	+0.013	*b*
6400.217...	*Fe*	8	+0.016	24	8	+0.015	+0.012	*d*
6400.528...	*Fe*	2	+0.026	24	11	+0.026	+0.020	
6408.233...	*Fe*	5	+0.021	25	8	+0.021	+0.017	*d*
6411.865...	*Fe*	7	+0.019	25	9	+0.019	+0.015	*d*
6420.169...	*Fe*	4	+0.025	25	10	+0.025	+0.020	
6421.570...	*Fe*	7	+0.023	25	8	+0.023	+0.018	*b*
6431.066...	*Fe*	5	+0.021	25	9	+0.021	+0.017	*b*
6439.293...	*Ca*	8	+0.018	24	4	+0.018	+0.014	
6450.033...	*Ca*	6	+0.020	25	10	+0.020	+0.016	
6455.820...	*Ca*	2	+0.036	44	12	+0.035	+0.029	
6471.885...	*Ca*	5	+0.028	44	10	+0.027	+0.022	
6494.004...	*Ca*	6	+0.020	47	9	+0.020	+0.017	
6495.213...	*Fe*	8	+0.019	48	11	+0.017	+0.014	*b*
6496.688...	*Fe*	2	+0.038	30	12	+0.037	+0.030	
6497.128...	*Fe*	4	+0.020	30	9	+0.020	+0.016	
6499.168...	*Fe*	1	+0.039	30	12	+0.037	+0.031	
6499.880...	*Ca*	4	+0.030	29	11	+0.030	+0.024	
6563.045...	*Hα*		−0.064	29	26	−0.064	−0.050	
6569.460...	*Fe*	5	+0.039	28	14	+0.037	+0.030	
6573.030...	*Ca ?*	1	+0.042	29	13	+0.041	+0.032	
6575.270...	*Fe*	2	+0.034	27	11	+0.032	+0.026	
6586.550...	*Ni*	1	+0.039	30	12	+0.037	+0.030	
6593.161...	*Fe*	6	+0.026	30	10	+0.024	+0.020	*b*
6594.121...	*Fe*	4	+0.029	30	10	+0.029	+0.022	
6597.807...	*Cr*	1	+0.041	30	17	+0.041	+0.031	
6633.995...	*Fe*	2	+0.045	30	18	+0.045	+0.034	
6643.876...	*Ni*	5	+0.033	30	12	+0.033	+0.025	

DISCUSSION

From a brief examination of the absolute displacements in the fourth column of Table I, it is evident that the larger displacements are associated with the longer wave-lengths. The mean displacements of the lines of iron in the violet and yellow-red regions from intensity 1 to 8 are given for each intensity in the following table. In the upper section the displacements are as measured. In the lower section, when reduced to a common wave-length, λ 5000.

TABLE II

DISPLACEMENTS AND WAVE-LENGTH

Region	Mean λ	Intensity								Mean
		1	2	3	4	5	6	7	8	
Violet.........	4017	0.022	0.020	0.014	0.014	0.013	0.011	0.008	0.006	0.014 Å.
Yellow-red.....	6121	.032	.030	.032	.024	.028	.024	.019	.016	.026
Violet.........	4017	.026	.024	.019	.018	.017	.013	.011	.008	.017 Å.
Yellow-red.....	6121	0.027	0.024	0.026	0.021	0.023	0.020	0.015	0.013	0.021

The means of the unreduced displacements are violet 0.014 Å., red 0.026 Å.; of the reduced displacements, 0.17 Å., and 0.021 Å., respectively—results that seem decisively in favor of an effect varying as the wave-length. The spectra of the two regions were of necessity photographed separately, and there was no over-lapping series of plates by which the relative displacements could be exactly determined. In each region the plates varied among themselves, due to slight changes in the position of the slit of the spectrograph in reference to the edge of the penumbra and to the fact that they do not always pertain to the same spot. In the means from a large number of plates, these accidental variations tend to neutralize each other. The violet region is represented by 34 plates and 81 lines, the yellow-red region, by 66 plates and 43 lines. In view of the probability that the effect of accidental variations is largely eliminated, the differences between the absolute displacements in the two regions may be considered to be fundamental and the decrease from 0.012 to 0.004 Å. in the mean difference when the displacements are reduced to a common wave-length indicates that the observed differences are due to the Doppler effect, and that we are dealing with real movements of gases in the reversing layer. The values used in the comparison were deduced from all the lines measured, and no account was taken of the effect arising from the different groups of iron lines or from the different spectral regions, other than that due to change in wave-length.

A fact equally striking is the systematic variation of the displacements with the intensities of the Fraunhofer lines. For lines of the intensities considered, the displacements progressively

decrease with the increase of line intensity. In Table III the mean displacements of 193 iron lines of intensities 1 to 8 are given under their respective intensities.

TABLE III

DISPLACEMENTS AND LINE INTENSITIES REDUCED TO λ 5000

Mean λ	Intensity								No. Lines	Mean Interval
	1	2	3	4	5	6	7	8		
4017	0.026	0.024	0.019	0.018	0.017	0.013	0.011	0.008	81	0.003 Å.
4092	0.030	0.026	0.026	0.025	0.018	0.016	0.006	0.007	69	0.003
6121	0.027	0.024	0.026	0.021	0.023	0.020	0.015	0.013	43	0.002
Wtd. mean	0.028	0.025	0.023	0.021	0.019	0.016	0.012	0.009	..	0.003
Vel. km sec.	1.68	1.50	1.38	1.26	1.14	0.96	0.72	0.54	..	0.18

For these lines the wave-lengths are longer on the limb edge of the penumbra than on the center edge when the spot is between the center of the sun and the limb, and the flow of the gases is outward and tangential to the solar surface, and on the assumption that, on the whole, the weaker lines originate at the lower level the velocity outward increases with the depth.

Though the lines of a given intensity in a given spectral region originate at some mean depth, it is not probable that the same mean depth would be found when widely differing spectral regions are compared. In Table IV the residuals are given for the three regions included in the investigation. These are obtained by deducting the mean displacements from the displacement for each of the three regions.

The consistently negative residuals for the violet region and the positive residuals for the yellow-red region point to a lower level in the region of longer wave-length for the lines of the same intensity. The mean level of lines in the red is about two units of intensity lower than in the violet; lines of intensity 4 in the red region being then at the level of lines of intensity 2 in the violet, and similarly for lines of other intensities. Such a difference is the natural consequence of the scattering of light by small particles, which varies inversely as the fourth power of the wave-length. From this it would result that we see into the sun to a greater depth

in the red than in the violet. If this difference in level is taken into account when the displacements in the violet and yellow-red regions are compared after reduction to a common wave-length, the agreement is very close. In Table V the first intensity refers to the violet, the second to the yellow-red, so that the comparison

TABLE IV

RESIDUALS

Intensity	Violet λ 4000	Blue-Green λ 5000	Yellow-Red λ 6100
1	−0.0018	+0.0014	+0.0014
2	− .0008	+ .0019	+ .0005
3	− .0043	+ .0031	+ .0036
4	− .0030	+ .0039	− .0004
5	− .0025	− .0006	+ .0040
6	− .0033	− .0004	+ .0037
7	− .0010	− .0055	+ .0032
8	−0.0013	−0.0019	+0.0041
Mean	−0.0023	+0.0002	+0.0026

is now between lines at approximately the same levels. The means of the unreduced displacements are 0.016 Å. and 0.024 Å., of the reduced displacements 0.0195 Å. and 0.0197 Å., respectively. The sum of the differences violet minus yellow-red is −0.001 Å.

TABLE V

DISPLACEMENTS AND WAVE-LENGTHS

Comparisons between Lines at Same Levels

Intensities	1, 3	2, 4	3, 5	4, 6	5, 7	6, 8
Violet	0.026	0.024	0.019	0.018	0.017	0.013
Yellow-red	0.026	0.021	0.023	0.020	0.015	0.013
Difference	0.000	+0.003	−0.004	−0.002	+0.002	0.000

For lines of intensities exceeding 10, the displacements are small or negative and then increase numerically with the elevation. The data are summarized in Table VI.

The negative sign indicates that the wave-lengths of the corresponding lines are shorter on the limb edge of the penumbra than

on the edge toward the center of the sun's image, and therefore denotes an inflow of the gases at the level of these lines with velocities increasing with the elevation.

TABLE VI

DISPLACEMENTS OF STRONG LINES (λ 5000)

Element	*Mg*	*Si*	*Al*	*Sr*	*Ca* (λ 4227)	H (H_δ)
Intensity..........	10	12	15–20	8	20	40
Displacement......	+0.002	0.000	0.000	−0.002	−0.002	−0.005
Vel. km/sec	+0.12	0.00	0.00	−0.12	−0.12	−0.30

Element	*Na* (D_1, D_2)	*Mg* (b_2, b_1)	H (H_γ)	*Ca* (H_2, K_2)	H (H_α)	*Ca* (H_3, K_3)
Intensity..........	20–30	20–30	20	....	40	....
Displacement......	−0.006	−0.012	−0.033	−0.044	−0.050	−0.063
Vel. km/sec	−0.36	−0.72	−1.98	−2.64	−3.00	−3.78

The series of displacements in the case of the iron lines when arranged according to the intensities forms a practical and convenient scale for determining the relative levels in the solar atmosphere at which the lines of other elements are produced. Extending the series to include *Fe* 00 and *Fe* 15–40, which then covers the entire range of iron lines, we have the following iron scale:

TABLE VII

THE IRON SCALE (λ 5000)

Intensity	00	0	1	2	3	4	5	6	7	8	10	15–40
Displacements..	0.034	0.030	0.028	0.025	0.023	0.021	0.019	0.016	0.012	0.009	0.004	0.000
Vel. km/sec	2.04	1.80	1.68	1.50	1.38	1.26	1.14	0.96	0.72	0.56	0.24	0.00

The extreme values at either end depend upon very few lines and are consequently of relatively low weight. On but one plate was it possible to measure the *Fe* lines of intensity 0 and 00. This gave the values 0.029 Å. and 0.036 Å. for the two intensities. The values found by extrapolation from the values 1–8 are 0.031 and 0.033 Å. The means of the observed and extrapolated values have been used to extend the scale. Three difficult iron lines,

intensities 15, 20, and 40, gave the value 0.000 as a mean of closely agreeing values. The displacements and the corresponding velocities are, throughout, the relative values for the two edges and are to be divided by two, so that the maximum velocity of outflow indicated by the means is 1 km per second. The maximum outward velocity observed in the case of any line was 2.2 km per second.

RADIAL VELOCITIES AND THE SOLAR VORTEX

When the velocities radial to the vortex axis and tangential to the solar surface are plotted as abscissae with elevations above the continuous spectrum background as ordinates upon any convenient scale, a very striking distribution is shown. Such a vertical section through the axis of the vortex is given in Fig. 2. In the lower portion representing the effective levels in the reversing layer, the velocities are mainly those given by the iron lines; in the upper portion, which refers to the chromosphere, the identification of the lines is given in connection with the plotted velocities. The ordinates are entirely arbitrary and are only meant to indicate the increasing velocities of inflow with increasing elevation above the region of velocity-inversion and the increasing velocities of outflow as the depth increases below this level. At the right are given some elevations taken from Jewell's table of which he says: "I have tried to give a fair and conservative estimate determined from several plates."[1] In the diagram are also given some preliminary and fragmentary results obtained in the course of an extended investigation of pressures in the solar atmosphere now being carried on, but upon which considerable reliance can be placed when it is a question of the variation of pressure with depth.

The distribution of the plotted velocities might lead one at first to consider them as representing the inflow and outflow of the gases in the case of an actual vortex. This comes partly from the apparent symmetry of the diagram, but the scale is very different above and below the level of inversion. The lower portion is some 3500 km thick while the upper is over 20,000 km in thickness.

[1] *Publications of the U.S. Naval Observatory*, 4, Appendix I, 299–307.

so that the actual height of the upper portion is several times that of the lower.

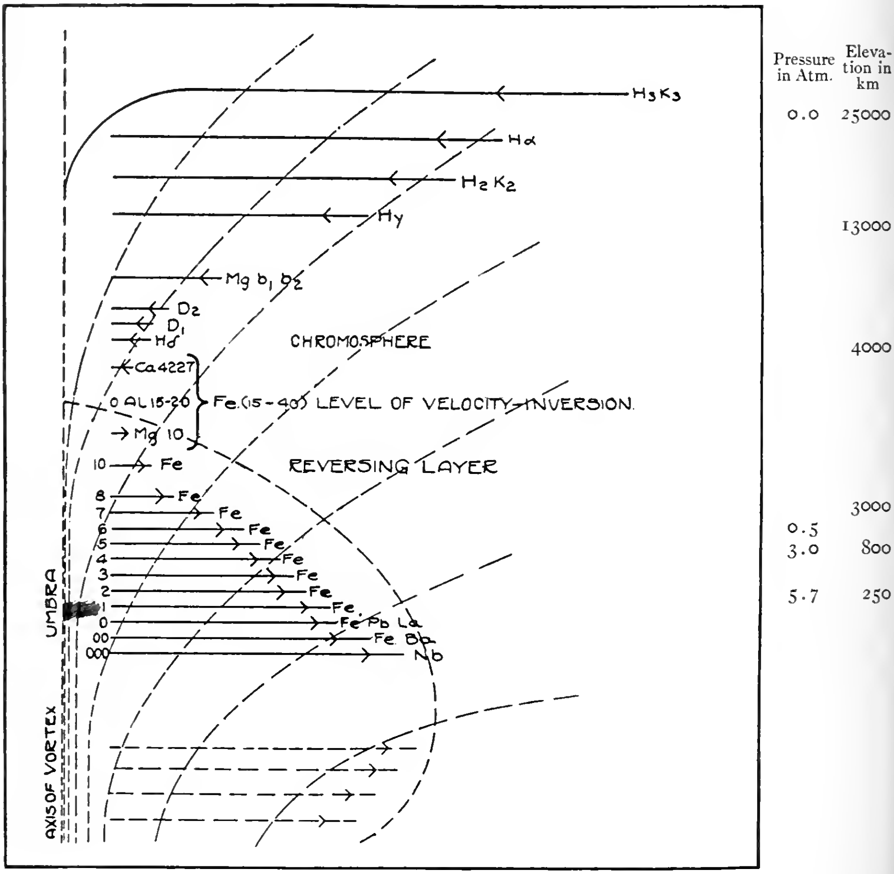


FIG. 2.—Vertical section of reversing layer and chromosphere showing the distribution of radial velocities in sun-spots. The lengths of the solid lines are proportional to the radial displacements of the corresponding Fraunhofer lines. The arrows indicate the direction of flow. The rounded head of the cyclonic disturbance is suggested by the broken-line curve enveloping the outward velocities. The broken lines with arrows refer to possible velocities below the accessible levels. The lines of force of the magnetic field are indicated in the usual way.

There are three lines of argument militating against the view that these velocities form the velocity system of an actual vortex.

In the first place, the composition of the material flowing outward in the lower portion is very complex, containing, as will be shown in a later paper, at least 27 elements—in fact, all elements the lines of which have been investigated in this paper—while the inflowing material is calcium, hydrogen, magnesium, sodium, and probably some of the higher level vapors of iron, aluminium, and strontium. It will be noticed that the level of velocity inversion is about where the customary division is made between the chromosphere and the reversing layer. Secondly, the masses involved in the inflow and outflow are quite different. An approximate value for the mass which flows across the boundary into the spot from the chromosphere may be found as follows. The velocity in this region decreases with increase of the density of the inflowing gases, so that the center of the mass of the inflowing material is located in the level where the velocity is low. For an approximation, the level may be assumed to be that of magnesium, b_1, b_2, where the velocity of inflow is 0.36 km per second. The pressure is very low at the level of this velocity, perhaps 0.1 terrestrial atmosphere, since in the upper reversing layer it is much less than 1 atmosphere and rapidly decreases upward. The mass resting upon an area of 1 sq. cm $=m=P/G=100/27.6=3.6$ grams, where G is the solar gravitation constant.

The mean radius of the penumbrae of the spots under investigation is 11,250 km, so that the velocities found are the velocities normal to the surface of a cylinder of this radius. The quantity of material crossing the boundary in one second may be found by considering a shell of the given radius whose thickness is numerically equal to the velocity per second inward across the boundary, and upon the cross-section of which the surface density is the mass of the homogeneous atmosphere per square centimeter, that is, $2\pi R m \Delta R$. In the present case $R=11{,}250$ km, $\Delta R=0.36$ km, and $m=3.6$ grams. The mass so calculated is $29\pi\times 10^{13}$ grams.

In like manner an approximate value of the mass that flows across the boundary outward into the reversing layer may be found. The velocity in the reversing layer increases with the increase of density, and both increase rapidly with depth, so that the center of mass of the outflowing material is in the level where

both velocity and density are high. It may be assumed to be at the level of *Fe* 00 where the velocity is 1 km per second. From the rapid increase of pressure with depth the pressure may be assumed as 10 terrestrial atmospheres. The mass of the reversing layer resting upon an area of 1 sq. cm is then $360-3.6=356$ grams, and the mass moving outward per second is $8\pi 10^{15}$ grams, and the ratio of the inflowing to outflowing mass is 276.

The third reason is based upon the energy crossing the boundaries above and below the level of inversion, in the chromosphere and in the reversing layer respectively. The ratio MV^2 (reversing layer)/MV^2 (chromosophere) reaches the approximate value 2000. It is true that these are approximate values and only roughly approximate, but from the order of the ratios it is evident that the outflow into the reversing layer is not fed by the inflow from the high level, and that both the mass and the energy supplied from above are only small fractions of that flowing out below, while in a true vortex there is an equilibrium between the two activities.

In the terrestrial cyclone and especially in the types represented by the tornado, hurricane, and water-spout, it is known that in the upper portion the air is moving outward. Moore, in describing the hurricane, says:

> In the cirrus cloud level, 10,000 to 12,000 m. above the sea, the clouds are observed to move rapidly outward in all directions. The outward moving clouds represent the radial velocities in the upper levels of the dumb-bell vortex while the inner wind velocities found on the sea-level occur in the plane which truncates the vortex at some distance above the theoretical lower boundary plane. . . . . The radial velocity diminishes up to the middle plane and then increases outward; the tangential velocity increases up to the middle plane and then gradually diminishes. Near the center the centrifugal force is so great, on account of the rapid rotation of the air in the inner rings, that there is practically a calm central core, something like 20 or 30 miles in diameter.[1]

The outward velocities in the reversing layer at the outer boundaries of the penumbrae of sun-spots may then, with great probability, be referred to a low-lying vortex, since the outflow is not supplied from the higher levels. In the upper part of a

[1] *Descriptive Meteorology*, pp. 162–163.

dumb-bell or trumpet-shaped vortex the radial velocities increase with higher levels and in the theoretical discussion the radial velocities approach infinite limits and become asymptotic to the plane of reference. In the reversing layer the velocities increase with the depth, and it is true that the radial velocities outward in cyclones increase with height above the middle plane, but in the nature of things these outward movements of the air after reaching a maximum must decrease with greater height, by propagating themselves into and by entraining the overlying air, so that the region of disturbance would be dome-shaped above and the surface of zero radial velocity would have the dumb-bell form. This spreading and rounded head is characteristic of the whirling columns of steam, smoke, and ashes that rise from volcanoes and great furnace stacks.

The complete radial velocity system in the upper section of a solar vortex is indicated in the diagram by including the dotted abscissae, and the section of the surface of zero radial velocity by the enveloping curve. From this point of view the level reached by the spectrograph is still above the region of outwardly decreasing velocities, and such a region is as yet inaccessible to the spectroscope directly, since the level reached by this method of sounding the solar atmosphere cannot be deeper than the level at which the weakest Fraunhofer lines originate. Iron and vanadium lines of intensity 00 still indicate by their displacements outward velocities increasing with the depth and it is not probable that lines of 2 intensities less, the weakest in Rowland's table, will reach the level of decreasing velocities. In the levels accessible to spectroscopic investigations the tangential velocities in the vortex have disappeared; their maximum value would occur at far lower levels. The same is true in the case of the vertical component, as both reach their maximum values in the middle plane of the vortex. This explains the failure to find definite indications of such velocities from displacements due to the Doppler effect. It is possible that by employing the lines of lowest level evidences of such velocities may be found in occasional spots.

The inflow of the chromospheric gases seems to follow, in part, at least, from the general downward movement of these gases.

This settling effect was shown for magnesium and sodium by Perot and Lindstreedt,[1] and for calcium in my paper on the general circulation of calcium.[2] The temperature in or immediately above the umbrae of spots is lower than in the surroundings, and this would produce in the overlying gases an added tendency to fall. As a result of one or both effects a downward flow would set in over the umbrae. Such an effect is indicated by the early observations of Respighi, who states as follows the results deduced from his observations made upon the border of the sun's disk:

1. In the neighborhood of the spots the chromosphere (*strato rosato*) is rather low, quite regular, and intensely bright.

2. Upon the exact locality of a spot or rather over its nucleus the chromosphere is generally very low and sometimes totally wanting.[3]

The downflowing gases would be caught in the current of the outflowing vapors in the upper portion of the vortex, and carried outward into the upper reversing layer, and the circulation once started would tend to continue. As in the case of fluids flowing into a sink, a whirl would in general be produced. The direction of this whirl would have no necessary hydrodynamical connection with the direction of rotation in the underlying vortex, since the tangential motion in the vortex ceases before reaching the lower reversing layer, and therefore is not propagated into the overlying region as the radial motions are. It is probably this upper whirl that has given the indications of inward flow and of vortex motion in spots that the visual observations, such as those of Secchi, occasionally show, and very probably the spectroscopic evidences of inflow and rotation of the high-level calcium vapor shown by my observations[4] refer to this pseudo-vortex. These observations, when considered from this point of view, are more easily interpreted. The wave-length of K_3 was measured at the center and over flocculi, penumbrae, and umbrae. The most striking feature of these

[1] *Comptes rendus*, **152**, 1367, 1911; **154**, 326, 1912.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 48; *Astrophysical Journal*, **32**, 36–82, 1910.

[3] *American Journal of Science*, 3d Series, **1**, 1871.

[4] *Contributions from the Mount Wilson Solar Observatory*, No. 54; *Astrophysical Journal*, **34**, 57–78, 131–153, 1911.

measurements is the remarkable agreement of the means for the separate regions with the general mean from 500 observations, which indicates a downward movement of approximately 1.1 km per second. A downward movement of the calcium vapor showed at all times in 88 per cent of the spots, varying in velocity from 0.68 km to 2.2 km with a general mean of 1.3 km per second. Four spots showed a motion of rotation of inflowing vapor with radial components for K_2 and K_3 of 1.24 km and 2.00 km per second, and tangential components of 0.98 km and 1.6 km per second, respectively. The small amount by which the downward velocity over spots exceeded that over the general surface, the fact that some spots gave no spectroscopic evidence of descent over the umbrae, and the small number of spots that showed vortex motion seemed surprising from the point of view that the observations refer to the real vortex, and especially in view of the fact that every spot is the seat of a magnetic field, as the results from Mr. Hale's discovery of the Zeeman effect in sun-spot spectra have shown. And there appears to be no other explanation than that the field is due to the existence of a vortex. But if a vortex, why do the surface indications so frequently fail to give evidence of it through spectroscopically observed displacements of the Fraunhofer lines or the vortical structure of the H_α flocculi in spectroheliograms? More rarely still do the conspicuous H_2 flocculi show any radial structure. The difficulties are in great measure removed by considering these as secondary and superficial effects depending upon elevation, the properties of the elements, and the field at these great distances from the underlying vortex. The cool region above the umbra acts as a sink into which the chromospheric vapors descend with or without the formation of a whirl, and from which they are discharged into the upper reversing layer. The whirl when formed may bear no relation to the direction of rotation, and only a very loose relation to the intensity of the underlying vortex. The conditions favoring the production of vortex structure in the inflowing gases are quite different at the levels of the H_α and H_2 flocculi. These levels are widely separated, the level of the H_α flocculi is probably some thousands of kilometers above the level of the H_2 flocculi, which are not far from the level of the upper

reversing layer. At the level of the H_α flocculi the conditions are: inward velocity high, magnetic field strong, inertia of ions small; and at the level of the H_2 flocculi the conditions are: inward velocity low, magnetic field stronger, inertia of ions large.

From an electrodynamic point of view, a view suggested by Professor Störmer, the high inward velocity of the hydrogen, due mainly to the general inflow, and the small inertia of the hydrogen ions are conditions favorable to the production of stream-line structure in the H_α flocculi by a magnetic field which may act mainly as a directing force upon the inward moving ions. At the level of the H_2 flocculi the relatively quiescent state of the gases and the great inertia of the calcium ions, the *Ca*-ion having forty times the mass of the *H*-ion, are conditions very unfavorable to the formation of stream lines at this level. The force exerted by the field upon the ions varies directly as their velocity which, at this level, is very small, since the H_2 flocculi are not far above the level of velocity inversion, and the velocity produced by the field varies inversely as the mass of the ion.

From a hydrodynamic point of view also the conditions are favorable to the production of stream lines in the H_α flocculi, as the hydrogen is flowing into the spot, but at the level of the H_2 flocculi the motion is small or nil, and stream lines are absent because there is little or no flow into the spot at this level. Bearing upon the quiescent condition of the gases producing the H_2 flocculi are some data I obtained when looking for evidence of vertical motion in the bright calcium vapor over the flocculi around spots. It had been assumed in the literature of the subject that this bright material was rising. The wave-lengths of the bright K_2 line over various regions when on the meridian were found to be as follows:[1]

Over general surface	3933.641
Over flocculi	.664
Over penumbrae	.665
Over umbrae	.682
In the arc	.667

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 54; *Astrophysical Journal*, **34**, 57–78, 131–153, 1911.

There is a rise of the calcium vapor over the general surface and a descent over the umbrae of 1.97 km and 1.3 km per second respectively, but over the flocculi and penumbrae an almost stationary condition obtains.

The occasional occurrence of vortical motion of the calcium vapor and the stream-line structure of the hydrogen flocculi may be coincident with the presence in spots of especially strong magnetic fields. A comparison of the field-strength in spots showing these phenomena with that in spots in which these phenomena are absent throws some light upon this interesting question.

In the "Summary of the Results of the Study of the Mount Wilson Photographs of Sun-Spot Spectra,"[1] Mr. Adams gives a list of lines for which the separations in the spot spectra have been measured. The separations given by the lines upon Plate T 219 are given in a separate column, as they were distinctly greater than for the other spots. The Greenwich numbers for the spots are 6393, 6441, 6511, 6577. From nine lines which were measured as doublets upon all the plates the ratio of the field given by T 219 to the mean of the other plates is 1.3. The umbra of this spot was very black and large and the total magnetic flux was much greater than for either of the other spots. It was 2.5 times the mean for the other spot, when the areas of the umbrae and field intensities are both taken into consideration. The spot is No. 6577, a spot remarkable for its numerous returns, as it persisted during seven rotations of the sun. Of this spot I said in my paper on the "Motion and Condition of Calcium Vapor over Spots and Other Special Regions": "In the case of Spot 6577, when it was near the west limb, the north edge of the penumbra gave the shorter wave-length. When it appeared at the east limb as 6592 the south edge gave the shorter wave-length."[2] In both cases the rotation of the calcium vapor was clockwise. The H_α spectroheliogram shows marked radial structure of the hydrogen flocculi. Spot 6441 was next in size to Spot 6577, and showed hydrogen structure and at times great

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 40; *Astrophysical Journal*, **30**, 86–126, 1909.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 54; *Astrophysical Journal*, **34**, 57–78, 131–153, 1911.

activity. It was with this spot that I obtained the series of exposures showing a dark hydrogen flocculus being drawn into the spot. Six of these exposures are shown in Plates XXXVII, XXXVIII, XXXIX, of Mr. Hale's original paper on the solar vortices.[1] Spot 6393 showed no definite structure on the H_a spectroheliograms, but large bright eruptive areas following in the train of the spot. Spot 6511 gave no definite structure on an excellent plate that showed remarkably interesting structure about other spots on the sun's disk at the same time.

In the type of vortex in which the data of the present paper seem to find their interpretation, there is an enormous flux of energy from the vortex into and below the lower reversing layer. The mechanical energy of motion is rapidly transformed into other forms, as the velocity decreases rapidly with distance. The question of how rapidly will be taken up in a later investigation. The form and distribution of this energy in the surrounding region is another interesting question. In the umbrae of sun-spots we now recognize the presence of hydrides and oxides, as shown by the investigations of Fowler, Hale, Adams, and Olmsted. Owing to the low temperature in spots, it is possible that these compounds undergo condensation also. They will be carried with the general flow outward into the reversing layer where decomposition will take place at the expense of a part at least of the energy of the moving gases. But the greater part of this energy will be available for raising the temperature of these low-lying regions, and not only will the temperature be raised, but there will probably be a heaping-up of very hot material around the spot, which would explain the presence and the formation of the faculae. Such a piling-up of air around the cyclone center is evidenced by the high barometer around the center of disturbance.[2] Secchi's sketch of a spot and faculae, when near the limb on March 4, 1866, is suggestive of such a heaping-up of hot material around a spot. In my paper on the "Motion and Condition of Calcium over Special Regions"

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 26; *Astrophysical Journal*, **28**, 100, 1908.

[2] Reye, *Die Wirbelstürme, Tornados, und Wettersäulen in der Erd-Atmosphäre*, p. 135.

I was led to infer in the explanation of the character of the H_2 spectroheliograms at different levels that the faculae may be considered as regions from which a disturbance capable of increasing their emissive power is propagated into the overlying gases, and that these regions may be of higher temperature absolutely, or apparently so from their being elevated above a portion of the surrounding vapor, and that there is probably a piling-up of calcium vapor over the faculae.[1] The result of the flux of material and energy outward from the spot would be to produce both of these effects, and the enormously increased emission on the part of the calcium vapor producing the bright H_2 and K_2 components would be caused by the high temperature prevailing around the spot. The relative emissive power of the H and K lines increases rapidly with temperature, as King has shown.[2] The intensely bright and quiet chromospheric layer observed near spots by Respighi may be referred to the high temperature of the region surrounding spots.

The measurement of the line displacements given in Table I has been made by Miss Ware. I wish to express my appreciation of the patience and skill required in the long series of measurements and of her interest in the progress of the work.

SUMMARY

1. The observations reported in the present paper are in entire accord with Mr. Evershed's hypothesis that the displacements considered are due to a movement of the solar vapors tangential to the solar surface and radial to the axis of the spot vortex.

2. The proportionality between the displacements and wave-lengths shows that the phenomenon is due to the Doppler effect and that we are dealing with an actual flow of the material of the reversing layer out of spots and of the chromospheric material into spots.

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 54, 40; *Astrophysical Journal*, **34**, 148, 1911.

[2] *Contributions from the Mount Solar Observatory*, No. 32; *Astrophysical Journal*, **28**, 389, 1908.

3. The increase of displacements indicating an outward flow with the decrease in the intensity of the lines of the reversing layer and the increase of the displacements indicating an inward flow with increasing intensity of the high-level lines of the chromosphere find their explanation in differences of level. The outward velocities increase with distance below a neutral level, the inward velocities increase with distance above this neutral level or the level of velocity inversion.

4. The vertical distribution of the velocities shows at high levels an inflow into spots and at low levels an outflow. A consideration of the quality and the quantity of the material and of the amount of energy involved in the two movements indicates that they do not of themselves form a vortex system.

5. The type of vortex indicated is that of the terrestrial tornado or hurricane. There is a whirling upward rush of material from the interior of the sun, which spreads out radially with rapidly decreasing velocity, tangential to the solar surface, and entrains with it the gases of the reversing layer. The actual vortex is deep-seated, the outflow into the reversing layer being a portion of the upper part of it, the inflow from the chromosphere being a secondary effect, a superficial indication of the underlying vortex in which the magnetic field originates.

6. The secondary phenomenon shows sometimes vortex motion of the high-level calcium vapor, visible evidences of vortex movements, and stream-line structure of the H_α flocculi, depending upon the strength of the magnetic field and the rotational energy of the underlying vortex.

7. The absence of stream-line structure in the H_2 flocculi follows from the quiescent state of the calcium vapor producing these flocculi. This shows no vertical motion, and the flocculi are near the level of zero velocity along the solar surface.

8. The flux of gases into and below the lower portion of the reversing layer would cause a piling-up of material, the temperature of which would be raised by the rapid transformation of mechanical energy into heat, and the increased emission of the calcium vapor would then be a temperature effect.

9. When the displacements of lines in the red and violet of

equal solar intensities are compared, the lines in the violet show the smaller displacements and therefore originate at higher levels, a consequence of the scattering of light.

10. The displacements of the iron lines arranged in the order of intensities from 00 to 10 form a regular descending scale for determining the relative levels in the solar atmosphere at which the lines of like intensities of other elements have their origin.

MOUNT WILSON SOLAR OBSERVATORY
December 23, 1912

CIRCULATION IN THE SOLAR ATMOSPHERE AS INDICATED BY PROMINENCES

SECOND PAPER

By FREDERICK SLOCUM

In the *Astrophysical Journal*, **33**, 108, 1911, I presented a preliminary paper on the above subject. The investigation discussed therein was suggested by the fact that, in the course of the study of the solar prominences photographed with the Rumford spectroheliograph of the Yerkes Observatory, so many of the prominences presented the appearance of being driven to one side by a horizontal current that it seemed worth while to make a special study of these cases, to see if they might not throw some light upon the circulation of the sun's atmosphere.

Illustrations and descriptions of the different types of prominences used as directional in that former, and also in the present investigation, were published in connection with the above-mentioned paper. In that investigation no discrimination was made between high and low prominences, so the conclusions pertain to the average height of all, namely $0\rlap{.}'7$ or 30,000 km. In many cases, however, low and high prominences in the same vicinity appear to be moving in opposite directions, which might indicate the existence of an upper and a lower current. In the present investigation, therefore, the prominences have been tabulated according to height.

I have recently brought the examination of the prominence plates down to the end of the year 1912. Upon all of these plates I have also studied carefully the structure of the chromosphere. This has frequently been described as resembling grain blown by the wind, and I hoped to get from this the prevailing current at the lowest level. I found, however, but few cases that showed a general tendency in one direction. Plate XIII shows a portion of the chromosphere as photographed in the light of calcium with the spectroheliograph. Much of the fine detail is lost in the repro-

duction, but it can be seen that the directions of the spikes and jets seem to be wholly at random.

Forty-six hundred prominences have been examined, of which over 1500 indicate direction. The data have been tabulated according to the limiting latitudes of each prominence, the direction of apparent motion, the maximum height, and type. The values were summarized by counting the number of directional prominences in each five-degree zone of latitude from the north to the south pole. When a prominence extended over two or more zones it was counted in each zone. The results were summed and the percentage of poleward direction computed.

TABLE I

Latitude	Northern Hemisphere			Southern Hemisphere		
	Number		Percentage Poleward	Number		Percentage Poleward
	−	+		−	+	
80–85	7	5	42	11	14	56
75–80	16	5	24	22	19	46
70–75	21	10	32	26	22	46
65–70	24	11	31	29	21	42
60–65	48	11	19	34	28	45
55–60	53	29	35	36	31	46
50–55	44	37	46	40	49	55
45–50	35	48	58	49	65	57
40–45	53	65	55	46	61	57
35–40	53	70	57	52	54	51
30–35	52	85	62	55	64	54
25–30	50	62	55	56	62	53
20–25	58	61	51	67	61	48
15–20	58	64	52	60	67	53
10–15	51	51	50	59	62	51
5–10	52	42	45	53	59	53
0– 5	50	39	44	39	60	61

+ indicates direction toward the pole.
− indicates direction from the pole.

Table I shows the tabulation for all prominences without regard to height. Similar tables were made for prominences between various limits of height. The minus sign indicates direction toward the equator, the plus sign toward the pole. In the zone from 60° to 65° of north latitude, for example, there were 48 cases of apparent motion toward the equator and 11 cases of motion toward the pole, giving 19 per cent of poleward motion.

that is, a decided tendency toward the equator. Again between 30° and 35° north latitude there were 85 cases toward the pole

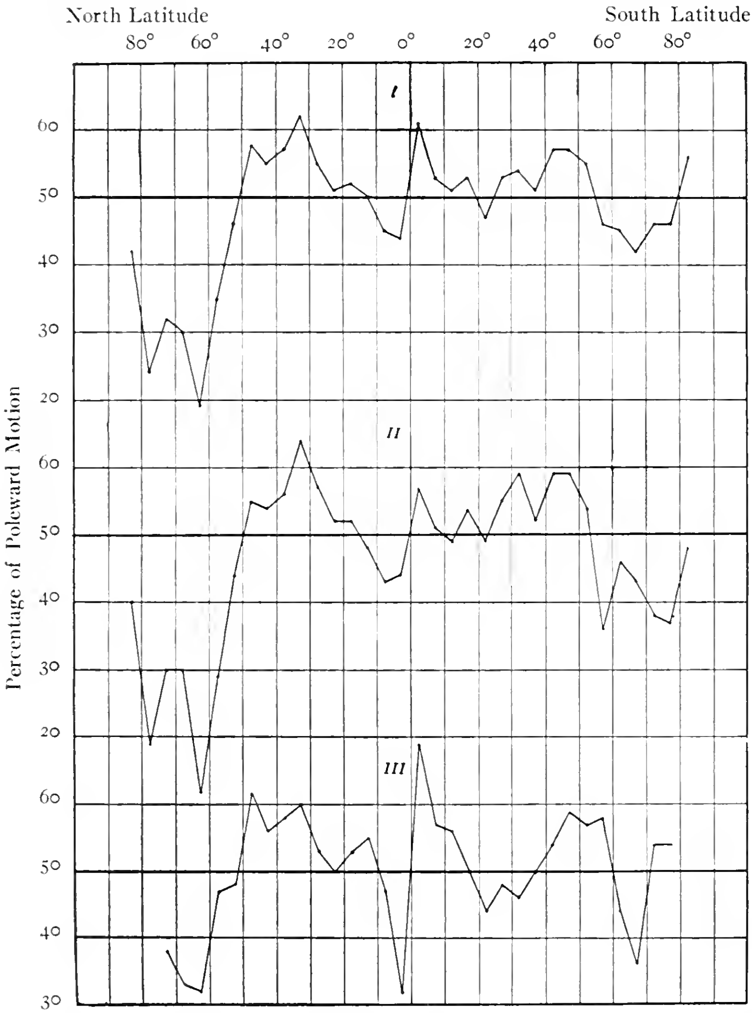


Fig. 1.—Percentage of poleward direction of solar currents as indicated by prominences.

I All prominences II Low prominences III High prominences

against 52 toward the equator, giving a poleward percentage of 62 per cent. A percentage of less than 50 per cent means a tendency

toward the equator, greater than 50 per cent toward the pole. The accompanying diagram shows graphically the results given in the table and also results for similar tabulations for high and low prominences, that is, for prominences less than $1'.0$, or 43,000 km in height and for those from $1'.0$, up.

The abscissas are solar latitudes; the ordinates, percentage of poleward motion. The X-axis represents percentage 50. Points above the X-axis indicate a motion toward the pole, below the axis, toward the equator.

Curve I, which is based upon 1524 prominences, confirms the facts brought out in the former paper, showing maxima in middle latitudes and minima in high latitudes. Moreover, the other two curves show practically the same phenomena, which signifies that the prevailing tendencies noted pertain to the whole realm occupied by prominences. If there is a counter-current, it must be below the chromosphere or above the prominences.

All three curves also show a minimum between $0°$ and $+10°$ and a maximum between $0°$ and $-10°$, which, if real, would signify a current right across the equator from $10°$ north to $10°$ south latitude. This would be somewhat analogous to the southeast trade winds in the Atlantic Ocean in July, which prevail from latitude $15°$ south to latitude $8°$ or $10°$ north.

The conclusions from this investigation are given in the following summary:

1. Many prominences, by their shapes or movements, seem to indicate the existence of a horizontal current in the solar atmosphere.

2. This current may have opposite directions at different altitudes in the same locality.

3. It may change its direction, just as the wind changes upon the earth. (See *Astrophysical Journal*, **35**, 301, 1912, Plates XV, XVI, and XVII.)

4. In middle latitudes the average tendency for movement is toward the poles.

5. In high latitudes the average tendency for movement is toward the equator.

6. This tendency is much more marked in the northern than in the southern hemisphere.

7. From latitude 10° north to 10° south the average tendency is from north to south directly across the equator.

8. The prevailing directions mentioned above are the same for prominences of all heights.

9. Upon a rotating sphere the circulation is undoubtedly spiral. The observations used in the present investigation take account only of the north and south components. The east and west components may eventually be added by an extended series of radial velocity measures of prominences.

10. Observations upon prominences within 5° or 10° of the poles are unreliable for determining direction of motion, as a prominence approaching the pole spirally may project so as to be moving apparently away from the pole.

YERKES OBSERVATORY
January 23, 1913

THE DETERMINATION OF AQUEOUS VAPOR ABOVE MOUNT WILSON[1]

By F. E. FOWLE

This paper gives the amounts of aqueous vapor above Mount Wilson, California, on certain days of the years 1910 and 1911 as determined by the method developed in the article entitled "The Spectroscopic Determination of Aqueous Vapor" in the *Astrophysical Journal* for April of last year.[2] A comparison is then made between the values thus determined and those obtained from the formula developed by Hann, Humphreys, and others.

OBSERVATIONS

On Mount Wilson, as a part of the regular observations for the determinations of the intensity of the solar radiation, each clear morning during the summer months, energy-curves of the solar spectrum are taken as the sun rises higher and higher above the horizon. In these curves (bolographs), the presence of water-vapor in the path of the sun's rays causes several deep indentations, or bands, which are deeper the greater the amount of water-vapor. With a height of the energy-curve on either side of the bands of some 10 cm, the depth in the bands may range from 3 to 8 cm. These magnitudes are so great that far higher accuracy is here possible than in atmospheric aqueous-vapor determinations by ordinary photometric measures of the photographic spectra sometimes used.

Across the top of a band produced in a bolographic energy spectrum of the sun by the atmospheric water-vapor a line is drawn and the transmissibility is measured by the ratio of the ordinate at the bottom of the band to the corresponding ordinate of the line drawn across the top. The loss of energy due to the general absorption of the atmosphere is eliminated from this ratio, since

[1] Published by the permission of the Secretary of the Smithsonian Institution.

[2] *Astrophysical Journal*, **35**, 149, 1912.

it affects in equal proportion both the enumerator and the denominator.

The calibration in the laboratory at Washington of these ratios for the bands ϕ and ψ' with the equivalent amount of water-vapor necessary to produce them was discussed in detail in the earlier paper. The results for ψ' were the more accurate. In the solar-

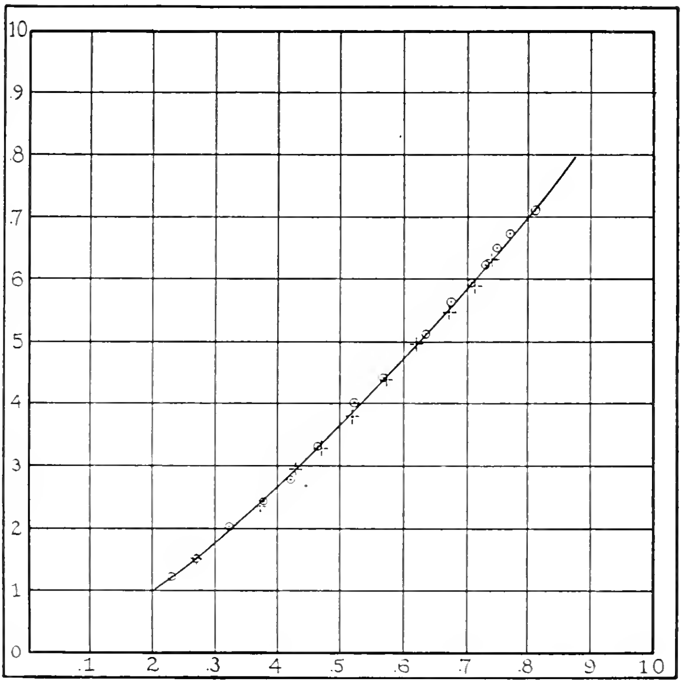


FIG. 1

Abscissae are transmissibilities for radiation by water-vapor in ρ ($\lambda = 0^{\mu}93$); ordinates are transmissibilities for radiation by water-vapor in ψ' ($\lambda = 1^{\mu}47$); circles represent 1910 data; crosses represent 1911 data.

spectrum work to be described the bands ρ, ϕ, ψ' were measured, ρ the most accurately,[1] ψ' the least accurately and only half as often as the others. In order to use the ρ ratios for the determination

[1] This difference in accuracy in laboratory and field observations of different bands is due to the very different distribution of energy in the solar prismatic spectrum from that in the Nernst-glower spectrum used in laboratory work.

of water-vapor, they must first be reduced to the corresponding ones for ψ' or ϕ. The full-line curve in Fig. 1 for this purpose is made by using the transmissibility in ρ as abscissae against those in ψ' as ordinates. The relation between the bands is so definite that the measures in ρ may at once be reduced to the corresponding values for ψ' and then by means of the calibration-curves (Fig. 7) of the earlier article, the amount of water-vapor corresponding to the depths of the three bands, ϕ, ψ', and ρ via ψ' is obtained.

REDUCTION OF OBSERVATIONS

The length of the path through the water-vapor in the atmosphere traversed by the solar beam is proportional to the secant of the zenith distance for the zenith distances used to within 1 per cent.[1] When the sun is in the zenith this length of path (air mass) is taken as unity. The observations are plotted as shown in Fig. 2, where the data of several days are treated, ranging from nearly the highest amount of absorbing vapor to nearly the lowest. As abscissae are used air masses, as ordinates the logarithms of the transmissibilities, since with these co-ordinates the resulting plots are nearly linear.[2]

From such plots were read for each day the transmissions at air masses 3.0 and 1.2, corresponding very closely to the beginning and to the end of a day's observations; the calibration plot (Fig. 7 of the earlier paper) was entered with these values and the corresponding amount of water in the form of vapor found; these values were then divided by the air masses, 3.0 and 1.2, respectively. The resulting water-vapor is stated as so much precipitable water, being the depth of liquid water which if in the form of vapor would be contained in the column of air of the same section reaching vertically to the limits of the atmosphere. This implies the same distribution of the vapor in successive atmospheric layers concentric with the earth for a vertical column as in the actually observed column which was not vertical.

[1] *Annals of the Astrophysical Observatory of the Smithsonian Institution*, **2**, 63.

[2] This is merely a convenient empirical device. The truly linear relation holds only for homogeneous rays.

These times for the determination of the vapor, namely, when the sun was at an altitude of 19° 27′ and 56° 26′, respectively (air

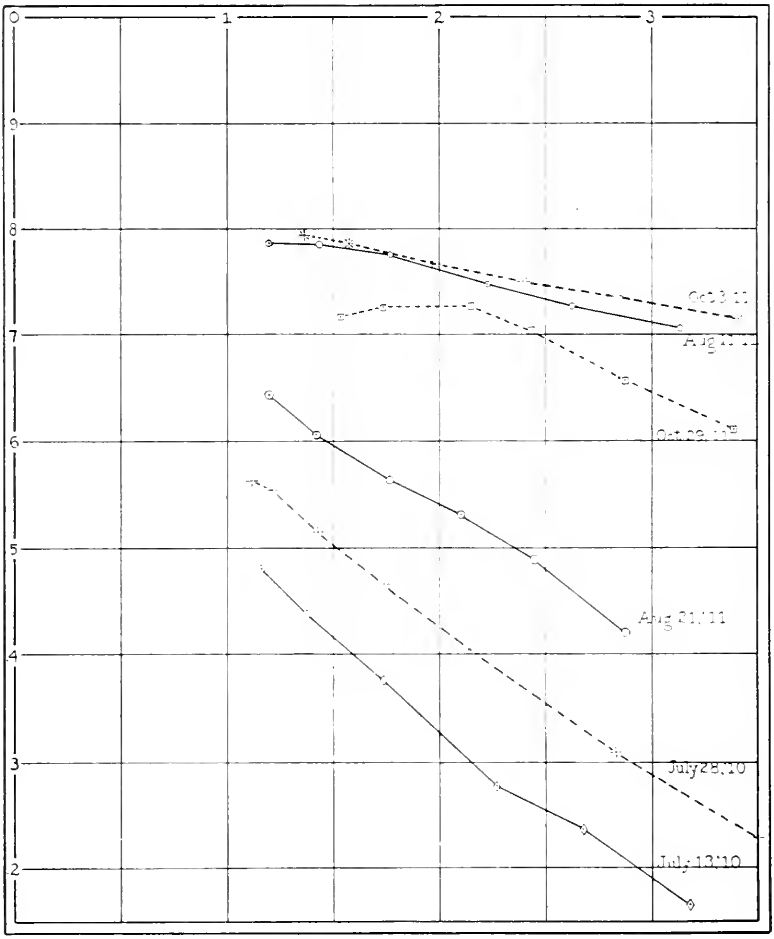


Fig. 2.—Illustrative of observations of water-vapor transmission and the method of reduction.

Abscissae are air masses (sec. zenith distance of the sun); ordinates are log transmission in Φ ($\lambda = 1^{\mu}.13$).

masses, 3.0 and 1.2), though perhaps ill chosen for meteorological purposes, were better adapted to the astrophysical purposes to which the data will finally be put. At latitude 34°13′ N. the

observations at air mass 3.0 correspond roughly to the following apparent times: May 1, 6^h54^m, June 1, 6^h37^m, July 1, 6^h34^m, August 1, 6^h47^m, September 1, 7^h14^m, October 1, 7^h47^m, November 1, 8^h25^m, all of the morning; for the air mass 1.2, roughly to 10^h A.M. The results thus obtained are given in the following table and the accompanying plot (Fig. 3). The table contains data only for 1910, merely to save space. The results for 1911 may be read from Fig. 3. Fuller data will be found in Volume **3** of the *Annals* of this observatory soon to be published.

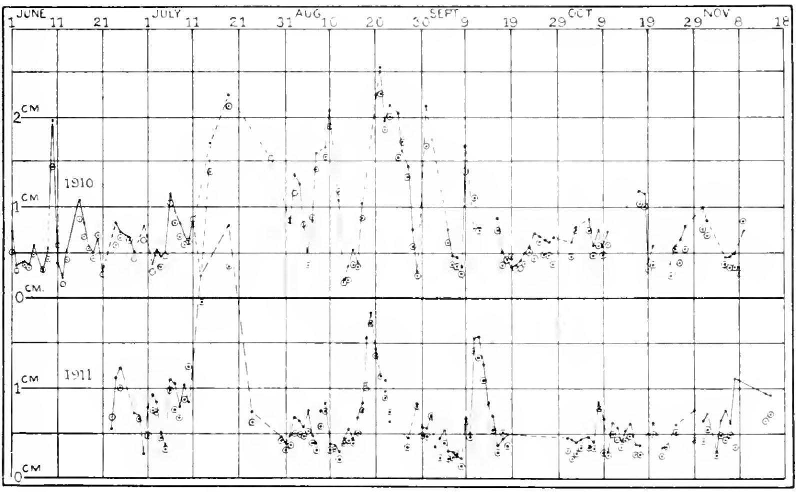


FIG. 3.—Aqueous vapor above Mount Wilson

Upper set is for 1910; lower for 1911; ordinates are depths of liquid water which, if in the form of vapor, would be contained in a column of the same section reaching to the limit of the atmosphere. Circles are the early morning values; lines connect the late morning points.

It may be noted that the values determined through ϕ are in general somewhat lower than those through ρ and ψ'. This may be because the slit correction (see earlier paper) is not complete. Again there is unfortunately a narrow absorption band very close to ϕ, due to the ultra-violet glass of which the prism of the spectroscope used at Mount Wilson is made, which may slightly influence the transmissibility for ϕ. The values for ψ' are on the average slightly lower than those for ρ. The calibration-curve (Fig. 1)

ATMOSPHERIC WATER-VAPOR, MOUNT WILSON, CALIFORNIA, 1910

Date 1910	Early Morning				Late Morning				Change	
	ρ	ϕ	ψ'	Mean *	ρ	ϕ	ψ'	Mean *	cm	Percentage
	cm	cm	cm	cm	cm	cm	cm	cm		
June 1....	0.53	0.47	....	0.51	0.83	0.63	0.71	0.72	0.21	0.22
2....	.33	.28	0.30	.30	.37	.33	.35	.35	.05	.17
4....	.41	.34	.37	.37	.42	.36	.38	.39	.02	6
5....	.38	.29	.34?	.33	.40	.24	....	.35	.02	6
6....	.55	.43	.46	.48	.65	.55	.57	.59	.11	23
8....	.32	.22	.38	.31	.37	.27	.26	.30	−.01	−3
9....	.46	.33	.50	.43	.56	.48	.62	.55	.12	28
10....	1.84	1.07	1.40?	1.44	2.02	1.73	2.22	1.99	.55	38
11....	0.71	0.50	0.56	0.59	0.42	0.32	....	0.39	−.20	−34
12....	.17	.12	.14	.14	.21	.17	0.24	.21	.07	50
13....	.44	.46	.37	.42	.55	.45	.56	.52	.10	24
16....	1.00	.76	.86	.87	1.11	1.01	1.12	1.08	.21	24
17....	0.76	.62	.70	.69	0.88	0.77	0.79	0.81	.12	18
18....	.57	.49	.56	.54	.64	.51	.52	.56	.02	4
19....	.48	.41	.40	.43	.54	.48	.45	.49	.06	14
20....	.77	.66	.67	.70	.71	.57	.69	.66	−.04	−6
21....	.31	.23	.23	.26	.32	.23	....	.29	.03	12
24....	.65	.54	.59	.59	.88	.73	.84	.82	.23	39
25....	.71	.63	.64	.66	.78	.73	.64	.72	.06	10
27....	.66	.57	....	.63	.71	.64	.67	.67	.04	6
28....	.47	.39	.40	.42	.58	.48	.47	.51	.09	22
30....	.69	.62	.61	.64	.84	.72	.82	.79	.15	24
July 2....	.31	.27	.28	.29	.39	.36	.32	.36	.07	24
3....	.56	.46	.48	.50	.58	.50	.51	.53	.03	6
4....	.37	.31	.31	.33	.51	.44	.47	.47	.14	43
5....	.57	.50	.34	.47	.57	.47	.47	.50	.03	6
6....	1.21	....	.87	1.04	1.22	1.05	1.19	1.15	.11	11
7....	0.92	.72	.78	0.81	1.02	0.91	0.95	0.96	.15	19
8....	.72	.62	.71	.68	0.93	.75	.76	.81	.13	19
9....	.65	.52	.61	.59	.75	.63	.71	.70	.11	19
10....	.66	.57	.64	.62	.67	.54	.62	.61	−.01	−1
11....	.87	.83	....	.86	.87	.73	.89?	.83	−.03	−4
15....	1.42	1.19	1.53	1.38	1.82	1.58	1.74?	1.71	.33	24
19....	2.19	....	2.10	2.14	2.38	1.97	2.45	2.27	.13	6
28....	1.62	1.31	1.67	1.53	1.61	1.37	1.59	1.52	−.01	−1
Aug. 1....	0.83	0.74	0.92	0.83	0.95	0.77	0.73	0.82	−.01	−1
2....	1.18	.94	1.33	1.15	1.41	1.19	1.47	1.36	.21	18
3....	....	....	....	....	1.31	1.07	1.35	1.24	....	....
4....	0.83	.73	....	0.80	0.89	0.73	0.73?	0.78	−.02	−2
5....	.40	.32	0.35	.36	.54	.43	.58	.52	.16	45
6....	.96	.80	.90	.89	.95	.73	.83	.84	−.05	−6
7....	1.51	1.26	1.45	1.41	1.67	1.42	1.70	1.60	.19	13
9....	1.70	1.43	1.53	1.55	1.78	1.52	1.66	1.65	.10	6
10....	1.91	....	....	1.91	2.17	1.89	2.22	2.09	.18	9
12....	1.28	0.94?	....	1.17	1.06	0.96	1.19	1.07	−.10	−9
13....	0.19	.15	0.17	0.17	0.15	.17	....	0.16	−.01	−6
14....	.22	.17	.20	.20	.42	.27	0.36	.35	.15	75
15....	.44	.33	.36	.38	.56	.46	.57	.53	.15	39
16....	.36	.30	....	.34	.43	.27	....	.38	.04	12
17....	0.94	.82	0.89	.88	1.12	0.98	0.98	1.03	0.15	17

* In taking the daily means, where ψ' has been lacking, double weight has been given ρ.

ATMOSPHERIC WATER-VAPOR, MOUNT WILSON, CALIFORNIA, 1910—*Continued*

Date 1910	Early Morning				Late Morning				Change	
	ρ	ϕ	ψ'	Mean *	ρ	ϕ	ψ'	Mean *	cm	Percentage
	cm	cm	cm	cm	cm	cm	cm	cm		
Aug. 21....	....	....	2.27	2.27	2.83	2.34	2.53	2.57	.30	25
22....	1.96	....	1.77	1.87	2.11	1.87	1.93	1.97	.10	5
23....	2.09	....	1.93	2.01	2.24	1.98	2.20	2.14	.13	6
25....	1.62	1.30	1.67	1.53	2.04	1.81	2.37?	2.07	.54	36
26....	1.83	1.48	1.85	1.72	1.75	1.56	1.53	1.61	−.11	−6
27....	1.48	1.12	1.34	1.31	1.40	1.26	1.67	1.44	.13	10
28....	0.63	0.49	0.60	0.57	0.77	0.63	0.84	0.75	.18	32
29....	.26	.21	.22	.23	.32	.21	.30	.28	.05	22
Sept. 1....	1.84	1.39	1.80	1.68	2.38	1.60	....	2.12	.44	26
5....	0.63	0.53	0.67	0.61	0.78	0.61	.86?	0.75	.14	23
6....	.40	.31	.39	.37	.52	.40	.53?	.48	.11	30
7....	.37	.31	.37	.35	.42	.41	.54	.46	.11	32
8....	.29	.21	.29	.26	.35	.35	.35?	.35	.09	35
9....	1.64	1.21	1.33	1.39	1.86	1.34	....	1.69	.30	22
11....	1.18	0.91	1.20	1.10	0.73	0.68	.89?	0.77	−.33	−30
12....	0.72	.57	0.66	0.65	.75	.65	.91?	.77	.12	19
16....	.80	.60	....	.73	.95	.73	....	.88	.15	21
17....	.39	.32	.38	.36	.53	.43	.58	.51	.15	42
18....	.44	.37	.43	.41	.48	.33	....	.43	.02	5
19....	.50	.39	.49	.46	.37	.30	.39	.35	−.11	−24
20....	.34	.27	....	.32	.38	.33	....	.36	.04	13
21....	.34	.27	....	.32	.43	.37	....	.41	.09	28
22....	.39	.34	.41	.38	.53	.44	.54	.50	.12	32
23....	.58	.42	.57	.52	.61	.49	.65?	.58	.06	12
24....	.47	.37	.46	.43	.77	.61	.75	.71	.28	66
25....	.63	.51	.71	.62	.73	.62	.73	.69	.07	12
26....	.50	.42	.53	.48	.66	.54	.73	.64	.16	33
27....	.51	.42	.50	.48	.66	.47	.72	.62	.14	29
28....	.41	.33	.40	.38	.75	.54	....	.68	.30	79
Oct. 2....	.53	.42	.47	.47	.64	.56	.64	.61	.14	30
3....	.83	.67	.74	.75	.92	.66	.76	.78	.03	4
6....	.82	.65	.72	.73	.95	.78	.89	.87	.14	19
7....	.51	.43	.49	.48	.62	.53	.61	.59	.11	23
8....	.64	.53	.60	.59	.77	.71	.82	.77	.18	31
9....	.58	.42	.48	.49	.63	.52	.60	.58	.09	19
10....	.66	.54	.60	.60	.77	.68	.78	.74	.14	23
17....	1.13	.93	1.04	1.03	1.43	.97	1.10	1.17	.14	14
18....	1.14	.90	1.00	1.01	1.14	1.08	1.22	1.15	.14	14
19....	0.37	.27	0.31	0.32	0.43	0.32	0.38	0.38	.06	19
20....	.40	.32	.36	.36	.63	.53	.62	.59	.23	64
24....	.27	.21	.24	.24	.38	.36	.43?	.39	.15	62
25....	.60	.49	.55	.55	.61	.53	.61	.58	.03	6
26....	.44	.34	.39	.39	.65	.63	.73	.67	.28	72
27....	.59	.49	.55	.54	.88	.69	.80	.79	.25	46
31....	.86	.67	.75	.76	1.11	.88	1.00	1.00	.24	32
Nov. 1....	.79	.61	.68	.69	0.88	.77	0.88	0.84	.15	22
5....	.46	.32	.36	.38	.49	.40	.47	.45	.07	19
6....	.32	....	....	.32	.46	....	....	.46	.14	44
7....	.36	.31	.36	.34	.50	.46	.53	.50	.16	47
8....	.29	.23	.27	.26	.41	.28	.34	.34	.08	31
9....	1.08	0.68	0.76	0.84	0.83	0.69	0.80	0.77	−.07	−8

* In taking the daily means, where ψ' has been lacking, double weight has been given ρ.

MEAN MONTHLY ATMOSPHERIC VAPOR FOR CLEAR DAYS

	June	July	Aug.	Sept.	Oct.	Nov.
	cm	cm	cm	cm	cm	cm
Early morning, 1910......	0.53	0.86	1.07	0.59	0.58	0.48
1911......	.78*	.86	0.61	.56	.37	.49
Late morning, 1910......	.62	.96	1.17	.70	.73	.56
1911......	0.76*	0.99	0.72	0.63	0.52	0.74

* Few observations.

MEAN YEARLY ATMOSPHERIC VAPOR FOR CLEAR DAYS

	cm
Early morning, 1910....................	0.71
1911....................	.55
Late morning, 1910....................	.82
1911....................	0.69
General mean.......................	0.69

CHANGE IN ATMOSPHERIC VAPOR DURING DAY OBSERVATIONS

RANGE	cm 0 to 0.05	cm cm 0.05 to 0.10	cm cm 0.10 to 0.15	cm cm 0.15 to 0.20	cm cm 0.20 to 0.30	cm above 0.30
1910........	22 days	19 days	34 days	8 days	12 days	4 days
1911........	14 "	21 "	21 "	8 "	9 "	7 "
Total...	36 "	40 "	55 "	16 "	21 "	11 "

	cm
Average change, 1910..................	0.13
1911..................	.14
Average increase, 1910..................	.11
1911..................	0.12

was determined from 1910 and 1911 observations; 1911 data would show the reverse effect.

COMPARISON WITH DETERMINATIONS FROM SURFACE HUMIDITIES

The formula in most general use for the determination of the aqueous vapor present in the atmosphere from measures of the humidity at the surface of the earth by the readings of the "wet and dry" thermometers, or otherwise, is that due to Hann.[1] If Q_w

[1] Hann, *Lehrbuch der Meteorologie*, pp. 224–226.

is the depth in centimeters of the precipitable water in the atmosphere above a place, e_w the vapor pressure in centimeters of the water-vapor at the surface, and K a constant, then

$$Q_w = \text{precipitable water} = Ke_w.$$

Hann's value for K is 2.3.

Humphreys has recently[1] collected considerable further data from balloon and kite observations and considers that a somewhat smaller value of K, namely, 2.0, would be preferable for "clear days" such as would presumably be suitable for bolometric observations. Hann's constant was determined from observations on "all sorts of days."

The above values of K are for determinations from sea-level surface-of-the-earth data; for the elevation of Mount Wilson (1.73 km) somewhat different values would be used. If e_o, e_w, and e_f are the pressures of the aqueous vapor in centimeters at sea-level, on a mountain, and in the free air, the two latter stations being at an altitude of h meters, then

$$e_w = e_o 10^{-\frac{h}{6500}}$$

and

$$e_f = e_o 10^{-\frac{h}{6000}\left\{1+\frac{h}{20000}\right\}}.$$

When $h=1730$ meters (Mount Wilson), then

$$e_w = 0.54 e_o$$

$$e_f = 0.49 e_o.$$

The more complete formula for Q is:

$$Q = 2.3 e_o \left(1 - 10^{-\frac{h}{5000}}\right).$$

Whence the total amount of precipitable water from sea-level to the limits of the atmosphere is, with Hann's constants,

$$Q_o = 2.3 e_o$$

and above the level of Mount Wilson

$$Q_w = 2.3 e_o (.45) = .45 Q_o,$$

whence

$$Q_w = 1.9 e_w.$$

[1] *Bulletin of the Mount Weather Observatory*, 4, 121, 1911.

A summation of the data collected by Humphreys for the air above the level corresponding to that of Mount Wilson with due allowance for the increase in the vapor pressure on a mountain over what it would be in the free air at the same altitude, viz.:

$$\frac{e_w}{e_f}=\frac{0.54}{0.49}=1.10\,,$$

gave

$$Q_w=1.7e_w\,.$$

However, if the data is summed by seasons, there results:

For spring	$Q_w=1.9e_w$
For summer	$=1.6e_w$
For fall	$=1.7e_w$
For winter	$=1.6e_w$.

In the following table are given the values of K necessary to get from the wet-and-dry determinations of the vapor pressure at the surface on Mount Wilson the amount of atmospheric water-vapor as measured by the spectroscope. Five sets of data are shown: the first two including some days graded as excellent in 1910 and 1911, respectively; the third set, days when the spectroscope indicated unusual constancy in the amount of aqueous vapor; the fourth and fifth, through two notable passages over Mount Wilson of very moist periods.

The following are the means from the various sets:

First	set:	early morning,	$K_w=1.94$,	late morning,	$K_w=1.55$
Second	"	" "	1.79	" "	1.36
Third	"	" "	2.82	" "	1.52
Fourth	"	" "	3.57	" "	1.96
Fifth	"	" "	1.53	" "	1.29
All 1910:		" "	1.94	" "	1.55
All 1911:		" "	2.29	" "	1.54
All:		" "	2.12	" "	1.54

Omitting the two abnormally large values of K_w, 10.4 and 11.8, the mean early morning value is

$$K_w=1.8\,.$$

COMPARISON OF ATMOSPHERIC WATER-VAPOR DETERMINATIONS BY THE SPECTROSCOPE AND BY SURFACE HUMIDITIES

Date	Q = Water by Spectroscope When		e_w = Surface Vapor Pressure When		$Q = Ke_w$ Values of K When	
	$m = 3.0$	$m = 1.2$	$m = 3.0$	$m = 1.2$	$m = 3.0$	$m = 1.2$
	First Selection, Excellent Days, 1910					
	cm	cm	cm	cm		
June 12.....	0.14	0.21	0.33	0.58	0.42	0.36
Aug. 13.....	.17	.16	.20	.41	.85	.39
Sept. 8.....	.26	.35	.20	.25	1.30	1.40
June 21.....	.26	.29	.54	.70	0.48	0.42
July 2.....	.29	.36	.45	.60	.64	.60
4.....	.33	.47	.38	.34	.87	1.38
Aug. 5.....	.36	.52	.30	.50	1.20	1.04
15.....	.38	.53	.43	.61	0.88	0.87
June 19.....	.43	.49	.41	.54	1.05	.91
July 5.....	.47	.50	.19	.40	2.47	1.25
June 6.....	.48	.59	.60	.71	0.80	0.83
Aug. 28.....	.57	.75	.34	.50	1.68	1.50
June 24.....	.59	.82	.70	.81	0.84	1.01
July 9.....	.59	.70	.35	.60	1.69	1.17
June 27.....	.63	.67	.53	.80	1.19	0.84
17.....	.69	.81	.63	.87	1.10	.93
July 7.....	.81	.96	.60	.95	1.35	1.01
June 16.....	.87	1.08	.75	.89	1.16	1.21
Aug. 17.....	.88	1.03	.085	.20	10.40	5.20
July 6.....	1.04	1.15	.28	.32	3.72	3.60
Aug. 2.....	1.15	1.36	.40	.62	2.88	2.20
June 10.....	1.44	1.99	.53	.71	2.72	2.80
Aug. 9.....	1.55	1.65	.75	.80	2.06	2.06
23.....	2.01	2.14	.71	.76	2.83	2.81
July 19.....	2.14	2.27	1.10	1.27	1.95	1.79
Aug. 21.....	2.27	2.57	0.60	0.85	3.78	2.99
	Second Selection, Excellent Days, 1911					
Oct. 3.....	0.27	0.40	0.32	0.55	0.84	0.73
Aug. 11.....	.31	.36	.38	.57	.82	.63
Oct. 10.....	.25	.28	.14	.36	1.79	.78
Aug. 1.....	.36	.48	.49	.62	0.74	.77
13.....	.39	.44	.43	.50	.91	.88
6.....	.39	.48	.75	1.00	.52	.48
29.....	.80	.81	.31	0.58	2.58	1.40
Sept. 14.....	.82	.81	.62	.63	1.32	1.29
July 6.....	.99	1.10	.29	.48	3.41	2.29
Aug. 19.....	1.71	1.84	.45	.57	3.80	3.23
July 13.....	1.96	2.24	0.67	0.91	2.93	2.46

COMPARISON OF ATMOSPHERIC WATER-VAPOR DETERMINATIONS BY THE SPECTROSCOPE AND BY SURFACE HUMIDITIES—*Continued*

Date	Q = Water by Spectroscope When		e_w = Surface Vapor Pressure When		$Q = Ke_w$ Values of K When	
	$m = 3.0$	$m = 1.2$	$m = 3.0$	$m = 1.2$	$m = 3.0$	$m = 1.2$
Third Selection, Constant Vapor by Spectroscopy, 1911						
	cm	cm	cm	cm		
Sept. 7.....	0.27	0.24	0.49	0.74	0.55	0.33
July 5.....	.31	.34	.62	.59	.50	.58
Oct. 10.....	.35	.28	(See above)	(See above)	1.79	.78
Sept. 10.....	.46	.50	.41	.50	1.12	1.00
Oct. 25.....	.53	.60	.52	.60	1.02	1.00
Sept. 9.....	.64	.67	.37	.65	1.73	1.03
1.....	.69	.70	.30	.45	2.30	1.56
Aug. 29.....	.80	.81	(See above)	(See above)	2.58	1.40
Sept. 14.....	.82	.81	(See above)	(See above)	1.32	1.29
Aug. 21.....	1.13	1.10	.18	.37	6.28	2.97
20.....	1.36	1.43	0.115	0.30	11.8	4.79
Fourth Selection, Group August 11 to August 23, 1911						
Aug. 11.....	0.31	0.36	(See above)	(See above)	0.82	0.63
12.....	.20	.29	0.24	0.53	.83	.55
13.....	.39	.44	(See above)	(See above)	.91	.88
14.....	.42	.56	.44	.63	.96	.89
15.....	.39	.42	.22	.33	1.77	1.27
16.....	.51	.67	.24	.40	2.13	1.67
17.....	.75	.83	.29	.38	2.59	2.18
18.....	1.+	1.57	.34	.50	....	3.14
19.....	1.71	1.84	(See above)	(See above)	3.80	3.23
20.....	1.36	1.43	(See above)	(See above)	11.8	4.79
21.....	1.13	1.10	(See above)	(See above)	6.28	2.97
22.....	0.90	0.97	.21	.57	4.29	1.70
23.....	0.73	0.62	0.11	0.40	6.64	1.55
Fifth Selection, Group September 7 to September 18, 1911						
Sept. 7.....	0.27	0.24	(See above)	(See above)	0.55	0.33
8.....	.12	.21	0.29	0.42	.41	.50
9.....	.64	.67	(See above)	(See above)	1.73	1.03
10.....	.46	.50	(See above)	(See above)	1.12	1.00
11.....	1.41	1.56	.39	.54	3.62	2.89
12.....	1.34	1.58	.52	.56?	2.58	2.82
13.....	1.10	1.27	.61	.65	1.80	1.95
14.....	0.81	0.81	(See above)	(See above)	1.32	1.29
15.....	.53	.69	.59	.76	0.90	0.91
16.....	.28	.35	.22	.43	1.27	.81
17.....	.51	.44	.26	.49	1.96	.90
18.....	0.35	0.50	0.32	0.50	1.09	1.00

However, means from figures having such a wide range, 0.33 to 11.8, have very little significance and are of very little service for the determination of water-vapor for any single day. It is perhaps interesting to note that this last mean value, $K_w=1.8$, lies just between the values of Hann and Humphreys, 1.9 and 1.7.

The best determinations of aqueous vapor by the spectroscope are probably those made nearer noon (smaller air masses) since they fall on the more accurately calibrated part of the curve connecting the transmissibility with the corresponding amount of water-vapor. On the other hand, the best determinations of K, which involves e_w, the surface humidity, are those of the early morning (large air masses), since the humidity at the earth's surface increases very rapidly toward noon. This increase is not nearly so marked away from the surface, although often indicated to some extent in the spectroscopic determinations as shown in some of the plots of Fig. 3. This greater increase in humidity at the surface over that in the free air is shown in the smaller values of K_w for the nearer-noon determinations.

From a consideration of the wide range of the values of K_w, 0.33 to 11.8, and the fact that sometimes the spectroscope shows a great amount of moisture in the free air when the surface humidity is very small, as for instance on August 20, 1911, it seems as if no other conclusion could be drawn than that any formula for determining the amount of atmospheric water-vapor from observed surface humidities is absolutely unsafe at least at Mount Wilson. Indeed it seems as if one ought to expect nothing else; for in the higher air the moisture in clear weather must be more or less conditioned by the quarter whence the wind comes, whereas near the surface the ground conditions and the surface convection currents will affect it.

A word or two as to the situation of Mount Wilson may be pertinent. The mountain rises fairly steeply from a more or less sandy plain near sea-level at the south to an altitude of about a mile (1.7 km); its surface is rocky, generally dry, with some few pines, stunted live-oaks, and low-lying shrubs. To the north and east rise somewhat higher, more or less bare, rocky mountains, yet farther north and east are extended desert regions; to the south-

east and southwest are low hills, then the Pacific Ocean some fifty miles away. During the months of these observations, there is very little rain, in some years none. During the night or the early morning a fog generally comes in over the valley to the south, at times high enough to cover the mountain. The pleasant-weather wind, when there is any, since calm conditions are very common, is generally westerly in the early morning; a southerly, more or less vapor-laden breeze is apt to set in toward ten o'clock to which is probably due the sudden increase then of the humidity near the surface and to some extent above.

SUMMARY

The quantity of precipitable water existing in the form of vapor between the top of Mount Wilson and the outer limits of our atmosphere during fair weather from June to November 1910, and 1911, has been determined by the spectrobolometric method described in detail in the *Astrophysical Journal*, **35**, 149, 1912.

The average quantity present was 0.69 cm and the range from 0.2 cm to about 2.8 cm of precipitable water. The difference in monthly means would be small but for a few exceptionally moist days in August; almost the dryest day indeed for 1910 was August 13 (0.17 cm) and the dryest for 1911, September 12 (0.12 cm).

A gradual but generally slow increase in atmospheric water-vapor often takes place during the observations which extend from about 7 A.M. to 10 A.M. This averages 0.12 cm. For about 40 per cent of the days this increase is less than 0.1 cm.

These spectrobolometric results were then used in a study of the formula of Hann which, with a coefficient determined from balloon and kite observations, has been in use for connecting surface humidities with the quantity of aqueous vapor in the atmosphere. This coefficient was redetermined by means of the data above discussed. The general mean for the coefficients (1.8) agrees closely with that derived by Hann (1.9) or from Humphrey's data (1.7). The range of values is, however, so great (from 0.33 to 11.80) that we must regard the formula, though applicable for mean conditions, as of no value for individual days.

ASTROPHYSICAL OBSERVATORY, SMITHSONIAN INSTITUTION
WASHINGTON, D.C.

THE PLANE GRATING FOR STELLAR SPECTROSCOPY

By J. S. PLASKETT

At the meeting of the committee of the Astronomical and Astrophysical Society of America on Co-operation in Radial Velocity Determinations at Mt. Wilson in 1910, the question of increasing the efficiency of the modern stellar spectrograph was discussed and various suggestions were offered. The writer, with the consent of Dr. King, director of the Dominion Observatory, decided to test a plane grating as the dispersion piece of a stellar spectrograph. Some earlier experiments on the use of gratings in stellar spectroscopy were made by Poor and Mitchell,[1] by Wadsworth,[2] and by others but without very encouraging results. The difficulty was probably due to the loss of light entailed by its distribution over several spectra and can be overcome by obtaining a grating giving a strong concentration in one order. Gratings have been ruled by Rowland giving a very strong first-order spectrum, and both Michelson and Ames considered that it should be possible to rule gratings giving a large proportion, as much as 75 per cent of the incident light, into one order. Even if this efficiency was not reached, grating spectra should compare favorably with those produced by three-prism dispersion especially toward the violet, where the dense flint glass used in most prism trains is strongly absorbing.

Consequently a plane grating to give the strongest possible first-order spectrum was ordered through the J. A. Brashear Co., in October 1910, was ruled by Dr. J. A. Anderson of Johns Hopkins University, and received in January 1912. The grating is a 5-inch plane with ruled surface 7.4×9.4 cm ($2\frac{7}{8} \times 3\frac{3}{4}$ inches), ruling 15,000 to the inch. The total number of lines is 55,875, which will be the resolving power in the first order if the whole aperture is used. It was estimated by Dr. Anderson to diffract about 50 per cent of the incident light into one first order, but, as will be seen later, this seems to be an overestimate.[3]

[1] *Astrophysical Journal*, **7**, 157, 1898.

[2] *Ibid.*, **7**, 198, 1898.

[3] See note at end.

Considerable thought was given to the optical design of the spectrograph, and three methods of producing a spectrum were combined in the one instrument. In the first of these, the usual method, in which a parallel pencil from a doublet collimator objective of 57 mm (2.25 in.) aperture is diffracted back, making an angle of about 30° with the initial direction, is employed, the spectrum being formed by a triplet camera objective of 63.4 mm (2.5 in.) aperture and 507 mm (20 in.) focus. In the second method the autocollimating or Littrow principle is used, the combined collimator and camera objective being a triplet of 63.4 mm (2.5 in.) aperture and 951 mm (37.5 in.) focus. The spectrum is formed in the plane of, and as close as possible to, one end of the slit. The same principle is used in the third method, the grating being replaced by a half-prism of 0.102 glass having angles of 31°.5, 58°.5, and 90°, and silvered on the side opposite the 58°.5 angle. The light is refracted into the prism on the hypotenuse and, when at minimum deviation, is incident normally on the silvered side and returned along its original path, except for the slight inclination necessary to bring it on the plate. The use of the half-prism for radial velocity work was proposed by Professor Campbell at the meeting above referred to, and I understand a stellar spectrograph has been constructed at the Lick Observatory from his designs which gives very promising results. Nevertheless, as no change was needed in the grating spectrograph except the substitution of the half-prism for the grating, it was thought worth while to make tests of this type.

The linear dispersions of the three forms are 33.0 Å, 17.5 Å, and 17.5 Å per millimeter at H_γ. These values are almost identical with those given by spectrograph I (33.4 Å) and spectrograph III S (17.5 Å), one- and three-prism spectrographs of the Dominion Observatory, thus enabling accurate comparisons of relative intensities to be easily made.

The mechanical design of the spectrograph follows the supported box form, first introduced by Campbell at the Lick Observatory, of which successful examples besides those at Mount Hamilton and in Chile are the Mellon spectrograph at Allegheny and the single-prism spectrographs at Ottawa and Ann Arbor. The box is

made of brass plates firmly screwed together at the angles and thoroughly braced internally. Owing to its relative compactness as compared with the Ottawa one prism, it was not thought necessary to introduce a third counterbalancing support as in the latter instrument, but the box is held flexibly and yet firmly in the braced T iron frame by two supports so placed as to reduce flexure to a minimum. The objectives, which define beautifully, and the reflecting slit were made by the J. A. Brashear Co., but all other parts of the instrument were constructed in the observatory workshop.

Preliminary tests and adjustments made in the laboratory, using the carbon arc as the light-source, showed that the field of the spectrum was practically flat over the 4 degrees included on the plate and that the definition of all three forms was excellent.

The method adopted for obtaining the comparative intensities of prismatic and grating spectra was to make exposures successively on the same celestial objects, both sun and stars being used. Five spectra, side by side on the same plate, were made of the object by each instrument, the exposure times in every case being proportional to the numbers 1, 1.5, 2, 3, 4. For example, on *Procyon*, 5 exposures were made with III S, 5 with the Littrow grating, 5 with the Littrow half-prism, and then 5 with III S, all three instruments having the same dispersion. A similar test was carried through with the first form of the grating spectrograph and the one-prism instrument, both of the same dispersion, and repeated for other stars and for the sun. As the linear dispersions were the same, the spectra of the same width, the observing conditions fairly constant, the exposure times the same, and the plates developed together for the same time, the intensity of the resulting negatives gives a direct comparison of the intensity of the spectra. As nearly equal intensities on the plates can be compared, little photographic error is likely to occur. The results of these comparisons are given in the subjoined table.

The most striking feature in the appearance of the grating as compared with the prismatic spectra—and this is also evident from the table—is the remarkable uniformity in intensity between H_β and λ 3850. This uniformity is perhaps the most useful property

RELATIVE INTENSITIES OF SPECTRA

Wave-Length	Sun			Procyon				Rigel				Mean (Stars)			
	G.	3 P.	½ P.	G.	3 P.	½ P.	1 P.	G.	3 P.	½ P.	1 P.	G.	3 P.	½ P.	1 P.
H_β	...	2.0	3.0	0.6	2.0	4.0	5.0	0.7	2.0	3.5	5.0	0.7	2.0	3.7	5.0
4800	0.9	3.2	4.0	1.0	3.0	5.5	6.5	1.0	5.0	5.0	6.0	1.0	4.0	5.2	6.2
4700	1.3	4.5	4.5	1.3	2.5	5.5	6.5	1.2	4.2	5.0	6.0	1.2	3.3	5.3	6.3
4600	1.5	5.2	4.5	1.3	2.0	5.0	6.0	1.2	3.0	5.0	5.6	1.3	2.5	5.0	5.8
4500	1.3	4.5	4.0	1.2	1.8	4.0	5.0	1.2	1.9	3.8	5.0	1.2	1.8	3.9	5.0
4400	1.1	3.0	3.5	1.1	1.4	3.0	3.9	1.1	1.5	2.8	4.3	1.1	1.5	2.9	4.1
H_γ	1.0	2.3	3.0	1.0	1.3	2.5	3.3	1.0	1.4	2.3	3.5	1.0	1.3	2.4	3.4
4300	1.0	2.0	2.8	1.0	1.1	2.5	3.0	1.0	1.1	2.3	3.0	1.0	1.1	2.4	3.0
4200	1.1	1.2	2.6	1.0	0.7	2.3	2.5	1.0	0.8	2.0	2.4	1.0	0.8	2.2	2.5
4150	1.1	0.7	2.0	1.1	0.3	2.2	2.2	1.0	0.3	1.8	2.0	1.0	0.3	2.0	2.1
H_δ	1.1	0.5	1.8	1.1	0.1	2.0	2.0	1.0	0.1	1.6	1.8	1.1	0.1	1.8	1.9
4050	1.0	0.2	1.6	1.1	...	1.7	1.7	1.0	...	1.4	1.5	1.0	...	1.5	1.6
4000	1.0	...	1.3	1.0	...	1.4	1.3	1.0	...	1.2	1.2	1.0	...	1.3	1.3
H	1.0	...	1.0	1.0	...	1.0	1.0	1.0	...	1.0	1.0	1.0	...	1.0	1.0
K	0.9	...	0.7	1.0	...	0.8	0.8	1.0	...	0.8	0.8	1.0	...	0.8	0.8
3900	0.8	...	0.2	0.9	...	0.5	0.6	0.9	...	0.4	0.4	0.9	...	0.4	0.5
3850	0.3	...	...	0.6	...	0.1	0.2	0.8	...	0.2	0.2	0.7	...	0.1	0.2
3800	...	...	...	0.2	...	...	...	0.6	...	...	...	0.4	...	...	...
3750	...	...	...	...	...	...	...	0.3	...	...	...	0.2	...	...	...

of the grating spectrograph. The contrast between grating and prism in this respect is very striking, as prismatic spectra are over ten times, grating spectra only one and a third times, as intense at λ 4700 as at λ 3900. This difference is due in the main to two causes: first, the increased dispersion toward the violet and diminished toward the red of prismatic spectra (dispersion at K one and a half times, at H_β two-thirds, that at H_γ), while diffraction spectra are nearly normal; and second, the strong absorption of the prism glass for the shorter wave-lengths. Calculations from Vogel's constants for 0.102 glass show that through 5 cm of this glass, about the mean length of path through the single and the half-prism, 33 per cent is transmitted at the K line as compared with 71 per cent at H_γ and 85 per cent at H_β. The effect of this absorption is strikingly shown in the table where the relative intensity of three-prism as compared with single- and half-prism spectra toward the violet is given.

Discussing first the relative intensity of three-prism and grating spectra, we find the former has the advantage from H_β to about λ 4300, while from λ 4200 down grating spectra are decidedly superior, three-prism spectra disappearing below H_δ. If a spectrum in which the region from λ 4300 to the ultra-violet is required, only the grating could be used, while from H_β to H_γ the prisms would have the advantage. In other words for early-type stars use the grating, for solar type, three-prism dispersion.

Comparing next grating with single- and half-prism spectra we find the advantage lies decidedly with the latter above the H and K lines, but if the K line is required, as is the case in many early-type stars, it can be obtained with the grating in the same time as with the one-prism spectrograph without making the lines between H_β and H_γ immeasurable by overexposure.

If now the prismatic spectra are intercompared, the superiority of one- over three-prism spectra is markedly shown. The former gives nearly three times the intensity above λ 4200 and a much greater ratio below, a difference of more than a magnitude in the stars within reach of the telescope. It must not be forgotten, however, that this difference is partially offset by the threefold greater resolving power of three prisms, although in photographic

spectra their high resolution cannot be effectively employed and it is of no value in early-type stars.

A final interesting comparison is that between the one-prism and the half-prism spectra. The former gives spectra about 20 per cent more intense than the latter, a result to be expected when the loss at the silvered reflecting surface is taken into account and when the larger aperture of the half-prism (63.4 mm as compared with 51 mm) and the consequent greater length of optical path are considered. It is evident that more intense spectra would be secured by using the regular one-prism instrument rather than the half-prism, making the camera the same length as collimator. The further advantage of narrower lines for the same slit-width would be obtained by increasing the length of collimator, camera remaining the same. The advantages of the half-prism instrument are its much simpler and more compact and self-contained mechanical form, avoiding some of the flexure and temperature difficulties occurring with the extended one-prism spectrograph.

In conclusion, it may be said that although the spectra obtained from the grating are disappointingly weak and show that the proportion of the incident light diffracted into the spectrum used is nearer 30 than the 50 per cent estimated by Anderson[1] or the 75 per cent considered possible, yet even under this handicap it can be used to advantage when the K line is required and if spectra of uniform intensity or of uniform dispersion are needed. It would also be useful in the red end where prismatic spectra are so unduly compressed. If a grating giving twice the intensity could be obtained it would be superior even to single-prism dispersion for most work.

The relative flexure of prism and grating spectrograph should also be considered. It is well known that a small change in the position of a prism when at minimum deviation does not displace the spectrum, while the angular movement of a diffracted pencil will be twice that of the grating. Tests of the single-prism spectro-

[1] I have learned since the above was written that Dr. Anderson's estimate referred not to the incident light but to the percentage returned by the speculum metal surface. As the reflectivity of speculum metal is about 65 per cent the two estimates are in good agreement.

graph showed very small flexures of the order of 2 or 3 km per second. A similar test of the half-prism spectrograph showed maximum flexure of about 7 km and of the grating spectrograph of about 20 km per second. In most exposures the displacement caused by the change of position in hour-angle will not much exceed one-tenth of these amounts, and it is evident that, with proper distribution of the comparison exposures, there should be no appreciable effect on the velocity measures. Hence the question of flexure need introduce no serious difficulties in the use of a grating spectrograph.

I have much pleasure in acknowledging the interest of the Director in this work and his willingness to supply the apparatus required.

Dominion Observatory
Ottawa
January 1913

THE RELATION BETWEEN BLACK-BODY AND TRUE TEMPERATURES FOR TUNGSTEN, TANTALUM, MOLYBDENUM, AND CARBON, AND THE TEMPERATURE VARIATION OF THEIR REFLECTING POWER

By C. E. MENDENHALL and W. E. FORSYTHE

The increasing use of various electrically heated metallic filaments in a variety of physical experiments, together with the difficulty of determining the true temperature of such filaments, suggested the desirability of securing data which might be useful in determining the true temperature of the radiating surface from the more readily observed optical black-body temperature. Furthermore, this question is intimately connected with that of the variability or constancy of the optical reflecting power of metals, and the observations up to the present time have supported the surprising generalization that the reflecting power of metals for visible wave-lengths was sensibly independent of temperature changes.

Accordingly, with these two points in view, the observations mentioned in the following pages have been made from time to time during the past two years. In the meantime Pirani and Meyer[1] have published data on some of the same metals, covering somewhat the same temperature range; but our results do not agree with theirs where they overlap, while our method seems essentially simpler and more direct.

Method.—For a discussion of the validity and accuracy of the method used by us, reference must be made to a previous paper;[2] the essential feature is the production of black-body radiation by forming a narrow V-shaped radiator either by folding a flat strip parallel to its length, or by combining two flat strips touching at one edge. Observations with a calibrated optical pyrometer taken in the mouth of the V will then give the true temperature of the inside

[1] *Verhandlungen der deutschen physikalischen Gesellschaft*, **14**, No. 5, p. 213; No. 13, p. 681.

[2] Mendenhall, *Astrophysical Journal*, **33**, 91, 1911.

surface, while observations on the flat side will give the "black-body" temperature of the outside surface. In the paper referred to it was shown that for metal strips of the dimensions used, the *real* difference of temperature between the outer and inner surfaces might be expected to be so small that it could be neglected; further evidence on this point will be brought forward in the present paper.

As further evidence of the reliability and accuracy of the method, the following observations of the true (T) and black-body (S) temperatures of the melting-point of palladium may be quoted:

	T	S
	1549° C.	1378° C.
	1548	1387
	1546	1384
Mean	1548−	1385 (weighted)

For the melting-point of palladium Day and Sossman have directly determined $T=1549°$ C., while Waidner and Burgess observed $S=1387°$ C. Other schemes of more or less closely realizing black-body radiation have been used by Pirani,[1] but none, it seems to us, as simple and at the same time as certain to realize the desired conditions, as that here used.

Temperature scale.—Our basis throughout is the high temperature scale adopted by the Bureau of Standards according to which the melting-points of gold and palladium are 1063° C. and 1549° C. respectively, while in determining the radiation scale in terms of which all our true temperatures have been expressed, Wien's law in the form—

$$\log E_\lambda = K - \frac{1}{T} \cdot \frac{c_2}{\lambda}$$

with the value $c_2=14{,}500$, has been used.

Since the wave-length used in observing was $0.658\,\mu$, the maximum value of the product λT is 2170, well within the limit $\lambda T=3000$ for which Wien's law is supposed to be essentially valid. The validity of Wien's law at extreme high temperatures should be subjected to further study, and such study is in fact now under way in this laboratory. As to the errors involved in applying Wien's

[1] *Verhandlungen der deutschen physikalischen Gesellschaft*, **8**, 427, 1912.

law with the only approximately monochromatic light transmitted by our red-glass screens, we have also tested this[1] up to 2500° C. and have found no systematic error due to this cause.

Pyrometers.—These were of the Holborn-Kurlbaum form already described,[1] though not of the spectroscopic type. Instead, red glass having a maximum transmission at $\lambda = 0.658\ \mu$ was used in the eyepiece; accordingly our *black-body* temperatures apply to observations with this wave-length. The pyrometers were calibrated from the melting-point of palladium directly determined with a platinum-wound black-body furnace.

Preparation of the strips.—The strips were about 3 cm long, and from 5 to 8 mm wide along the side of the V, while the angular opening was about 10°. The metal strips varied in thickness in different cases from 0.02 to 0.04 mm. The tantalum was rolled from heavy sheet obtained from Siemens and Halske, and the molybdenum rolled from a ductile specimen kindly furnished by the General Electric Co. In each of these cases the V was formed by folding. The tungsten was of the ductile variety also furnished by Director Whitney of the Research Laboratory of the General Electric Co., ground down to the desired uniform thickness, the V being formed by two flat strips touching along one edge. We were unable to obtain carbon of the kind used in incandescent lamps, in the form of broad strips flat enough to make a satisfactory V, though the General Electric Co. very kindly made some strips for us of both metalized and untreated carbon. We were therefore forced to use a very compact fine-grained plate carbon, of which we had two varieties, one from Conradti and the other from the National Carbon Co. No difference was found between them. It was found impracticable to use carbon strips thinner than 0.08 mm, and on this account and because of the relatively low thermal conductivity of carbon, it was impossible to neglect the real temperature difference between the inner and outer surfaces of the V. The simplest way of eliminating this difference seemed to be to determine the apparent difference of temperature for a constant inside temperature, with several thicknesses of wall, and to graphi-

[1] *Physical Review*, **33**, p. 74, 1911.

cally extrapolate to zero thickness. The curves by which this was done are shown in the corner of Fig. 4, the three thicknesses being 0.08, 0.12, and 0.20 mm. In carrying out the extrapolation, consideration was given to fitting an individual curve to the group as a whole.

Condition of the surfaces.—In order to realize black-body conditions on the inside of the V it is desirable to have the side walls plain and fairly well polished. The initial surface of our metals was either that given by polished rollers or that obtained by polishing with the finest grade of emery paper. The latter treatment was also given the carbon surfaces. After a little use at high temperatures, however, the surfaces of all the materials are changed more or less by sublimation, and in the case of carbon probably also by slight graphitization. Our results are intended to apply to such *used* surfaces. In the case of carbon the effect of heating (of course *in vacuo*) is to make the surface less compact, undoubtedly permanently lowering the reflecting power. In the case of the metals, while the surface is altered considerably in appearance, especially with tungsten, this alteration seems to consist mainly in the development of *facets* which destroy the flatness of the surface, though the individual facets seem to be highly polished. These changes seem to us to be such as would alter the *specular* reflecting power, but not seriously change the emissive or absorbing power of the surfaces. We found that more consistent results were obtained if the filaments were given a preliminary heating of a minute or so, at a fairly high temperature before observations were begun.

Arrangement of apparatus.—All the filaments were heated *in vacuo*. The vacuum vessel was provided with an arrangement for taking up the expansion of the filaments, and thus preserving the shape of the V, a very important consideration. The vessel was continually exhausted by a Gaede pump, the pressure being usually a few thousandths of a millimeter, as measured on a McLeod gauge. When using tantalum it was found desirable also to heat calcium in the vacuum before heating the filament. Pyrometer observations were made through glass windows, the correction for which was carefully determined.

Errors.—1) Calibration errors. These (assuming the validity of Wien's law) we estimate to be not more than $\pm 3^\circ$ at the extreme high temperatures.

2) Correction due to glass window. This was determined for *clean* glass with a maximum uncertainty of not more than 2° at the highest temperature, but much greater errors doubtless entered with a few individual observations through failure to detect slight deposits of sublimed metal or carbon on the inside of the windows.

3) Failure to realize black-body conditions on the inside of the V, due to warping or welding together of the sides of the V, or to the opening of a crack at the back of those V's made from two pieces. No observations were taken under obviously bad conditions. These errors *always* tend to diminish the apparent difference in temperature between inside and outside of the V, hence in general the larger differences are more probably correct. In general the longer a V is used, especially at high temperatures, the more care must be taken to avoid the welding together of the sides of the V.

4) Variable conditions of outside surface of the V. We are inclined to minimize the magnitude of such uncertainties.

5) A real difference of temperature between outside and inside of wedge. Besides the evidence already referred to showing that this difference is small, we have made observations with different thicknesses of molybdenum filaments varying from 0.028 mm to 0.038 mm, without being able to detect any difference due to this cause.

6) The variation of temperature across the flat side of the V is usually small, and may be made quite negligible by properly varying the thickness of the strip from edge to center. Our observations were made on the middle of the flat side; in a few cases there was quite a considerable variation, from 25° to 50° across the face of the strips at very high temperatures.

7) Errors due to unsteady electrical and temperature conditions. The much greater consistency of the observations on molybdenum as compared with those on tantalum and tungsten is partly due to better control and partly to greater care in avoiding errors 3) and 6).

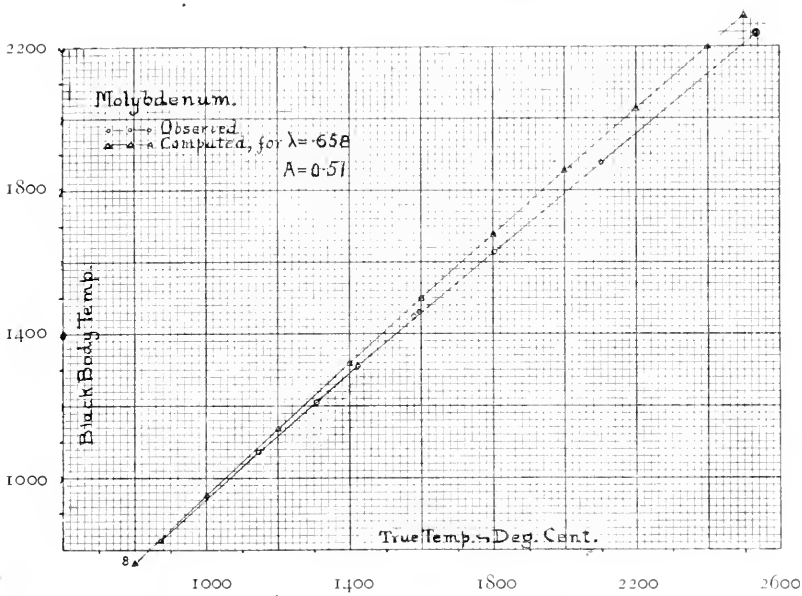


FIG. 1.—Relation between true temperature (T, abscissae) and black-body temperature (S, ordinates), for molybdenum, both observed and computed.

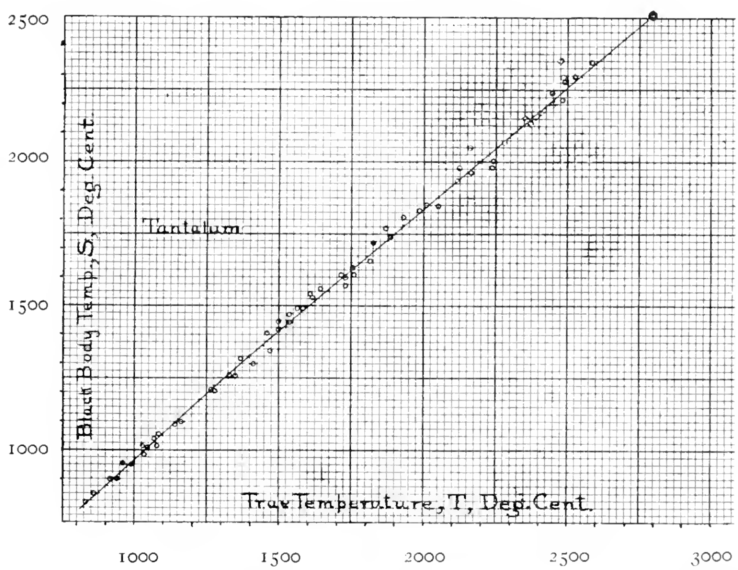


FIG. 2.—Relation between true temperature (T, abscissae) and black-body temperature (S, ordinates), for tantalum; observed values only.

Results.—The results are shown in Figs. 1–4, and the table gives the co-ordinates of a number of points on the lines, giving in each case the graphical average. Considerable weight is given to the melting-points, because they depend partly on independent data already published by one of us,[1] and because in general

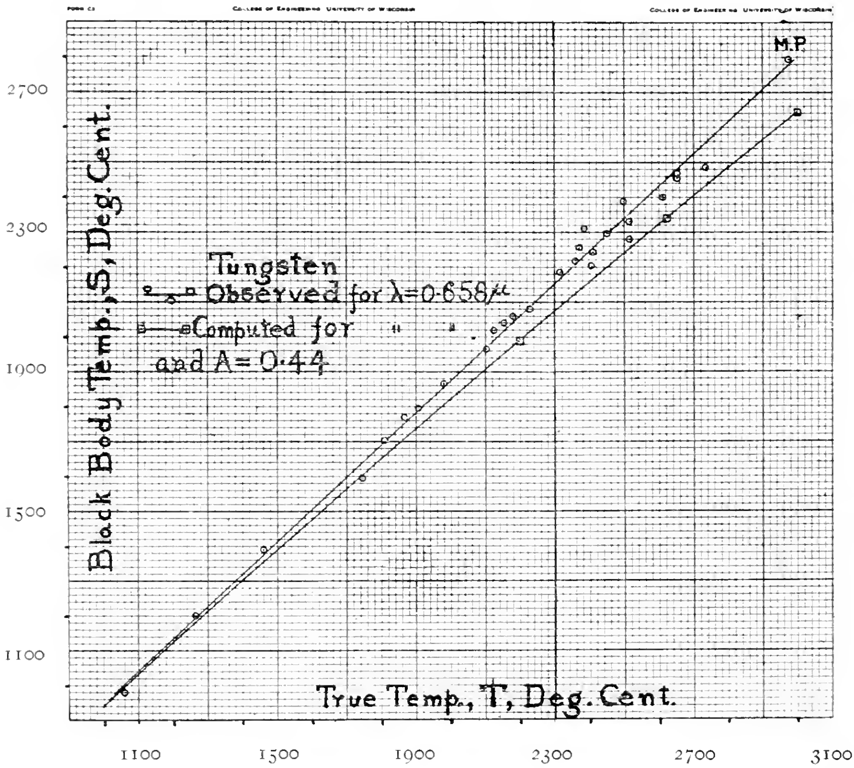


FIG. 3.—Relation between true temperature (T, abscissae) and black-body temperature (S, ordinates), for tungsten, both observed and computed.

black-body conditions were better with the fresh filaments used for melting.

In drawing the line through the observed points, some attention has been paid to the fact that the most probable error (3) tends to make $T-S$ too small. It will be noticed that a slight curvature of the T, S-curve is always indicated, especially for the lower range of

[1] *Astrophysical Journal*, **34**, 353, 1911.

temperatures, though in the case of molybdenum the relation is linear from 1500° up, and for tungsten a linear relation from 1000° to 3000° is all that the observations warrant.

Variation of optical reflecting power with temperature.—The equation

$$\log A = \log(1-R) = \left(\frac{1}{T} - \frac{1}{S}\right)\frac{c_2}{\lambda} \log e,$$

an immediate consequence of Wien's law first suggested by Holborn and Henning, has been used by Wartenberg, Pirani, and

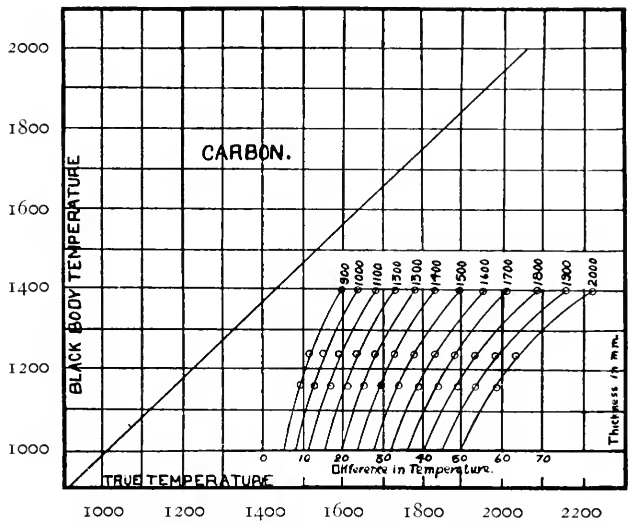


FIG. 4.—Relation between true temperature (T, abscissae) and black-body temperature (S, ordinates), for carbon, observed; curves used for determining corrections due to thickness of strip.

others to compute T from S for various metals, assuming R to be the same at high temperatures as at low. Concerning the constancy of R there is the evidence of Holborn and Henning[1] for platinum up to the melting-point; of Hagen and Rubens[2] for Pt and $Pt+Rh$ up to 1450° C. (for $\lambda = 0.78\ \mu$) and the approximate figures of Wartenberg for palladium and rhodium. None of this evidence seems to us at all conclusive; and in fact Holborn and Henning's *computed* value of the melting-point of platinum, when compared with the value at present accepted, really indicates an increasing

[1] *Berl. Akad. Ber.* (1905), p. 311. [2] *Preuss. Akad. Ber.*, **23**, 467, 1910.

rather than a constant reflecting power. Our curves seem to be quite inconsistent with the assumption of a constant reflecting power, and to point definitely to a change in the optical reflecting power with temperature, an *increase* for molybdenum, tantalum, and carbon, and a *decrease* for tungsten. The case is especially

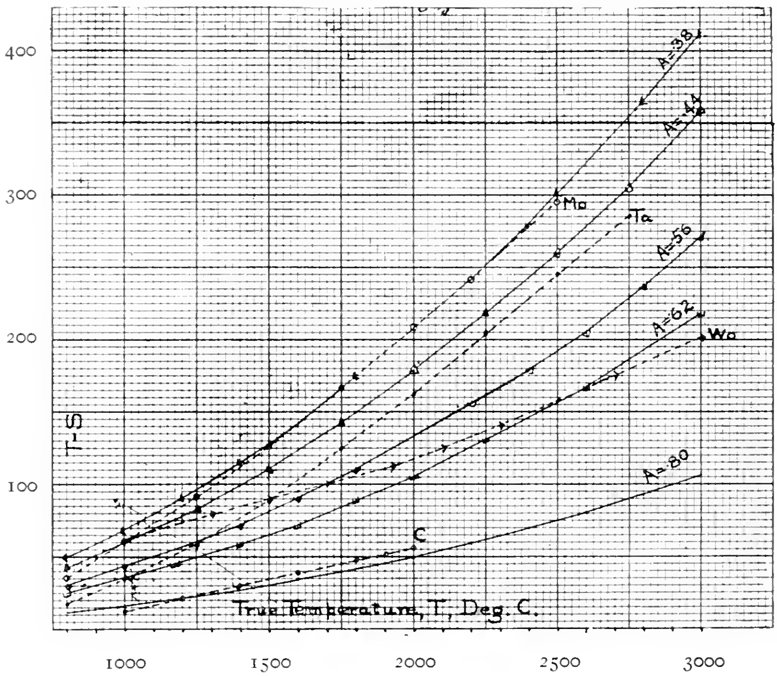


FIG. 5.—Broken curves, observed values of $T-S$ and T for the several metals; solid curves, values of $T-S$ and T computed for the several reflecting powers noted, by the equation

$$\frac{1}{T} \cdot \frac{1}{S} = \frac{\lambda \cdot \log(1-R)}{C_2 \log e}$$

clear for *Mo* and *Wo*—for as may be seen from Figs. 1 and 3, the T, S-curves at 800 to 1000° agree with those computed from the low-temperature reflecting power, but depart markedly from these computed curves at high temperatures. The absorbing power used for ductile tungsten at ordinary temperatures is based on the recent measurement of Littleton,[1] allowance being made for the change in λ from $0.589\,\mu$ to $0.658\,\mu$, in accordance with data

[1] *Physical Review*, **35**, 306 (1912).

quoted by Burgess.[1] While our curve indicates a change in R for tantalum, it must be said that a T, S-curve computed for $R=0.51$ comes more nearly fitting the experimental curve than will any curves, computed with R constant, for the other metals. But this value of R does not agree with Wartenberg's[2] observations at ordinary temperatures.

TABLE I

BLACK-BODY AND TRUE TEMPERATURES

Black-Body Temperature	Corresponding True Temperature			
	Tungsten	Tantalum	Molybdenum	Carbon
800° C.	. . . . . .	. . . .	840	. . . .
1000	1068° C.	1035	1068	1012
1200	1273	1255	1295	1222
1400	1486	1485	1530	1430
1600	1700	1720	1770	1638
1800	1910	1955	2010	1847
2000	2126	2190	2250	2056
2200	2345	2430	2490	. . . .
2400	2565	2665	. . . .	. . . .
2600	2783	. . . .	. . . .	. . . .
2700	2890	. . . .	. . . .	. . . .

The observations in their bearing on reflecting power are summarized in Fig. 5—from which it may be seen that the reflecting powers vary as follows:

Mo	1000° C.	0.56
	2400° C.	0.63
Ta	1100° C.	0.40
	2600° C.	0.52
Wo	1100° C.	0.55
	2900° C.	0.34
C	1000° C.	0.14
	2000° C.	0.21

While our individual observations, except in the case of molybdenum,[3] are subject to large accidental errors, the graphical average

[1] *Measurement of High Temperatures* (1912), p. 497.

[2] *Verhandlungen der deutschen physicalischen Gesellschaft*, **12**, p. 105, 1910.

[3] Most of the points plotted on the molybdenum curve are the means of several *sets* of observations taken at different times and on different strips, but the variations (with one exception) are of the order of 1° to 3°. This degree of consistency among the observations is largely due to careful control of the heating current, in such a way as to maintain S (apparent temperature outside of V) constant, while several observations of T (temperature inside of V) were made.

we consider to be fairly accurate, and on the whole to indicate very decidedly the variations in reflecting power stated above.

Summary.—Data have been given in the form of curves and tables from which the true temperature (T) of radiating surfaces of tungsten, tantalum, molybdenum, and pressed carbon may be determined from observations of the black-body temperature (S) using $\lambda = 0.658\,\mu$. From these curves it is concluded that the optical absorbing power of these substances varies with temperature instead of being constant as hitherto considered.

DEPARTMENT OF PHYSICS
UNIVERSITY OF WISCONSIN
February 1913

ON THE PRESSURE-SHIFT OF IRON LINES

By H. G. Gale and W. S. Adams

In the course of an investigation of the displacements of the iron lines at a pressure of 8 atmospheres made in 1910 and 1911[1] we found that most of the lines could be classified according to the amount of their pressure-shifts into several groups. The first of these, group *a*, consists mainly of the low temperature "flame" lines. The average displacement for this group at λ 5000 is between 0.003 and 0.004 Ångström. For group *b* the displacements are large, and for groups *c* and *d* larger yet, averaging in the case of group *d* about 0.02 Ångström per atmosphere. The lines of groups *c* and *d* become very wide and diffuse under pressure, and in the case of group *d*, at least it was impossible to make more than a rough estimate of the amount of shift.

A most important addition to these groups was later made by St. John and Ware[2] who in their article on "Tertiary Standards with the Plane Grating" showed that certain lines in the iron spectrum are displaced to the violet by pressure. Their results were obtained by comparing measurements of the iron arc lines on photographs taken on Mount Wilson and in Pasadena, at a difference in altitude corresponding to a difference of about one-fifth of an atmosphere in barometric pressure. These lines were classified as group *e* and the shifts were found to be relatively large.

On account of the diffuseness of the lines of groups *c* and *d* at a pressure as high as 8 atmospheres it seemed desirable to repeat the measurements of these lines at much lower pressures, including also the lines of group *e* which are quite unmeasurable at considerable pressures.

For this purpose Mr. Swaim kindly obtained a number of photographs at pressures ranging from a partial vacuum of 5 or 10 cm

[1] *Astrophysical Journal*, **35**, 10, 1912.

[2] *Ibid.*, **36**, 14, 1912.

Wave-Length	Group	No. of Measures	Δ per Atm.	Δ 8 Atm.
4859.928	*c*	5	+0.017	+0.100
4871.512	*c*	5	+0.018	+0.080
4872.332	*c*	5	+0.017	+0.094
4878.407	*c*	5	+0.016	+0.087
4890.948	*c*	5	+0.016	+0.070
4891.683	*c*	5	+0.014	+0.052
4919.174	*c*	5	+0.016	+0.072
4920.685	*c*	5	+0.014	+0.082
4957.480	*c*	5	+0.013	+0.083
4957.785	*c*	5	+0.016	+0.086
4994.316	*a*	5	+0.001	....
5012.252	*a*	5	+0.001	....
5051.825	*a*	5	+0.004	....
5065.207	*e*	4	−0.016	....
5074.932	*e*	7	−0.007	....
5083.518	*a*	3	+0.003	....
5097.175	*e*	4	−0.008	....
5110.574	*a*	6	+0.005	....
5125.300	*d*	5	+0.026	....
5133.870	*e*	8	−0.020	....
5162.449	..	7	+0.060	....
5167.678	*a*	8	+0.003	....
5195.647	*e*	5	−0.007	....
5227.362	*a*	10	+0.004	+0.031
5233.122	*d*	10	+0.022	+0.11
5266.738	*d*	10	+0.021	+0.13
5269.723	*a*	10	+0.004	+0.027
5340.121	*d*	8	+0.022	+0.14
5341.213	*a*	8	+0.004	+0.028
5365.069	*e*	6	−0.021	....
5367.669	*e*	6	−0.020	....
5370.166	*e*	6	−0.008	....
5371.734	*a*	6	+0.004	+0.029
5373.905	*e*	4	−0.013	....
5383.578	*e*	5	−0.013	....
5393.375	*d*	5	+0.020	+0.14
5397.344	*a*	6	+0.005	+0.029
5404.357	*e*	5	−0.012	....
5405.989	*a*	6	+0.004	+0.027
5411.124	*e*	6	−0.020	....
5415.416	*e*	6	−0.020	....
5424.290	*e*	6	−0.025	....
5429.911	*a*	6	+0.003	+0.029
5445.259	*e*	6	−0.018	....
5447.130	*a*	6	+0.004	+0.031
5463.174	*e*	6	−0.009	....
5463.494	*e*	6	−0.007	....
5466.609	..	5	+0.050	....
5476.500	*a*	6	+0.004	+0.029
5476.778	*d*	6	+0.020	+0.11
5487.959	..	6	+0.078	....
5497.735	*a*	6	+0.004	+0.030
5501.683	*a*	6	+0.003	+0.030
5507.000	*a*	6	+0.004	+0.031
5543.414	*e*	6	−0.012	....
5555.122	*e*	6	−0.018	....

Wave-Length	Group	No. of Measures	Δ per Atm.	Δ 8 Atm.
5558.209	*d*	5	+0.026	....
5562.933	*e*	4	−0.004	....
5563.824	*d*	5	+0.022	....
5565.931	*e*	4	−0.009	....
5586.991	*d*	6	+0.024	+0.12
5594.884	*e*	4	−0.018	....
5598.524	*e*	4	−0.018	....
5603.186	*d*	6	+0.022	+0.15
5615.877	*d*	6	+0.024	+0.13
5624.769	*d*	5	+0.024	+0.16
5659.052	*d*	6	+0.022	+0.15

to 1 or 2 atmospheres. They were taken in the second order of a 21-foot concave grating at the Ryerson Laboratory with the pressure-box which was used by us in our previous work at the Pasadena Laboratory of the Mount Wilson Observatory. The photographs were measured for the most part by Miss Lasby of the Mount Wilson computing staff.

The measures at present include about 180 lines in the region between λ 4800 and λ 5700, and will be extended both toward the red and toward the violet. A discussion of the results naturally will await the completed measures but some of the more interesting lines are given in the adjoining table. Among these is a complete list of lines in this region which show displacements toward the violet under pressure, as well as a few lines which show enormous displacements toward the red amounting to over twice that of the average for the group *d* lines. The shifts have been reduced to Ångströms per atmosphere. The last column contains the displacements found by us previously at 8 atmospheres.

These results amply confirm the violet displacements found by St. John and Ware although the numerical values are considerably smaller. The lines of group *e* are characterized by great diffuseness and widening at a pressure as low as even one atmosphere. They are, however, more nearly symmetrical than are the lines of group *d* which are always very diffuse on the red side.

A comparison of these results for a pressure of one atmosphere with our previous results for 8 atmospheres indicates that the present values are somewhat larger. In the case of the lines of group *d*, the agreement, however, is quite as good as could be

expected for lines of this character, and perhaps the same may be said of the lines in group *c*. It seems certain that the displacements of lines which belong to groups *c*, *d*, and *e* may be measured much more accurately at low pressure than at pressures of several atmospheres.

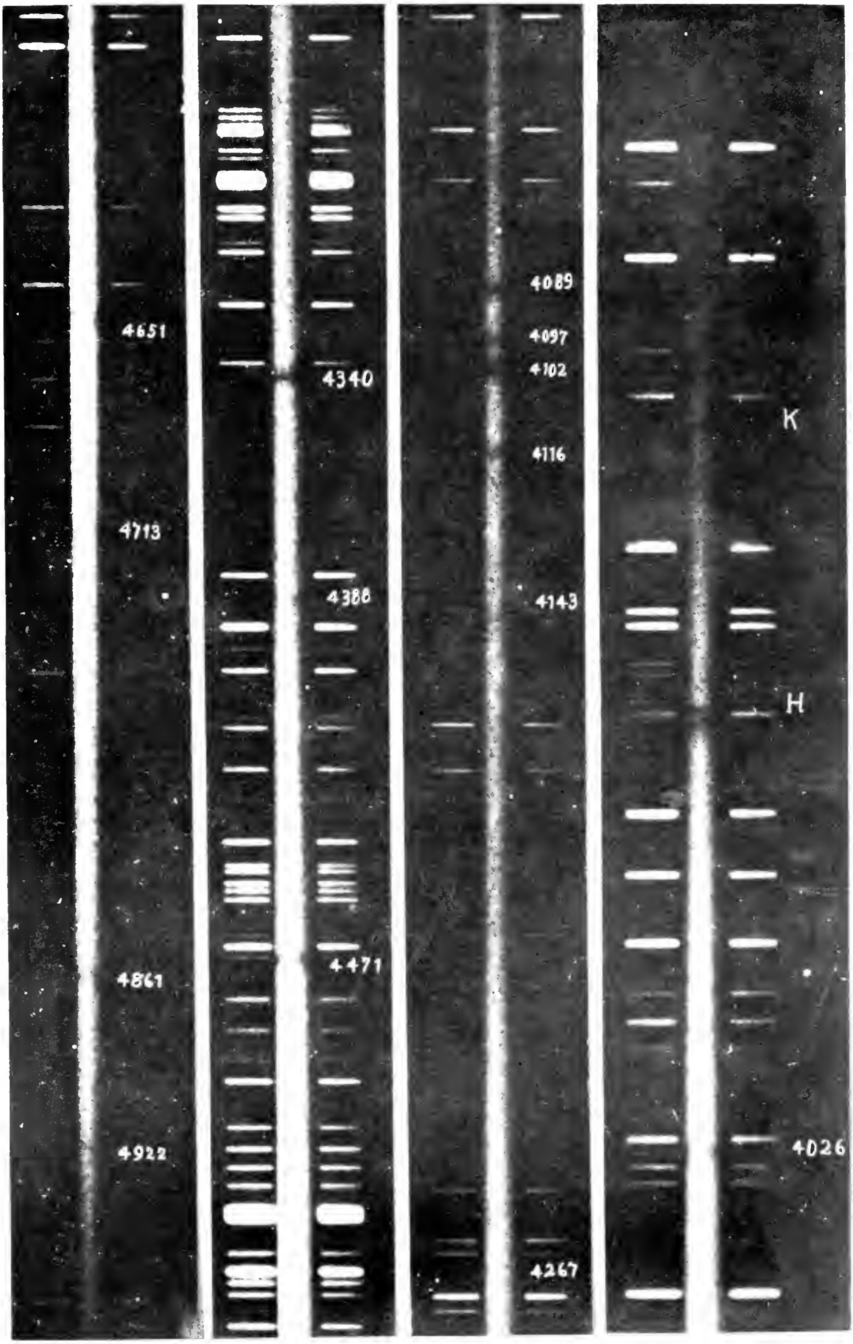
INDEX TO VOLUME XXXVII

SUBJECTS

INDEX TO VOLUME XXXVII

AUTHORS

PLATE I



SPECTROGRAM OF *o Camelopardalis*
With Comparison Spectrum of Titanium and Iron
Enlargement is 15

PLATE II

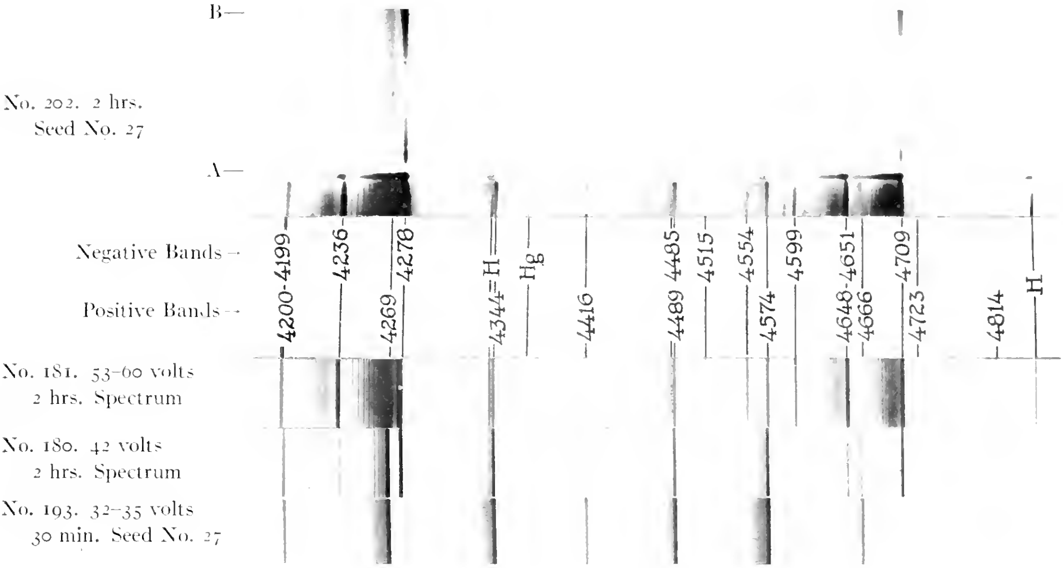


FIG. 2.—Air

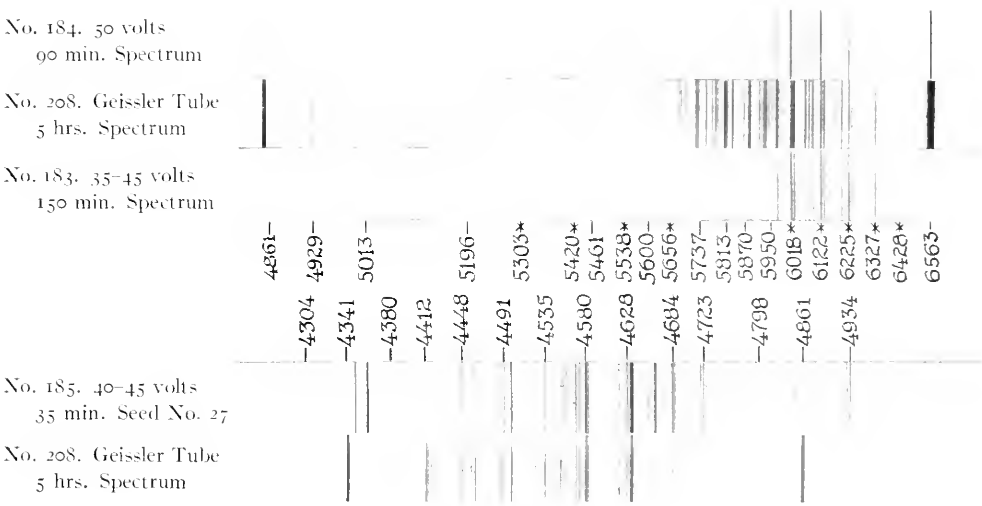


FIG. 3.—*H*

PLATE IV

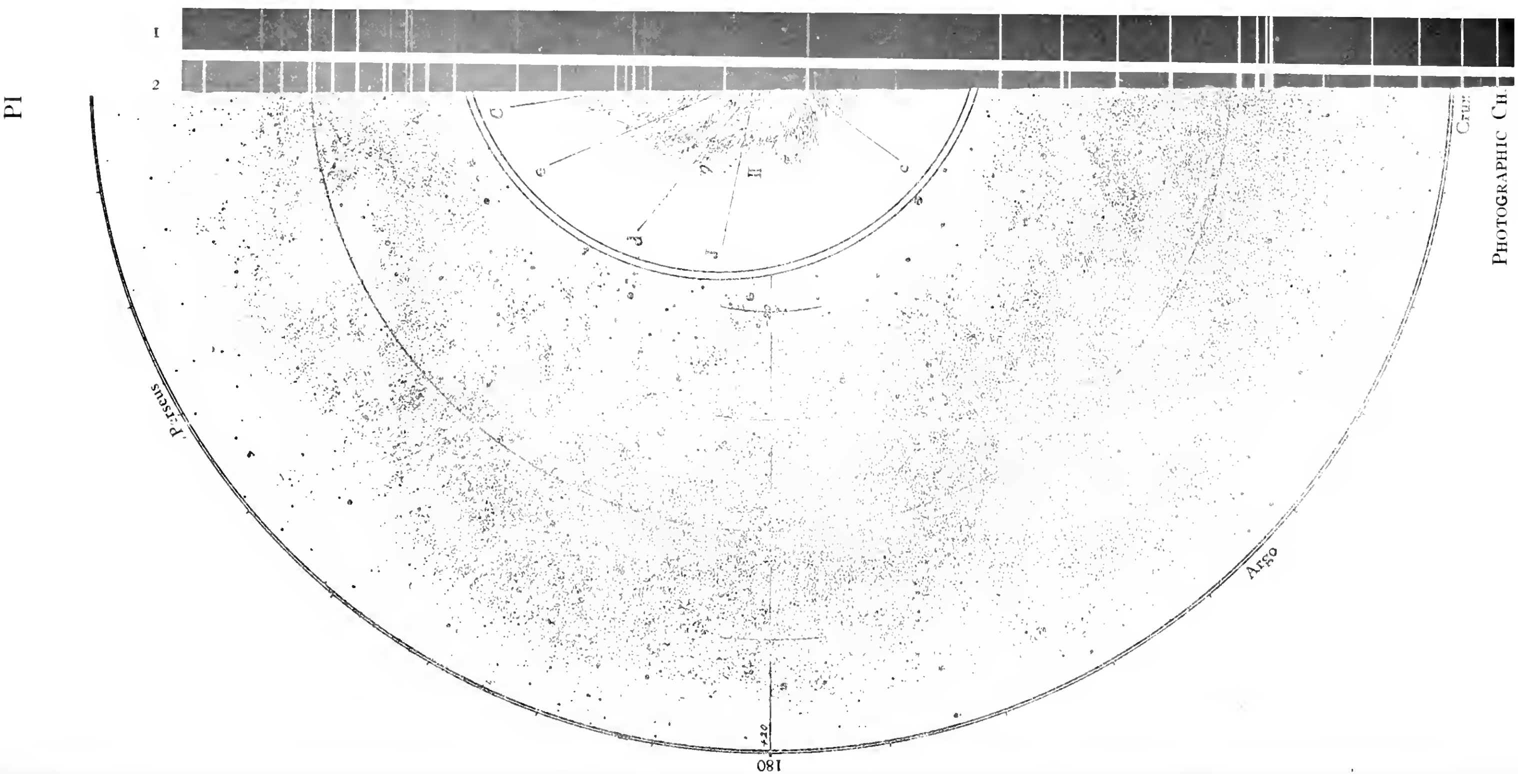
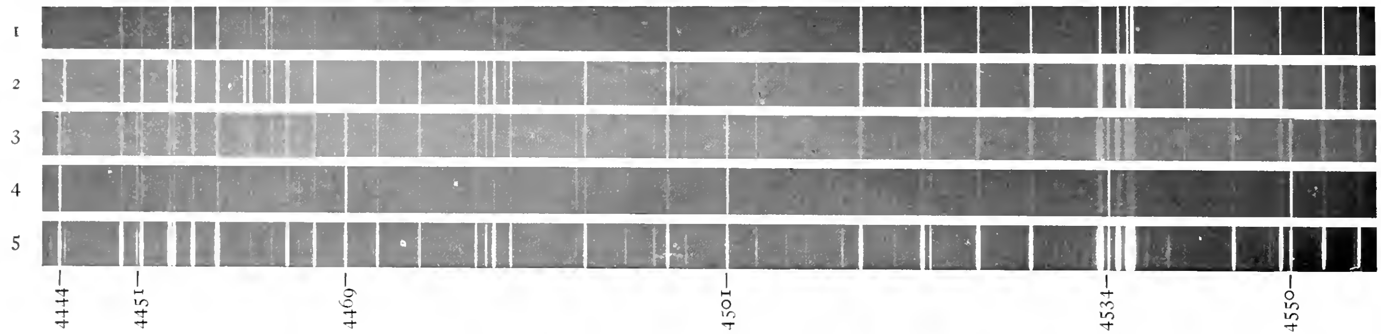


PLATE III

Photographic Chart of the Milky Way

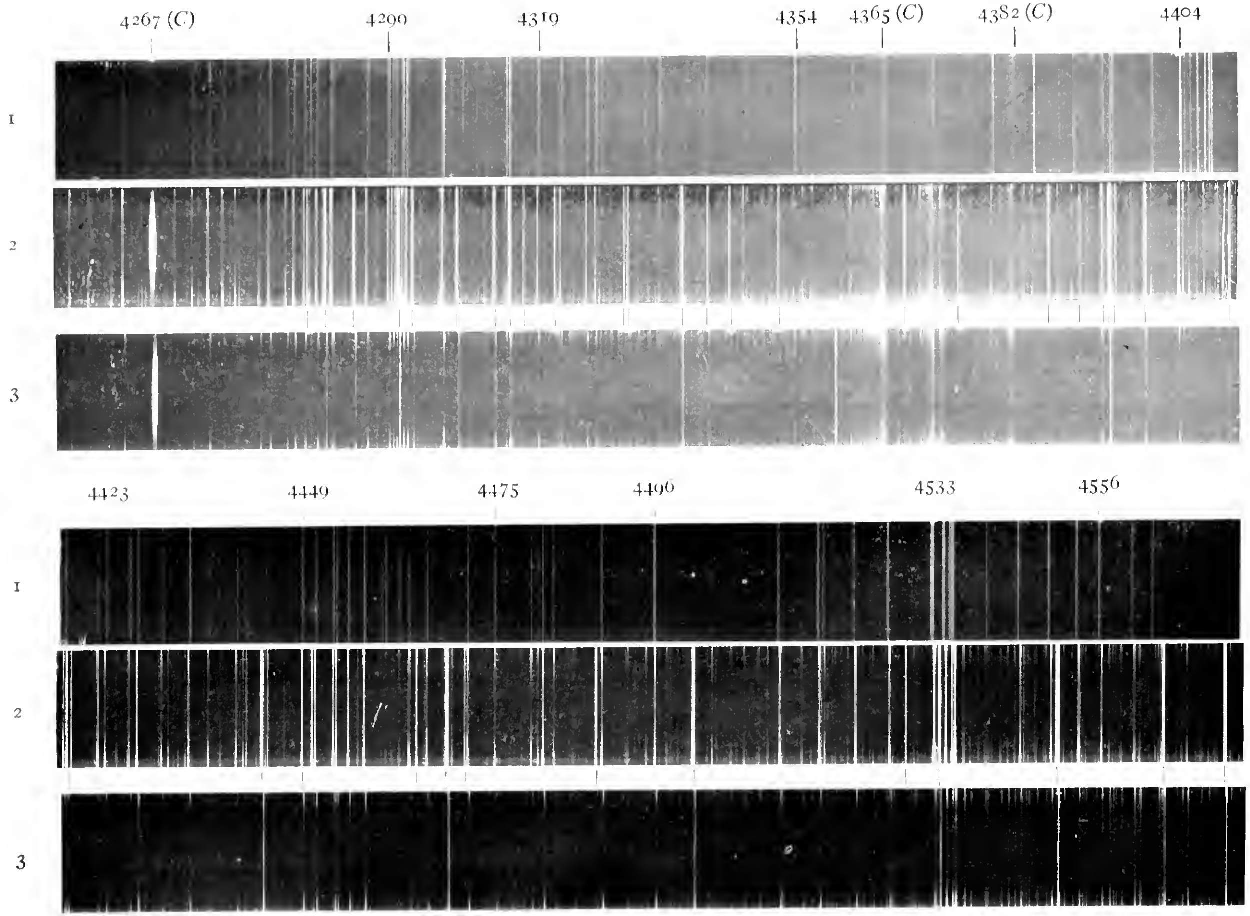
PLATE IV



TITANIUM SPECTRA GIVEN BY ELECTRIC FURNACE AND ARC, SHOWING DEVELOPMENT OF ENHANCED LINES WHEN TUBE BURNS THROUGH

1. Regular furnace spectrum for about 2400° C
2. Regular furnace spectrum for about 2600° C
3. Spectrum when slit was covered just after tube burned through
4. Spectrum given chiefly by conditions following break of tube
5. Spectrum of carbon arc containing titanium, taken on same plate as 4. The stronger enhanced lines are marked

PLATE V



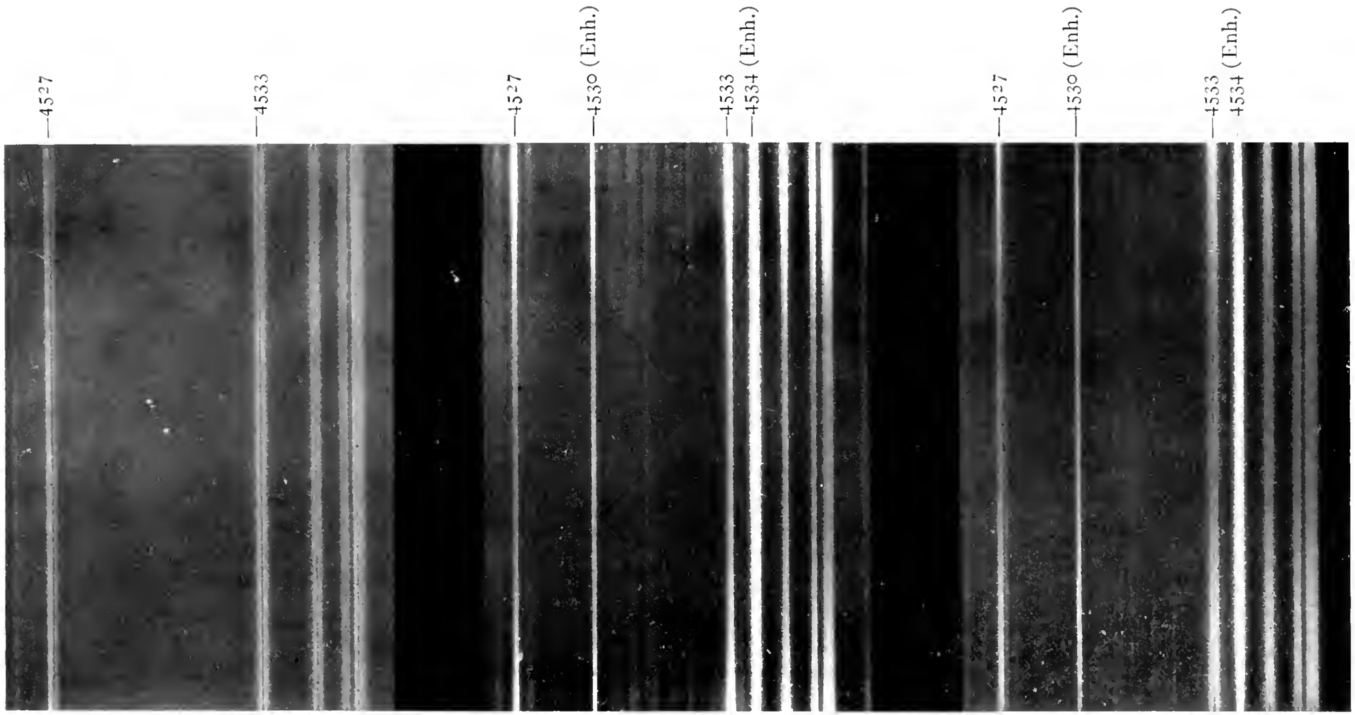
SPECTRA OF THE ELECTRIC FURNACE PHOTOGRAPHED WITH LONG SLIT, SHOWING HIGH RELATIVE INTENSITY OF ENHANCED LINES IN CENTER OF TUBE WHEN TUBE BURNS THROUGH

Scale: 1 mm = 1 Å

1. Regular furnace spectrum with unbroken tube

2. and 3. Spectrum given after tube burns through. Titanium enhanced lines are marked on margin of 2

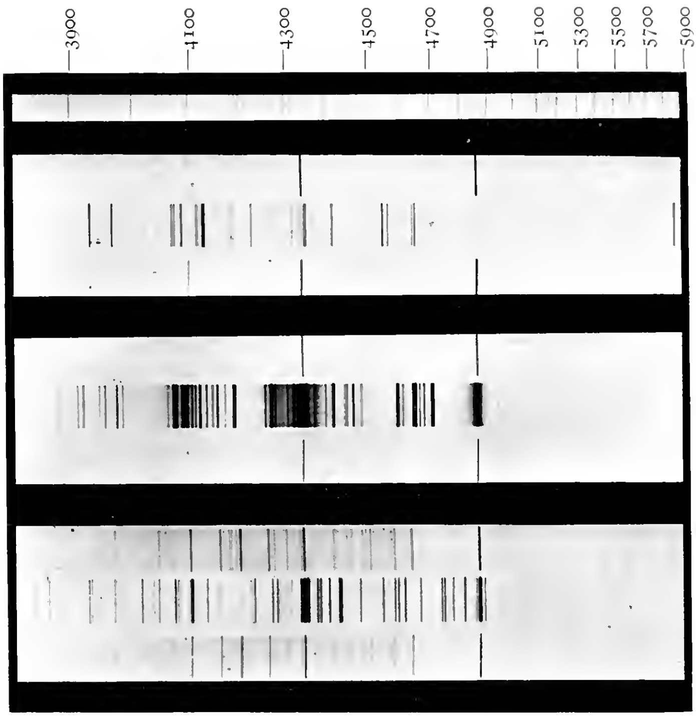
PLATE VI



SPECTRA TAKEN WITH LONG SLIT BEFORE AND AFTER THE BREAK OF TUBE IN ELECTRIC FURNACE, SHOWING DISSYMMETRY OF LINES AFTER BREAK, TOGETHER WITH APPEARANCE OF ENHANCED LINES

1. Regular furnace spectrum with unbroken tube
2. Spectrum when plate was exposed at moment of break
3. Spectrum combining effects of 1 and 2, exposure covering period before and after break of tube

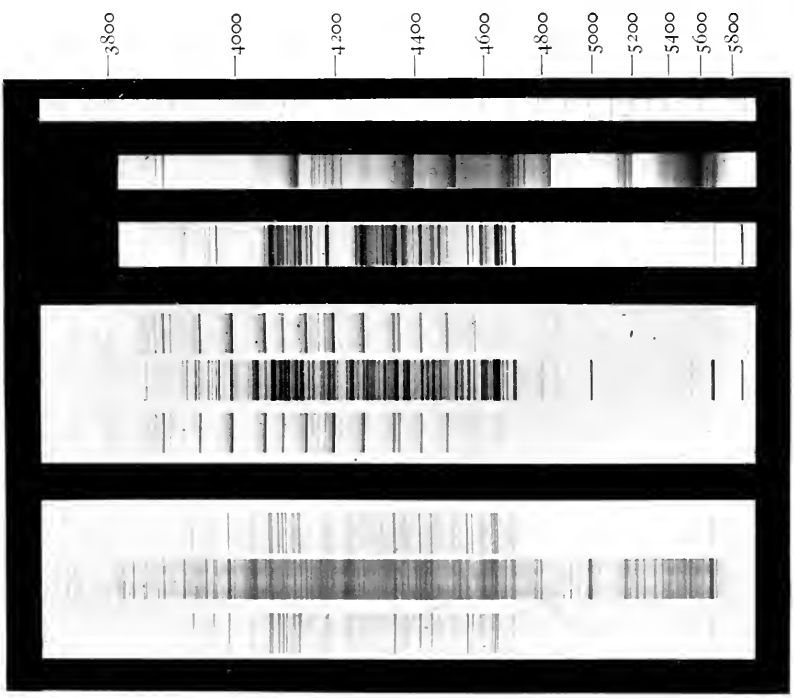
PLATE VII



3

8

9



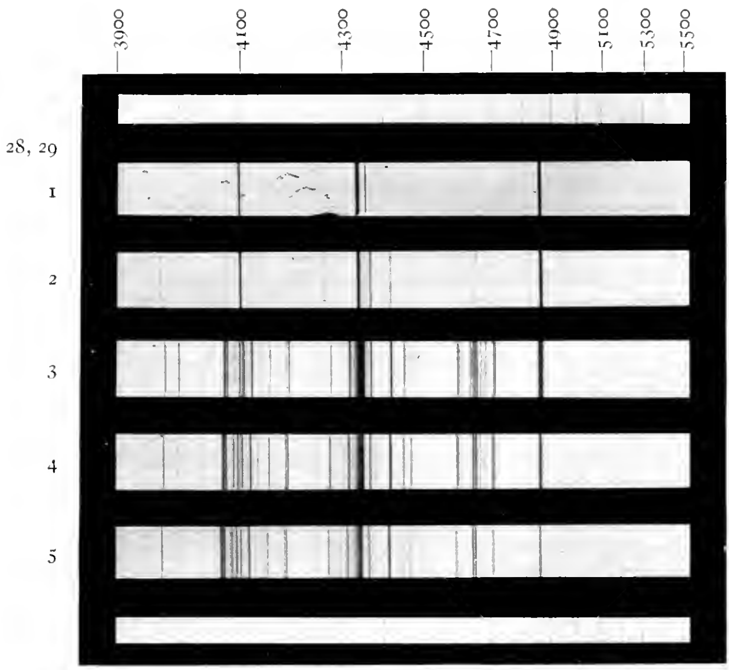
6a

6b

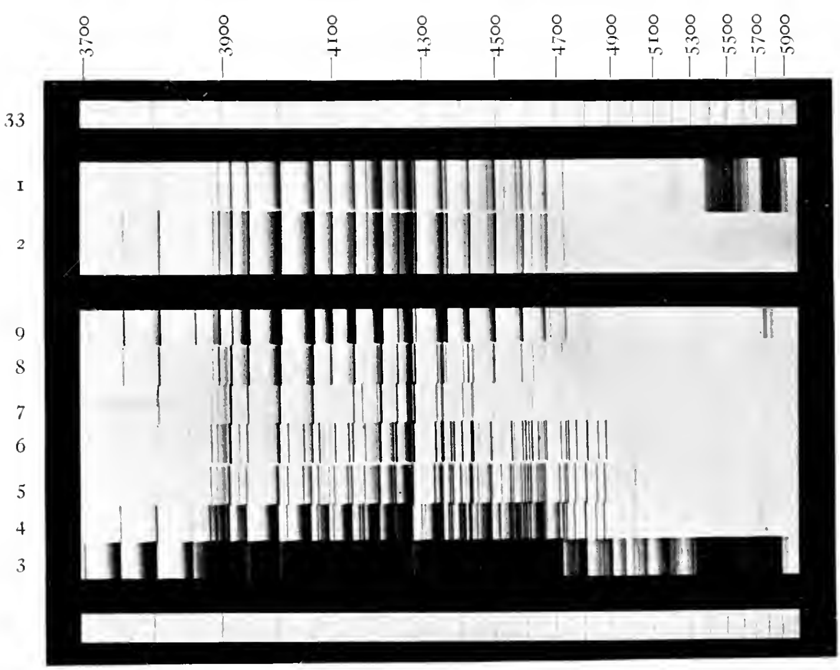
13a

13b

PLATE VIII



3900
4100
4300
4500
4700
4900
5100
5300
5500
28, 29
1
2
3
4
5



3700
3900
4100
4300
4500
4700
4900
5100
5300
5500
5700
5900
33
1
2
9
8
7
6
5
4
3

PLATE IX

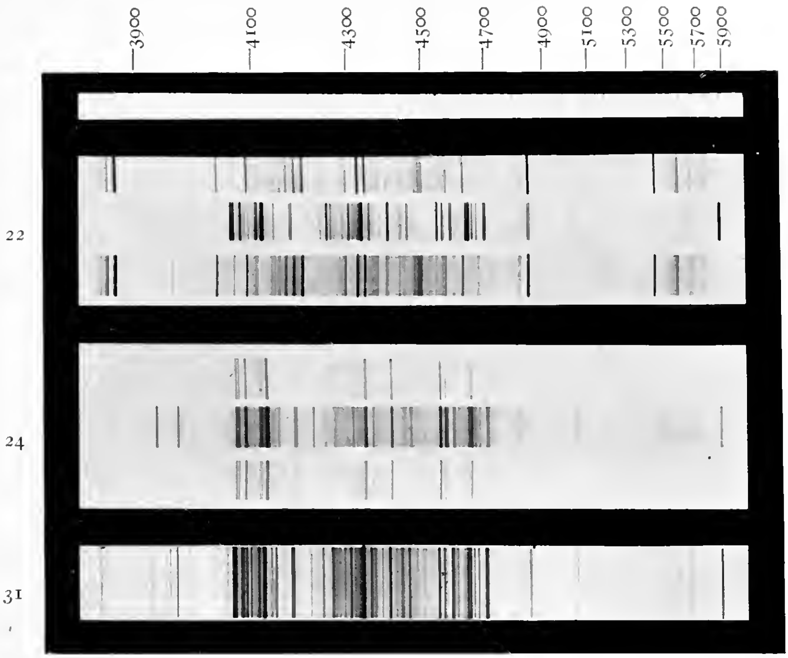


PLATE VII

Three hydrogen tubes: (3) quite pure, (8) containing carbon monoxide and other impurities, and (9) a small amount of argon. The middle spectra show the effect of the field. Notice the enhancement of the hydrogen lines in (8) and (9), in which large-sized capillaries were used; also the manner in which impurities are brought out by the field.

(*6a*, *6b*) Carbon monoxide, field off and on respectively. (13*a*) nitrogen with carbon monoxide as impurity, field off, outside spectrum; on, middle spectrum. In the latter the lines are due to nitrogen and oxygen. (13*b*) shows the coincidence of these lines with the air lines of the iron spark spectrum.

PLATE VIII

(28, 29) Effect of increasing the field in the case of hydrogen. (1) field off, (2) to (5) progressive increase of field. The hydrogen lines at first broaden, then fade out; oxygen and sodium gradually appear. In the third exposure the hydrogen lines have reached the stage shown in a former tube (Plate VII, 8) with larger capillary in a stronger field.

(33) Nitrogen. (1) No field; (2) maximum field. In (2) one of the band systems has disappeared and a few of the stronger lines of the nitrogen line spectrum have come up—the effect noted by Chautard. (3) to (6) inclusive, progressive increase of a weak field through a point where the blue argon spectrum partially replaces the nitrogen band spectrum. Argon will not generally appear in the presence of nitrogen. (7), (8), (9) all with field off—three successive three-minute exposures with a few seconds intervening. Although the intensity of the argon lines immediately falls off, the nitrogen band spectrum reappears but gradually—a unique phenomenon.

PLATE IX

(22) Effect of the field on a tube containing both carbon monoxide and hydrogen. Oxygen and sodium lines are equally strong in the middle spectrum (field on).

(24) Effect of an unusually small capillary in bringing up sodium lines; field on in both cases—central exposure smaller capillary. The same effect is produced in (31) on the larger capillary by an exceptionally strong field.

PLATE X

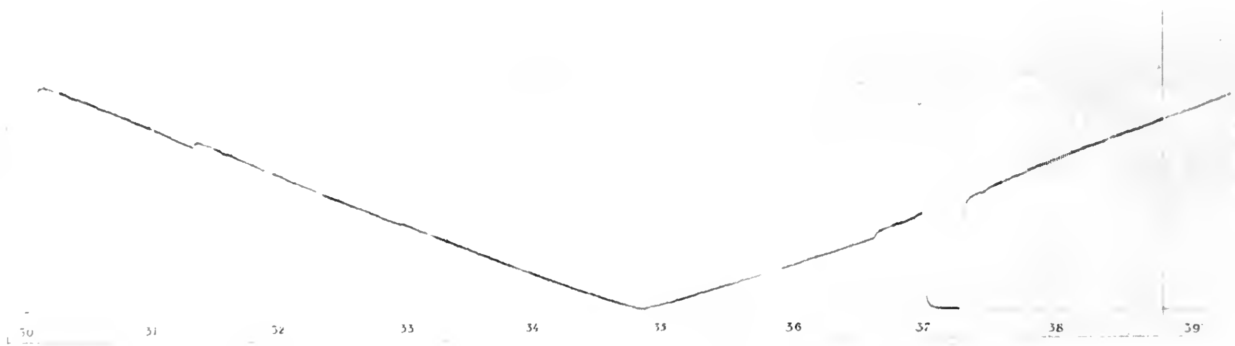
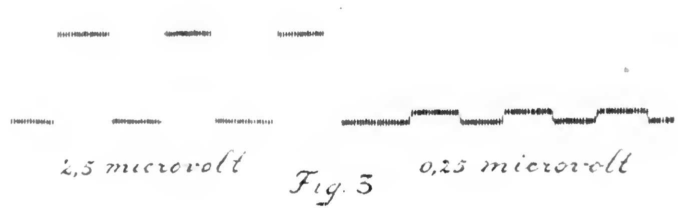


FIG. 1



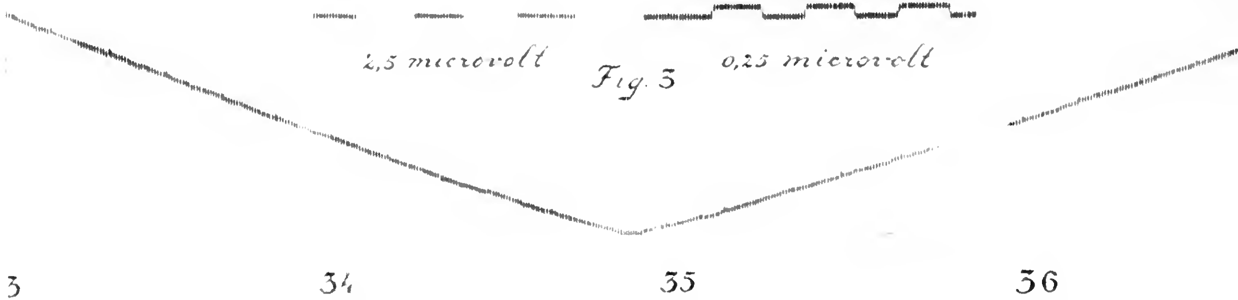


FIG. 2

FIG. 1.—Photographic record of the total solar radiation during $9\frac{1}{2}$ minutes, comprising the annular phase of the eclipse of the sun on April 17, 1912, as obtained near Maastricht with a bolometric arrangement ($\frac{1}{10}$ original size).

FIG. 2.—Middle part of the curve of Fig. 1 ($\frac{5}{7}$ original size).

FIG. 3.—Check of galvanometer condition on eclipse day.

PLATE XI

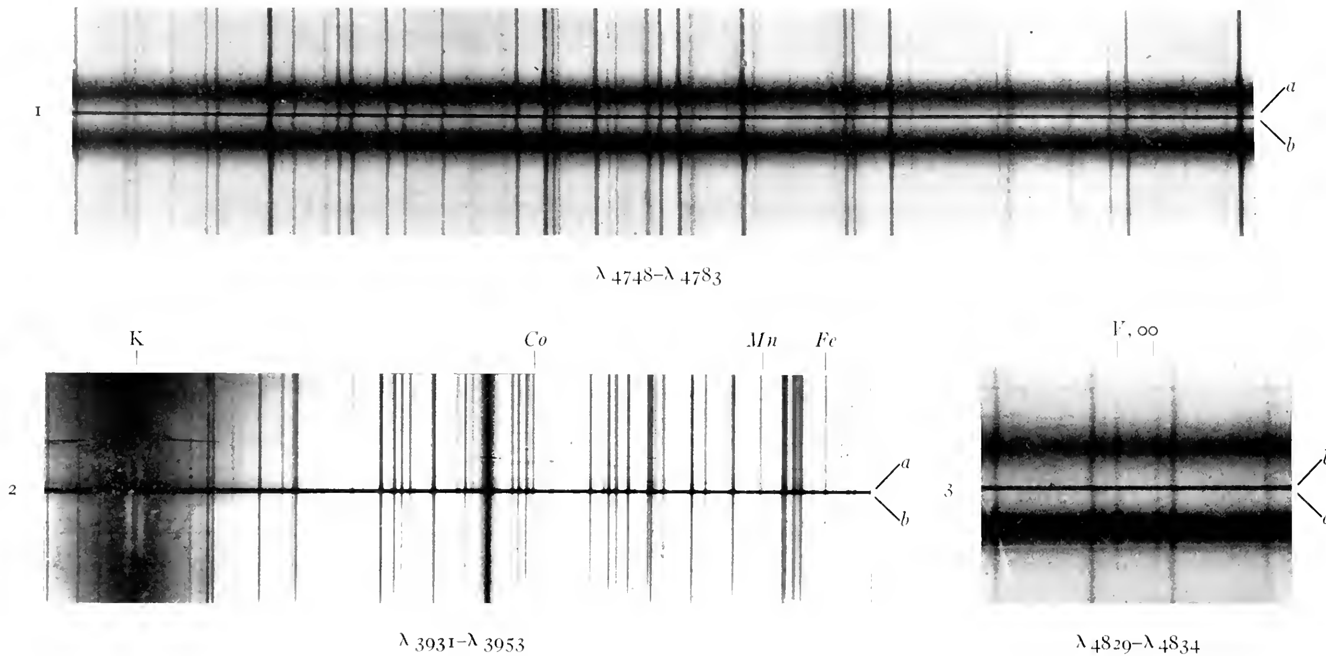
Fig. 1

Curve Showing the Variation of the Total Solar Radiation during the Annular Eclipse of the Sun on April 17, 1912, from Measurements Made with a Thermopile

Fig. 2

Middle Part of the Radiation-Curve Above, on a Ten Times Larger Scale

PLATE XII

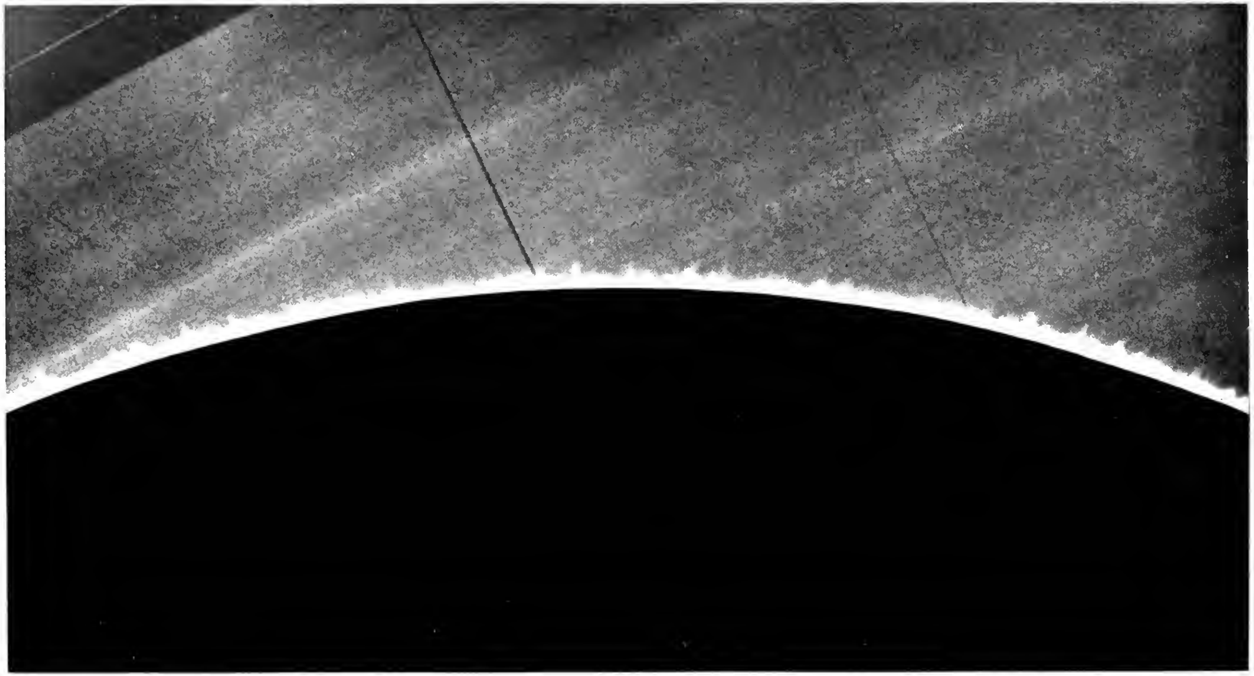


SPECTRUM OF THE EDGE OF THE PENUMBRA

a, Edge directed toward the limb. *b*, Edge directed toward the center of the disk

1. Relative displacements of strong and weak lines, indicating different velocities of outflow
2. Displacement of the K line of calcium, showing inflow
3. Large displacement and great strengthening of *V* lines of intensity oo

PLATE XIII



Spectroheliogram of Chromosphere, May 23, 1912

Scale: 1 mm = 3060 km